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THE METHODS OF PETROGRAPHIC-MICROSCOPIC RESEARCH

THEIR RELATIVE ACCURACY AND RANGE OF APPLICATION

BY

FRED. EUGENE WRIGHT

WASHINGTON, D. C.

PUBLISHED BY THE CARNEGIE INSTITUTION OF WASHINGTON

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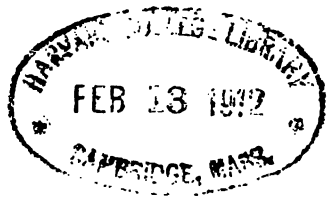

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The Methods of Petrographic-Microscopic Research

Their Relative Accuracy and Range of Application

BY

FRED. EUGENE WRIGHT

CONTENTS.

	Page.
Introduction	7
The petrographic microscope	10
The Gauss equations for paraxial rays	14
Refraction at a single spherical surface	14
Refraction through a lens	15
Principal planes	18
Nodal planes	18
The focal length of a lens system of two components	19
Spherical aberrations	21
Monochromatic light	21
Spherical aberration proper (for the principal axis)	21
Refraction at a single spherical surface	21
Aplanatic points of a single refracting spherical surface	23
Spherical aberration in simple lenses	24
Spherical aberration for object points near the axis (sine condition)	26
Astigmatism	29
Curvature of the field (Petzval condition)	30
Distortion (tangent condition)	30
Chromatic aberrations	31
Diaframs in the microscope	34
Magnification	40
The field of the microscope	42
The physical significance of the numerical aperture	42
Useful and empty magnification in the microscope	47
The optical elements of the microscope lens system	49
The objective	49
The ocular	52
The condenser	52
The Nicol prisms	56
The petrographic microscope as an accurate measuring device	57
The adjustment of the petrographic microscope	61
The stereographic, orthographic, gnomonic and angle projections	63
Biot's or Fresnel's rule	68
Chapter I	69
Color	69
Pleochroism and absorption	71
Cleavage and crystal habit	71
Character of the principal zone	71
The combination wedge	71
Optical character of birefracting minerals	74
Plate perpendicular to the acute bisectrix	78
Plate perpendicular to an optic axis	78
Plate parallel to the plane of the optic axes	79
Dispersion of the optic axes	82
Dispersion of the bisectrices	82
Chapter II	83
Refractive indices	83
The immersion method	92
The Becke line method	95
The direct measurement of refractive indices	98
Chapter III	101
Birefringence	101
The Newton color scale	108
Summary	114

CHAPTER IV 115
Extinction angles 115
Mathematical formulæ for the intensity of light 115
Methods 132
Parallel polarized light 132
Sensitive-tint plate 133
Bravais-Stöber plate 134
The Koenigsberger plate 134
The double combination wedge 134
Calderon calcite wedge 135
Quarter-undulation plate of H. Traube 135
Twinned selenite plate 136
Artificially twinned quartz plate 136
Bi-nicol ocular 137
Bertrand plate 137
The quartz half-shade plate of S. Nakamura 139
Bi-quartz wedge-plate 139
Methods involving rotation of the upper nicol 141
Convergent polarized light 141
The relative sensitiveness of the different methods 142
Experimental tests 145
CHAPTER V 147
Optic axial angle 147
Convergent polarized light 147
The Mallard constant 148
The Becke drawing-table method 152
Methods with the double-screw micrometer ocular; also with the coordinate micrometer ocular 155
Observations in convergent polarized light 162
Muscovite 163
Anhydrite 164
The condenser apertometer 172
Summary statement of course of procedure in measuring the optic axial angle of a mineral plate in convergent polarized light 172
Michel-Lévy Method 173
Methods with the universal stage 173
Parallel polarized light 175
The determination of the crystal system of a given mineral by means of the universal stage 177
The accurate determination of the position of an optic axis in the field of vision 178
Section nearly perpendicular to the optic normal β 186
Section nearly normal to the obtuse bisectrix 187
Probability of encountering sections suitable for optic axial angle measurements 189
Methods based on the relative birefringence of known sections 190
Extinction angles of plates in zones whose axes lie in the plane of the optic binormals 191
Measurement of the optic axial angle on the total refractometer 193
The relative accuracy of the different methods 194
Measurements with the axial-angle apparatus 194
Measurements with the Becke drawing-table 194
Measurements with the double-screw micrometer ocular 196
Measurements with the universal stage 197
Recapitulation 199

INTRODUCTION.

The most efficient and useful tool in petrographic research is the petrographic microscope, and since its introduction by Sorby, half a century ago, the science of petrology has been largely dependent on it for further development. During these years the microscope itself, together with microscopical methods, has undergone frequent change and improvement to meet adequately the requirements made by the rapidly advancing science. As a result, the modern petrographic microscope is admirably suited to the purpose for which it is intended, namely, the examination of thin sections of rocks. It is satisfactory and convenient, and for most observers serves merely as a means to an end; and only when new conditions arise is the need for special microscopical devices felt.

During the past six years the work with artificial silicate preparations in the Geophysical Laboratory has imposed new and difficult problems to be solved by the microscope. Not only are such preparations very fine-grained, but the degree of accuracy of each measurement must be definitely known if it is to be applied without reserve to geophysical problems. To meet these new conditions it has been necessary to devise new methods, involving extensive alterations in the microscope, and also to test the different methods available for the determination of the optical constants of minerals in the thin section and to ascertain their relative accuracy and general applicability. As a result of these tests, the methods best adapted for work with artificial and all fine-grained preparations are now fairly well known and their application is in large measure a matter of routine.

Minerals are determined under the microscope by means of their crystallographic and optic properties; the more accurately such properties or constants can be ascertained for any given mineral, the more reliable and satisfactory is the determination. The optical properties thus made use of in the practical determination of minerals under the microscope are, briefly, *refractive index, birefringence, optic axial angle, optical character, extinction angle, color, and pleochroism.* By means of these properties alone it is not only possible to ascertain the crystal system to which a given mineral belongs, but also by a short process of elimination to determine definitely the mineral in question. This process has been carried to such refinement in certain instances, as in the isomorphous series of the plagioclase feldspars, that by optical properties alone the actual chemical composition of the particular plagioclase under observation can be obtained with considerable accuracy.

The optical and crystallographic characters which thus serve in the microscopical examination of minerals can be grouped into two classes, those of the first class (*crystal habit, color, pleochroism* and *absorption, optical character of elongation, optical character of the mineral,* and *dispersion of the optic axes and bisectrices*) being ascertained usually by direct observation

without measurement, while for the second class (*cleavage angles, extinction angles, optic axial angle, refractive indices,* and *birefringence*) numerical values obtained by actual measurement are required. The properties included in the first group are used as a general rule only in an essentially qualitative way in microscopic work and can usually be recognized at a glance. For these the ordinary petrographic microscope suffices. But in the second group, the quantitative element predominates, and, as such, demands accurate, precise measurements. In ordinary petrographic work, however, these properties are only very roughly measured and are then expressed in general terms, as *optic axial angle, large; birefringence, strong,* etc., actual numerical data being rarely given. But with the increased knowledge of rocks and minerals, the demand for data which are precise and quantitative in character, rather than qualitative, has become more imperative, with the result that at present a thorough petrographic investigation should contain accurately determined optical constants of each of the rock-forming minerals examined and, in critical instances, the probable error of each determination.

This passage from qualitative to quantitative work implies consequences of profound importance; an additional burden is placed on the working geologist and the time and energy required for the investigation of a particular problem are much greater under the present régime than formerly; but at the same time this transition indicates that in one phase of geology, at least, the step from reconnaissance work to the higher level of precise and detailed work is being taken.

During the past decade the results of comparative and critical studies on the relative merits and accuracy of a number of these methods have been published at different times by the writer, but there is still lacking a connected presentation of the entire subject, so far as it has been carried, in which the different methods are coordinated and the significance and usefulness of each particular method are made to appear in their proper relations to other available methods.

In microscopic work with fine-grained preparations, where the average diameters of component grains are expressed in hundredths and often thousandths of millimeters, the different methods are frequently taxed to the limit, and all available resources must be called into play to obtain quantitative results of even a fair order of accuracy. In such investigations however, the quantitative element is essential and methods must be applied which are capable of furnishing quantitative data of a known degree of accuracy. It may be stated, as a result of experience, that on clear, individual grains measuring 0.01 to 0.03 mm. in diameter, all the optic properties ordinarily employed in the petrographic-microscopic investigation of minerals in the thin section can be determined with a satisfactory degree of accuracy.

In the following pages the different methods best adapted for the microscopic examination of fine-grained and artificial preparations will be considered with special reference to their degree of accuracy and range of general application.

Experience has shown that in the determination of any one of the optical constants, a particular method or device may be satisfactory under certain conditions, but less so and even worthless under other conditions. For this

reason it is necessary, in judging of a method or of a piece of apparatus, to note its general applicability and its accuracy under the different conditions which may arise. A device which is so constructed that its sensibility is variable and may be changed to meet the different possible conditions is obviously better than one which gives satisfactory results for only a fixed set of conditions. The question of adjustable sensibility is especially important in methods based on differences in light intensity or on the interference phenomena of white light; and several of the devices described below (the double combination wedge, the bi-quartz wedge plate, the artificially twinned quartz wedge) have been constructed with this special end in view.

In the preparation of the descriptions, constant use has been made of the standard works on microscopical petrography, especially of the Mikroskopische Physiographie d. Mineralien, ed. 4, by Rosenbusch-Wülfing; of the Traité de Technique Minéralogique et Pétrographique, 1, by Duparc-Pearce; and of Rock Minerals by J. P. Iddings. For the sake of brevity, it has been found expedient, in not a few of the methods noted below, to indicate merely their salient features and to refer to the standard works or the original articles for their more complete description and elucidation.

My thanks are due to Mr. E. S. Larsen, formerly of the Geophysical Laboratory, and to Dr. H. E. Merwin and Dr. C. N. Fenner, of the Geophysical Laboratory, who have aided me from time to time in testing the different methods described below. I am also indebted to Dr. P. G. Nutting for advice in several optical matters; and especially am I under obligations to Dr. H. Kellner, of the Bausch and Lomb Optical Company, Rochester, N. Y., who has read the introduction and has made many valuable suggestions in regard to the presentation of the optics of the lens system of the microscope.

THE PETROGRAPHIC MICROSCOPE.

Although not strictly germane to the subject of optical methods, it has been found convenient to outline in this introductory section the essential features of the petrographic microscope and in particular to emphasize the principles underlying its construction. In the following descriptions constant reference will be made to the details of the optical system as sketched in this section, which, though necessarily incomplete and disconnected, serves to facilitate the presentation and to direct attention to certain principles which are essential to accurate work.

In the preparation of this section the following books and papers have been found especially useful:

Abbe, E. Die Lehre von der Bildentstehung im Mikroskop. Edited by O. Lummer and F. Reiche, 1910.
Beck, Conrad. The theory of the microscope. Cantor Lectures, Roy. Soc. of Arts, 1908.
Beck, Conrad. The consideration of the equivalent planes of optical instruments. Proc. of Optical Convention, I, 9–18, 1905.
Chwolson, O. D. Lehrbuch d. Physik, II, 1904.
Czapski, S. Theorie d. optischen Instrumente, Aufl. 2, 1904.
Dick, Allan. Notes on the polarizing microscope, London, 1894. Also Min. Mag., **8**, 160–163, 1888.
Drude, P. Lehrbuch der Optik, 2d edition, 1906.
Ferraris, G. Die Fundamental-eigenschaften der dioptrischen Instrumente. German translation by F. Lippich, 1879.
Hastings, C. S. Light. New Haven, 1901.
Heath, R. S. A treatise on geometrical optics, 2d edition, 1895.
Hockin, J. Journ. Roy. Micros. Soc. (2), **4**, 337, 1884.
Leiss, C. Die Opt. Instrumente, etc., 1899.
Lummer, O. Müller-Pouillet, Lehrbuch d. Physik, 10th edition, II, 2, 1907.
Pelletan, A. Optique Appliquée, Paris, 1910.
Rosenbusch-Wülfing. Mikroskopische Physiographie, I, 1, 1904.
Schuster, A. The theory of optics, 2d edition, 1909.
Southall, J. P. C. Principles and methods of geometrical optics, 1910.
Spitta, E. J. Microscopy, 1907.
Wright, A. Principles of microscopy, 1906.

The petrographic microscope is designed to serve two purposes.

(1) That of an ordinary microscope to magnify and to render visible details which would otherwise escape attention.

(2) That of an instrument for measuring certain optical properties of minerals in the thin section.

The first requirement is one common to all microscopes, and on it an extraordinary amount of care and thought have been spent in recent years, with the result that the lens systems of modern microscopes are highly corrected and satisfactory in practically every respect. The calculation of such lens systems is an exceedingly intricate task and one which does not primarily concern the working petrologist, who uses the microscope solely as a means to an end. Having been furnished with the optical system, it is essential, however, that he understand, in a general way at least, the functions of the different parts of the microscope if he desires to obtain the best possible results from his instrument. In the examination of thin sections of rocks and even of artificial silicate preparations, the details which must be resolved are not, as a rule, of such a nature as to require magnifications above 250 diameters. This is in part due to the fact that a practical limit is set to the thinness (0.01 to 0.02 mm.) to which rock sections can be

ground, and this limit precludes the isolation and determination of the optical properties of a single grain, measuring much less than 0.01 mm. in diameter, especially if it be birefracting. In aggregates of minute and overlapping mineral grains or fibers, it is extremely difficult to single out one of the individuals and to determine its optical constants. The optical behavior of aggregates of minute superimposed mineral plates is often deceptive and may lead the observer to erroneous conclusions in regard to the optical properties of the substance under examination. In practical determinative work it is important, therefore, to make all determinations on separate grains which are clearly distinguishable and which alone are responsible for the optical phenomena observed.

In view of the relatively low magnifications used in petrographic microscopic work, the strict attention to details of illumination and manipulation required in work with high powers is less essential to success in petrology than in biology or bacteriology, and for this reason the petrologist is often inclined to disregard them altogether and fails in a measure to utilize the possibilities of his instrument. In case, however, the microscope is to be used for determinative work with fine-grained preparations, it is essential that these details be observed in a general way at least, and that the working parts of the microscope fit accurately and be in adjustment—the accuracy of the results being dependent in large measure on the accuracy of the construction and the adjustment of the instrument. For this reason it has seemed advisable to sketch in brief outline the general principles underlying the construction both of the optical and of the mechanical system of the microscope.

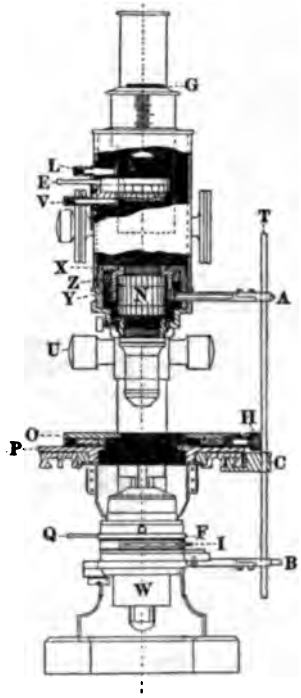


FIG. 1.—Section through microscope of Plate 1, Fig. 3, showing working parts. The letters *T, A, B, C, H, W, Q, F, O, U, V, E, L,* and *G* refer to the same parts as in Plate 1, Fig. 3. *I* = lower iris diafram; *N*, upper nicol; *X, Y, Z*, brass parts effecting rotation of upper nicol; *P*, rotating stage supporting movable plate *O*.

The optical system of the ordinary microscope consists of four essential parts:

(*a*) The reflecting mirror or reflector (Plate 1, Fig. 4).

(*b*) The substage condenser *C*.

(*c*) The objective or object-glass *O*.

(*d*) The ocular or eye-piece *E*.

The petrographic microscope comprises in addition to these:

(*e*) The lower nicol or polarizer *P* (Plate 1, Fig. 4).

(*f*) The upper nicol or analyzer *A*.

(*g*) The Bertrand lens *B*.

Each of these parts has a definite task to perform and contributes its share to the efficiency of the whole. These parts are supported by a *mechanical system*, the important parts of which are (Plate 1, Fig. 3):

(*a*) The stand or body for carrying the parts (*b*) (*c*) (*d*).

(*b*) The draw tube to which are attached the objective, the ocular, the Bertrand lens, the analyzer, and an iris diafram, and into which, at *M*, certain optical wedges and plates can be inserted.

(*c*) The stage *O*, a rotating and mechanical platform upon which the object to be examined is placed.

(*d*) The substage, which contains the condenser, an iris diafram, and occasionally other devices for illuminating the object through the central aperture of the stage.

For satisfactory work, it is essential that both the mechanical and the optical systems fulfill certain requirements which are postulated by the fundamental principle of construction that the scientific observer, in any line of research, should have definite control over the different parts of his instrument, and should be in a position to adjust and to test the adjustment of each part, if his observations are to be free from unknown factors which may seriously affect the accuracy of the final result. For this reason each scientific instrument should be designed with reference to its adjustment and to the testing of the same. Simple adjustment facilities should be included on the instrument and a definite order of procedure for the adjustment given which the observer may follow step by step with the assurance that each successive step in the adjustment will not disturb the preceding step and thus necessitate a fresh start from the beginning. Unfortunately the importance of adjustment facilities is often disregarded by makers of scientific instruments, and the observer is compelled either to devote many hours or days to proper adjustment tests or to trust blindly to the adjustment by an unknown mechanic and to assume that the instrument has continued in adjustment notwithstanding the gradual seasoning of the whole and the vicissitudes of transportation and rough handling. In accurate work such conditions are obviously intolerable. Good adjustment facilities are as essential in a scientific instrument as good construction; by their use the observer is able not only to detect and to determine accurately the various instrumental errors which may occur, but often also to decrease appreciably the effect of these on the final result, and thus greatly to increase the accuracy of his observations.

In the petrographic microscope the adjustments are relatively simple and can be made without elaborate apparatus; for this reason the importance of proper adjustment is often disregarded by the petrologist who accepts the instrument as he finds it and allows unknown factors to creep into his observations on the assumption that these are negligible in their effect and can be disregarded. This is unfortunate, as it introduces an element of uncertainty into his results and destroys their intrinsic value to just that extent.

In the *mechanical* construction of the petrographic microscope the following requirements are usually considered important:

(1) Firm, rigid stand for the support of the optical system.

(2) Optical system centered; optic axis of the system to pass through the center of rotation of the stage.

(3) Simple device for centering the objective; the centering screws to be parallel with, and not diagonal to, the cross-hairs of the ocular in order that the observer may have field coordinates as guides. To center the stage instead of the objective is wrong in principle as it displaces the one point to which the optical system is tied.

(4) Easy passage from parallel to convergent polarized light.
(5) Easy passage from low to high powers.
(6) Bertrand lens centered and adjusted to proper focus.
(7) Properly constructed coarse and fine adjustment screws for focussing the objective, the fine adjustment screws to record intervals of 0.001 mm. and to be free from lost motion.
(8) Satisfactory arrangement for raising and lowering the substage condenser.
(9) Accurately constructed mechanical stage on which lateral movements of 0.01 mm. can be measured directly.
(10) Degree circle of stage to be accurately divided and provided with vernier to read to 5′ at least.
(11) The ocular, the upper nicol carriage, the Bertrand lens support—in short, all moving parts—to fit accurately, so that on insertion they invariably return to exactly the same point.

Mechanically, the petrographic microscope is judged not only by the above criteria, but also by the ease and rapidity with which its different parts can be manipulated and by the rigidity and wearing qualities of the whole. The external finish of the instrument, although important from the standpoint of the sales agent, is secondary to the above considerations.

The optical system of the microscope consists of several different working parts, each one of which has a definite function to fulfill and is constructed with that end in view. The calculation of these component lens systems is an extremely complex affair and requires long training and experience to accomplish satisfactorily. The minute objects in the thin section are often highly magnified by the microscope and the slightest defect in its design or construction is at once felt in the inferior quality of the image produced. Fortunately for the petrologist the detailed knowledge necessary to design the optical system of a modern microscope is not essential to good work with the completed instrument and will not be considered even in outline in this paper. The functions of each part of the instrument, however, should be understood, in a general way at least, if accurate results are to be obtained on fine-grained preparations.

To facilitate the presentation of this part of the subject, which is somewhat involved, a few introductory paragraphs, recalling several important definitions and the simple lens formulas, may well be inserted at this point, especially as constant reference will be made to them in the pages to follow.

THE GAUSS EQUATIONS FOR PARAXIAL RAYS.

REFRACTION AT A SINGLE SPHERICAL SURFACE.

Light-waves travel more slowly in glass than in air, and if incident on a plane glass surface their direction of propagation is deflected according to the sine law $n \sin \alpha = n' \sin \alpha'$, where α = angle of incidence measured from the normal to the plate, α' = angle of refraction; n = refractive index of first medium, n' = refractive index of second medium. In applying this formula to the refraction at the spherical surface S (Fig. 2), of a single lens,

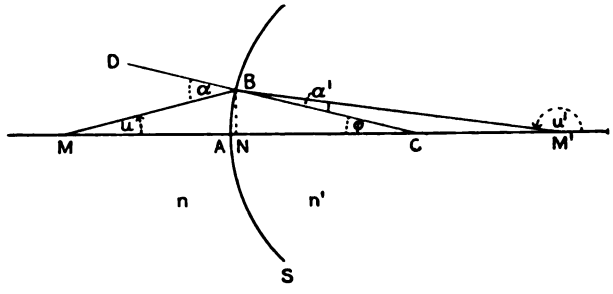


FIG. 2.

let n be the refractive index of the first medium and n' that of the second medium. Assuming that the rays travel from left to right, we find that a ray from M incident at B is deflected along BM' while the *central ray* MAM' passes through the spherical surface without deflection. From the triangles MBC and $M'BC$ it is evident that

$$\frac{\sin \alpha}{\sin \phi} = \frac{MC}{MB} \text{ and } \frac{\sin \alpha'}{\sin \phi} = \frac{M'C}{BM'}$$

Hence, by division

$$\frac{\sin \alpha}{\sin \alpha'} = \frac{n'}{n} = \frac{MC}{M'C} \cdot \frac{BM'}{BM}$$

For small-slope angles u, u' we may substitute, as a first approximation, MA for MB and $M'A$ for $M'B$ and obtain the equation

$$\frac{MC}{M'C} \cdot \frac{M'A}{MA} = \frac{n'}{n} \text{ or } \frac{x-r}{x'-r} \cdot \frac{x'}{x} = \frac{n'}{n} \qquad (1)$$

which on rearrangement becomes

$$\frac{n(x-r)}{x} = \frac{n'(x'-r)}{x'} \text{ or } \frac{n}{x} - \frac{n'}{x'} = \frac{n-n'}{r} \qquad (1a)$$

if the origin of coordinates be at A, and $MA = -x$, $M'A = x'$, $AC = r$.

The action of the lens is therefore to converge to the axial point M' waves of light emerging from the axial point M. The relation between the points M and M' is reciprocal; they are said to be *conjugate* points or foci.

For a second point near the axis (Fig. 3) and at a distance from C equal to CM, equation (1) can be applied directly and the point p' on the axis PCP' located. Assuming the distance pM to be small, we may substitute

as a first approximation PM for pM and $P'M'$ for $p'M'$; in other words, a small surface element normal to the axis is pictured, by refraction at the spherical surface, point for point, as a small surface element normal to the axis. The planes PM and $P'M'$ are called *conjugate* planes.

From Fig. 3 and equation (1) we obtain

$$\frac{P'M'}{PM}=\frac{CM'}{CM} \text{ or } \frac{y'}{y}=\frac{x'-r}{x-r}=\frac{n}{n'}\cdot\frac{x'}{x} \tag{2}$$

if $PM=y$ and $P'M'=-y'$.

This equation states that the *lateral magnification*

$$\beta=\frac{y'}{y}=\frac{n}{n'}\cdot\frac{x'}{x} \tag{3}$$

is constant for any pair of conjugate planes and only varies from pair to pair.

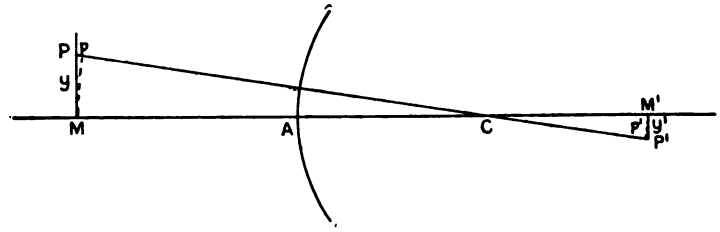


FIG. 3.

Similarly, from Fig. 2 we find with the same degree of approximation the *angular magnification*

$$\gamma=\frac{\tan u'}{\tan u}=\frac{x}{x'} \tag{4}$$

On combining (4) and (3), we have

$$y'.n'.\tan u'=y.n.\tan u \text{ or } \beta\gamma=\frac{n}{n'}\ * \tag{5}$$

which states that the product of the lateral and the angular magnifications is constant. In equation (5) neither x nor x' appears.

REFRACTION THROUGH A LENS.

In the above paragraphs we have considered the refraction at a single spherical surface S, and found that an image $P'M'$ is produced of a luminous object PM (Fig. 3). This image is similar to the original object and may serve in turn as a luminous object for a second refracting surface S_2 (Fig. 4), which produces an image $P''M''$, similar both to $P'M'$ and PM. Strictly speaking, $P'M'$ is not a luminous object similar in every respect to PM, but one from which only a limited cone of light is emitted, not sufficient to fill the entire aperture of S_2, as is the case with the luminous object PM and the refracting surface S_1. For a series of two or more centered refracting surfaces, therefore, as in actual lenses, the image M' from the first surface becomes the object or axial point for the second surface, and is pic-

*The Smith-Helmholz or Lagrange-Helmholz equation; see J. P. C. Southall, Geometrical Optics, 268, 1910.

tured by it in M''. This process repeats itself for each new refracting surface, but in every case a point in the object is pictured as a point in the image, with the result that a point of the original object is pictured as a

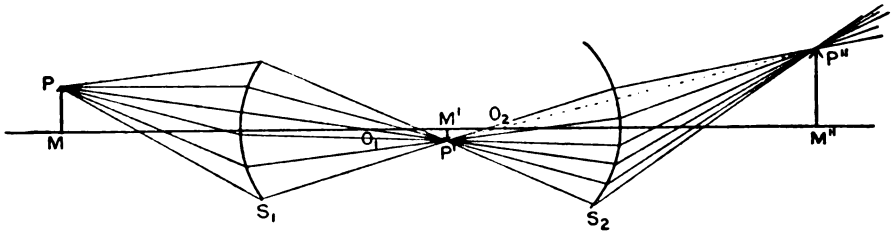


FIG. 4.

point in the final image and a small surface element normal to the axis is pictured as a small surface element normal to the axis in the final image. For this entire system the equation (5) can therefore be written

$$y' \cdot n' \cdot \tan u' = y \cdot n \cdot \tan u \qquad (5a)$$

where n and n' are the refractive indices of the original object space and the final image space respectively.

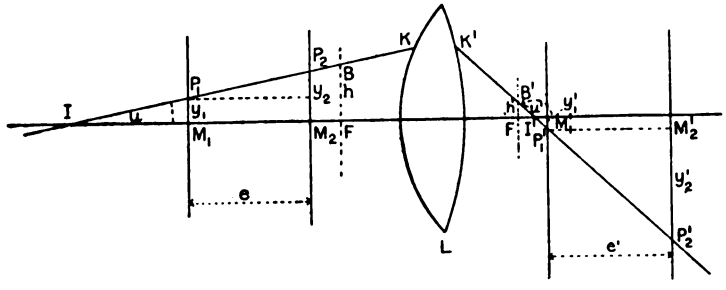


FIG. 5.

If in a lens system L (Fig. 5), two pairs of conjugate planes M_1 and M'_1, M_2 and M'_2 be given and also the lateral magnification for each pair

$$\beta_1 = \frac{y'_1}{y_1} \text{ and } \beta_2 = \frac{y'_2}{y_2}$$

the behavior of the system for any ray or point can be readily found. Thus any entering ray IP_1P_2 must pass on emergence through the points P'_1 and P'_2 of the conjugate planes M'_1, M'_2, and these points suffice to fix its direction; from these relations the fundamental lens formulæ can be easily derived. In Fig. 6, let a ray P_1P_2 be incident parallel with the axis; then, for this ray, $y_1 = y_2 = h$, and for the emergent ray $\frac{y'_1}{y'_2} = \frac{\beta_1}{\beta_2}$; similarly, let the incident ray Q_1Q_2 emerge parallel with the axis; then $y'_1 = y'_2 = h'$ and $\frac{y_1}{y_2} = \frac{\beta_2}{\beta_1}$; from the figure it is evident that

$$\frac{F'M'_1}{M'_1P'_1}=\frac{M'_1M'_2}{P'_2M'_2-P'_1M'_1} \text{ or } \frac{d'}{y'_1}=\frac{e'}{y'_2-y'_1};\ \therefore\ d'=\frac{e'\beta_1}{\beta_2-\beta_1}$$

In similar manner it may be proved that $d=\frac{e\beta_2}{\beta_2-\beta_1}$.

These equations prove that the distance d' is independent of y, and the distance d, of y'; in other words, all rays incident parallel with the axis pass

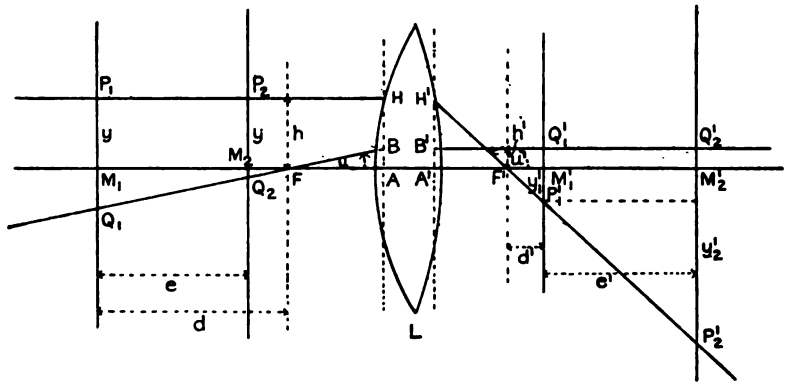


FIG. 6.

through F', the focal point of the second medium, while all rays emerging parallel with the axis pass through F, the focal point of the first medium.

From Fig. 6 it is evident that

$$\tan u'=\frac{-y'_2+y'_1}{e'}=-\frac{h}{e'}\left(\frac{y'_2}{h}-\frac{y'_1}{h}\right)$$

or

$$\frac{h}{\tan u'}=\frac{e'}{\beta_1-\beta_2}=f' \tag{6}$$

where f' is the *principal focal length* in the second medium. In like manner we may prove that

$$\frac{h'}{\tan u}=\frac{e\beta_1\beta_2}{\beta_2-\beta_1}=f \tag{6a}$$

in which f is the principal focal length in the first medium. Equations (6) and (6a) define the focal length of an optical system as the ratio of the height of an incident ray parallel with the axis to the tangent of the angle which it includes with the axis on emergence.

If in the first medium or object space the measurements be referred to F as origin of coordinates, and in the second medium or to image space F' as origin of coordinates, the positive direction in each coordinate system being from left to right and above the axis, then from Fig. 5 we have

$$\tan u=\frac{BF}{FI}=-\frac{h}{x} \text{ and } \tan u'=-\frac{B'F'}{F'I'}=-\frac{h'}{x'} \tag{7}$$

On substituting these values in (6) and (6a), we obtain

$$xx'=ff' \tag{8}$$

From the similar triangles MQF, ABF, and $H'A'F'$ in Fig. 6 and from equation (8) we obtain the equations

$$\frac{y'}{y}=\frac{f}{x}=\frac{x'}{f'} \tag{9}$$

These two equations (8) and (9) are simple and fundamental, and express the behavior of the central rays in any given lens system. They can be derived in a number of different ways and are applicable to all lens systems with focal points. As noted in the derivation they are strictly valid only for central, paraxial rays.

Substituting the values from (7) and (9) in 5*a*, we have

$$\frac{f}{f'}=-\frac{n}{n'} \tag{10}$$

In a lens system the refractive index of the first medium (object space) is often equal to that of the last medium (image space), and for this case $f=-f'$. The principal focal lengths are equal and of opposite sign.

PRINCIPAL PLANES.

The two conjugate planes for which the lateral magnification $\beta=\frac{y'}{y}=1$ are located at $x=f$ and $-x'=-f'$ (equation 9), or at distances FA and $F'A'$ (Fig. 7) from the focal points equal to the principal focal lengths; they are

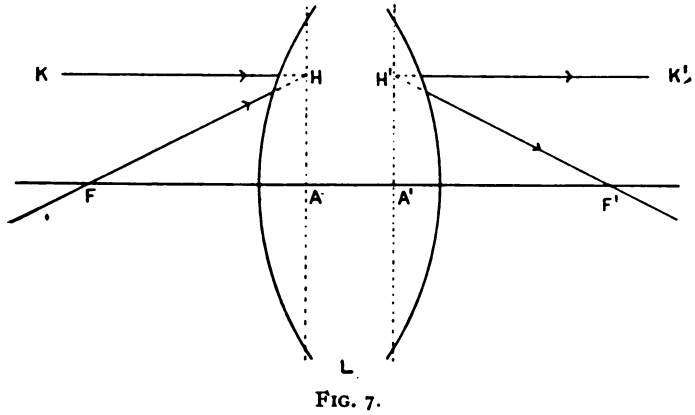


FIG. 7.

called the principal or Gauss planes and are useful in representing graphically the effect of an entire lens system consisting of any number of component lenses.

NODAL PLANES.

The two conjugate axial planes for which the angular magnification $\gamma=\frac{\tan u'}{\tan u}=1$ are called the nodal planes. Their position is readily found from equations (5*a*), (9), and (10) to be $x=-f'$ and $x'=-f$. (Fig. 8, N and N')

For the usual case in which object and image appear in the same medium (air), and $f = -f'$, the nodal planes coincide with the principal planes, which

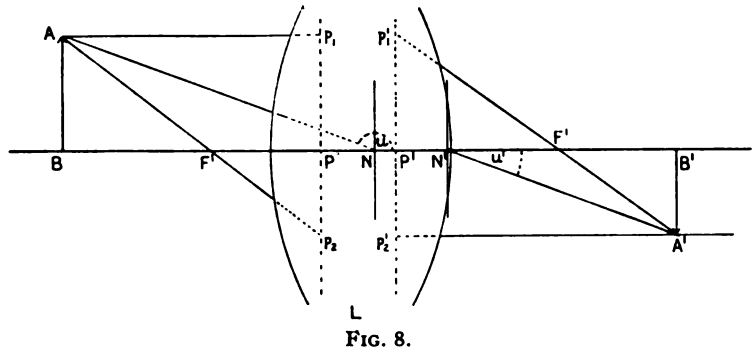


FIG. 8.

are then called the *equivalent* planes of any given lens system, as their distance from the focal points is directly the *equivalent focal length* (E. F.)* of the lens system.

THE FOCAL LENGTH OF A LENS SYSTEM OF TWO COMPONENTS.

The microscope may be considered a lens system consisting of two components, the objective and the ocular, each one of which contributes its share to the magnification attained; it may, however, be looked upon as a single system of definite E. F. with its focal planes in a definite position. To calculate the E. F. of the combined system, let L_1 (Fig. 9), represent the objective lens and L_2, the ocular; let F_1 F'_1 and f_1, f'_1 be the foci and the focal lengths of L_1, and $F_2F'_2$ and f_2, f'_2, the foci and focal lengths of L_2; let the distance $F'_1F_2 = D$ (the optical tube length). Then the incident rays H_1E, H_2G parallel with the axis pass through F'_1, the posterior focus of L_1, which in turn serves as object point for the lens L_2, and this in turn converges the emergent rays to F', the posterior principal focus of the combined system. In similar manner the rays FS and FR pass through F_2, the anterior focus

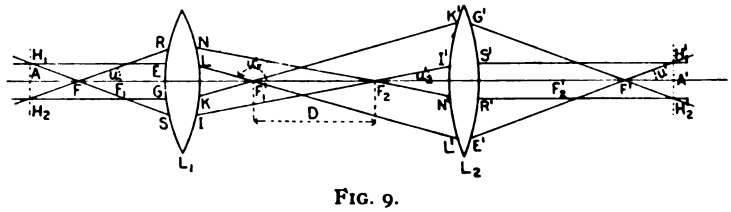


FIG. 9.

of the lens L_2, and emerge parallel with the axis; the point F is consequently the anterior focus of the entire system. The rays H_1E and H_1S from the point H_1 are brought to focus by the system at H'_1, but as $H_1A = H'_1A'$, the planes H_1H_2 and $H'_1H'_2$ are the principal planes and the distances AF and

*This abbreviation has been suggested by Dr. H. Kellner as a concise expression for designating the distance of the focal point of a lens from its equivalent focal plane. This distance (aequivalente Brennweite) is one of the fundamental characteristics of a lens and determines at once the magnification produced by the lens under different conditions. The E. F. of a lens should not be confounded with its *back focus* (Schnittweite) or distance from the surface of the lens to its focal point. In place of the expression *back focus*, the term *focus distance* might well be substituted.

$A'F'$ are the principal focal lengths of the system. To express these relations in mathematical form we observe that the distance F'_1F_2 or D is conjugate to F'_2F' with respect to the lens L_2. Accordingly $F'_2F'=\frac{f_2f'_2}{D}$. Similarly the distance $FF_1=\frac{f_1f'_1}{D}$.

To find the equivalent focal lengths we have by definition $f=\frac{h'}{\tan u}$ and $f'=\frac{h}{\tan u'}$; also by definition the rear focal length of $L_1, f'_1=\frac{h}{\tan u'_1}$ where $h=H_1A$, the distance of the ray H_1E from the axis. For the conjugate points F'_1 and F' the relations (6), (7), and (8) are valid, from which the equation $\frac{\tan u'}{\tan u'_1}=\frac{D}{f'_2}$ is readily derived, provided both F'_1 and F' are in air, as is the case in the microscope. This equation combined with the above expression for u', becomes

$$f'=\frac{h}{\tan u'}=\frac{f'_1f'_2}{D}$$

Similarly

$$f=\frac{h'}{\tan u}=-\frac{f_1f_2}{D} \qquad (11)$$

The four quantities FF_1, F'_2F', f', and f define the optical behavior of the system in the formation of images and will be used frequently in the following pages.

SPHERICAL ABERRATIONS.

MONOCHROMATIC LIGHT.

The assumptions made in the above paragraphs on the Gauss or first order equations, that only paraxial rays (the narrow bundle of rays immediately adjacent to the axis) are to be used, is not applicable to the lens system of the microscope. Here it is necessary for a number of reasons to employ lenses of larger opening. From each point in the object a pencil of rays of wide angle emerges and of this the objective should collect as much as possible and bring it to sharp focus at a point in the image if the definition is to be satisfactory. This condition should hold for all points of the object. The image, in short, should be similar in every respect to the object and as bright as possible. In actual lens designing and lens construction there are practical difficulties in the way of fulfilling this condition accurately in all details, and the lenses which the observer receives are more or less encumbered with defects or aberrations (deviations from the theoretically perfect) which it is impossible to eliminate entirely. If monochromatic light be used, these aberrations may be conveniently grouped under five heads: (1) Spherical aberration proper. (2) Spherical aberration of oblique rays (sine condition). (3) Astigmatism. (4) Curvature of field (Petzval condition). (5) Distortion (tangent condition).

SPHERICAL ABERRATION PROPER (FOR THE PRINCIPAL AXIS).

(a) REFRACTION AT A SINGLE SPHERICAL SURFACE.

In Fig. 10 let HA be a spherical glass surface (refractive index = 1.50) and P a luminous point on the axis PA. A light-wave impulse starting from P travels with equal velocity in all directions and the shape of the wave-front at any given instant is that of a spherical shell concentric to P. The spherical arcs in Fig. 10 represent the different positions which a part of the wave impulse sent out by P reaches after equal successive time intervals. On entering the glass the wave travels more slowly and the wave-front is no longer a spherical surface, but a warped surface, the normals (ray directions) of which are represented by the arrows of Fig. 10. Optically, it takes the wave impulse the same time to travel from b B (Fig. 10) in air that it does to travel from A to A' in the glass (the ratio of b B to AA' being by definition the refractive index of the glass = 1.5). Thus the wave impulse from A reaches A' at the same instant that the thrill from b reaches B. Similarly the impulses from $A, b,$ and c reach the points A'', B', and C respectively, and the surface containing these points is the wave-front. When the impulse reaches H, it thrills A^{VII}, B^{VI}, C^{V}, etc. at the same instant. The wave-front for the section represented in Fig. 10 is accordingly H, C^{V}, B^{VI}, A^{VII} . . . H, which is no longer a simple circular arc.

The ray directions or normals to the wave-front can then be readily found by drawing the secondary Huygens spherical wave-fronts as indicated in Fig. 10, and finding the points at which the wave-front is tangent to these circles. A simpler construction is that of Weierstrass as represented in Fig. 11, in which C is the center of the refracting sphere of radius R and refractive index n', while the refractive index of the enveloping medium is n. By con-

struction the radii of the outer and inner circles are $R_1 = \frac{n'}{n} R$ and $R_2 = \frac{n}{n'} R$. Then any ray DB incident on the sphere at B intersects the outer circle at

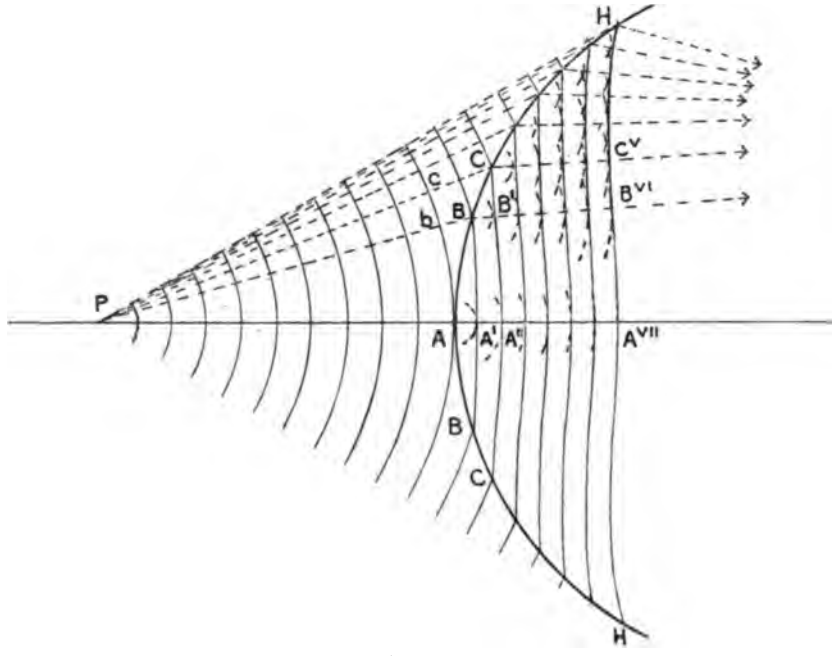


FIG. 10.

A. The line $BA'D'$ joining B and the point of intersection of the radius CA with the inner circle is then the refracted ray from the incident ray BD.

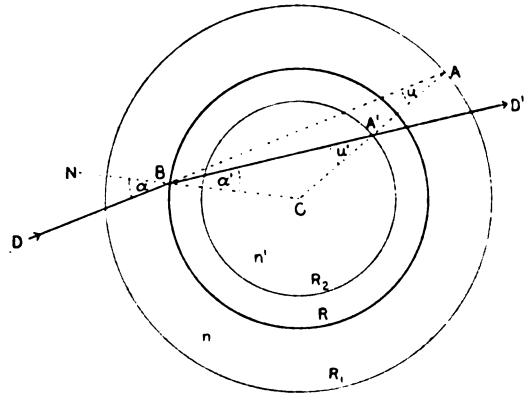


FIG. 11.

This is evident from the similarity of the two triangles CBA and CBA' whereby the angle $CBA = BA'C$ or $\alpha = u'$ and

$$\frac{\sin u'}{\sin \alpha'} = \frac{BC}{A'C} = \frac{n'}{n} \qquad \frac{\sin \alpha}{\sin \alpha'} = \frac{n'}{n}$$

(b) APLANATIC POINTS OF A SINGLE REFRACTING SPHERICAL SURFACE.

The wave-front developed in Fig. 10 by waves emerging from the axial point P and entering the glass surface is not spherical but is a warped surface whose normals or ray directions intersect the principal axis at different points. This deviation of the refracted wave-front from the spherical shape is called spherical aberration and its effect is to produce a wave-front whose rays appear to come from very different points and not the common point P, as was the case while the waves were still propagated in air. The shape of the refracted wave-front is directly dependent on the position of P and changes its shape as P is moved nearer to, or farther away from, the spherical surface. It can be shown, moreover, that for waves converging to one particular position of P, the refracted wave-front is still spherical and converges to a definite point P'. In Fig. 12, let a spherical wave converging in

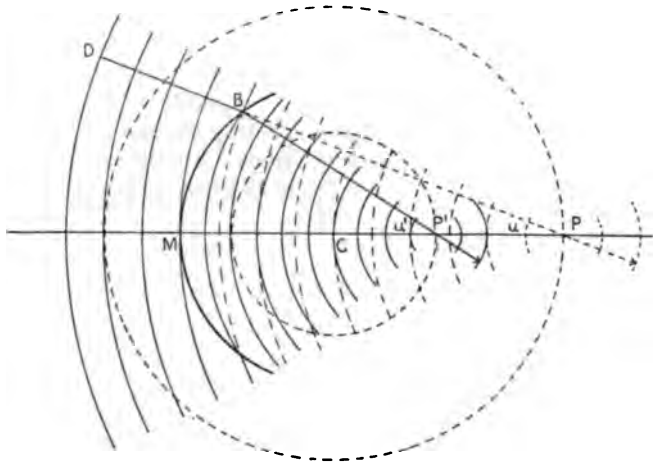


FIG. 12.

air toward the point P be incident at the glass spherical surface BM. Let the distance $CP=\frac{n}{n'}R$ (as in Fig. 11) and the distance $CP'=\frac{n'}{n}R$, where R is the radius of the spherical surface BM. Then the Weierstrass construction shows that any ray DBP is refracted to the point P', the optical path BP' in glass being equal to the optical path BP in air. As the ray DB is any ray converging toward P, the construction is valid for the entire area of the spherical surface and not alone for paraxial rays. The refracted wave-front is accordingly strictly spherical with A' as the center. The points P and P' are conjugate points and for them the relation is readily obtainable from Fig. 11.

$$\frac{CP}{CP'}=\frac{n'\sin u'}{n\sin u}=\text{constant} \qquad (11)$$

which is valid for all possible angles u; these two points are free, therefore, from the spherical aberration resulting from the use of other than central

rays. From Fig. 13 it is evident that the same relation holds true for the points K and K'; the arc $K'A'$ is accordingly brought to focus, point for point, in the image KA. If a small surface element alone be considered, the relation $\frac{A'C}{AC} = \frac{A'K'}{AK}$ can be written $\frac{A'C}{AC} = \frac{A'k'}{Ak} = \beta$ (the linear magnification); or from equation (11)

$$\frac{A'C}{AC} = \frac{n \sin u}{n' \sin u'} = \beta$$

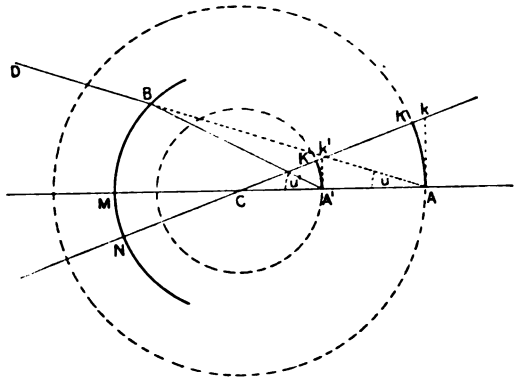


FIG. 13.

This is the condition which must be fulfilled, if a small surface element normal to the axis at A is to be imaged by wide ray pencils, point for point. The points A and A' are called the aplanatic points of the refracting surface for the axis ACM, since they alone satisfy the above conditions. In the construction of oil-immersion objectives these two points are of great importance.

(c) SPHERICAL ABERRATION IN SIMPLE LENSES.

In Fig. 14 let L be a simple collective lens and P a luminous point on the

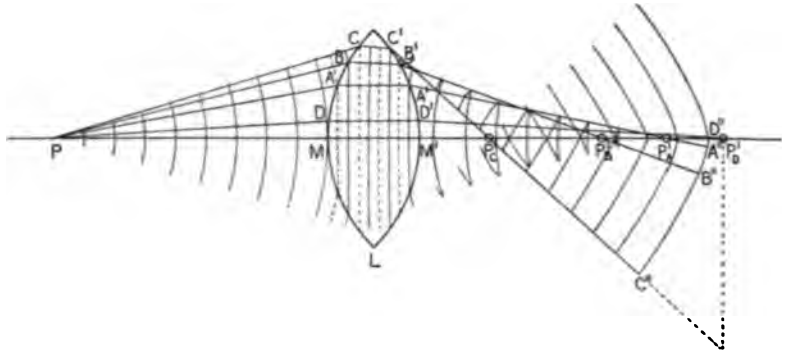


FIG. 14.

principal axis PM. Let the concentric arcs about P represent the positions, after equal time intervals, of a wave impulse starting from P and expanding in all directions. On reaching the lens the speed of propagation of the wave is retarded and its wave-front assumes the shape represented in the figure. On emerging into air again, the original velocity is regained, but the wave-front is still further distorted into a complex warped surface whose radii of curvature (ray directions) do not converge to a single point on the axis, the axial part of the wave meeting the axis at P' while the marginal rays intercept the axis at $P''c$, a point much nearer the lens than P'. This deviation of the emerging wave-front from the spherical form is called *spherical aberra-*

tion. In Fig. 15, *a*, the spherical aberration of a simple collective lens is pictured; here the marginal rays, as in Fig. 14, intercept the axis at *B* before the central rays; in Fig. 15, *b*, the spherical aberration of a simple negative element is given. If the spherical aberration in the lens Fig. 15, *a*, is equal and opposite to that in Fig. 15, *b*, it is evident that on combining the two lenses (Fig. 15, *c*) compensation will result and the resultant spherical aberration of the combination for the central and marginal rays will be *nil.* In Fig. 15, *c*, glasses of different refractive index (positive element crown and negative

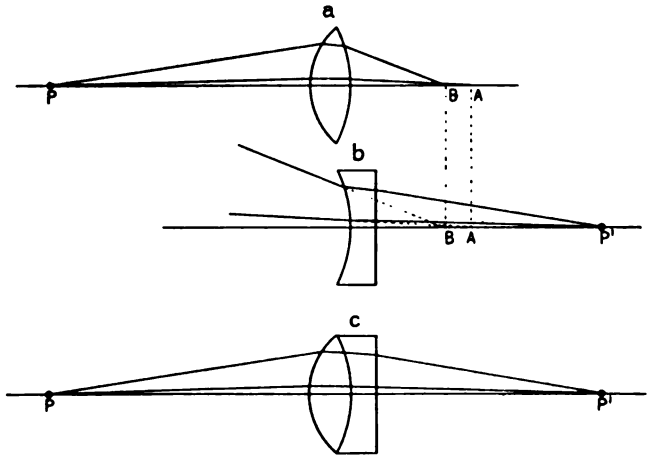


FIG. 15.

element of higher refracting flint glass) have been chosen to compensate for the spherical aberration. It is possible, however, by a proper selection of the shapes of the component lenses, to correct for spherical aberration even with elements of the same refractive index as indicated in exaggerated form in Fig. 16.

If the compensation be not fully attained and the marginal ray still intercepts the axis at the point 2 nearer the lens than the central ray *P'* as in Fig. 17, *a*, the lens is said to be spherically *undercorrected;* and *overcorrected* if the marginal rays intercept the axis at a point 2 beyond *P'* as in Fig. 17, *b*.

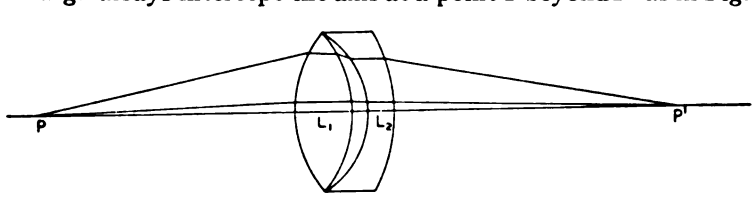


FIG. 16.

It is thus possible, by combining a positive and a negative element of proper shapes and refractive indices, to produce a lens free from spherical aberration (axial and marginal rays coincide in the image point). This does not signify, however, that the rays from points or zones intermediate between margin and axis are automatically corrected. These may be undercorrected or overcorrected (Fig. 17, *c*, *d*). This residual spherical aberration

is called *spherical zones*. If the angular aperture (angle under which the rays meet) is large, the effect of zonal aberration becomes serious. Objectives of high aperture should be corrected for axial, marginal, and at least one other zone, as indicated in Fig. 17, *e*, in which only rays 2 and 4 are still aberrant. If this residual spherical aberration is slight and not disturbing to the eye the correction for spherical aberration is, for practical purposes, satisfactory. For low apertures sufficient correction can be obtained by the use of two lenses, one positive and one negative. The higher the aperture the more lenses are necessary.

In the practical calculations for correction of spherical aberration, advantage is taken of the fact that the eye is an imperfect optical instrument such that an area which subtends less than one to two minutes of arc appears as

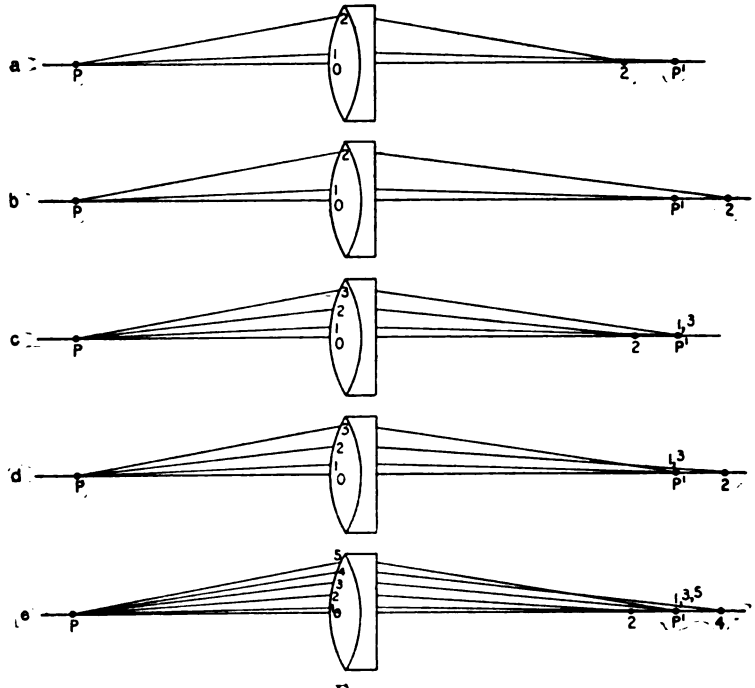


FIG. 17.

a point to the eye. In the image, therefore, a point which subtends less than one minute of arc appears to the eye sharply defined. To carry the corrections below this limit would be labor wasted, as the eye would fail to appreciate the improvement.

SPHERICAL ABERRATION FOR OBJECT POINTS NEAR THE AXIS (SINE CONDITION).

When spherical aberration is corrected for a point on the axis an object point near the axis is not then necessarily sharply imaged. The rays entering the lens along the chief axis AM (Fig. 18) are brought to focus at the point A', while the marginal rays AN converge to the point A''. In the

image, the point A appears drawn out like the tail of a comet, which may be directed away from or toward the margin of the field. This aberration is usually called *coma*, the fuzziness being caused by the difference in magnification by the different zones of the objective. The first approximation for the removal of coma is the fulfillment of the Abbe sine condition, which states that the ratio of the sines of slope angles between any two incident

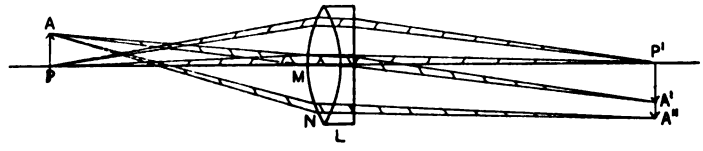


FIG. 18.

rays and the two resulting emerging rays is constant (in other words, the optical paths for the different rays from object point to image points are equal). The condition that all rays from the axial point P unite in P' (Fig. 19) postulates that for these conjugate points the system is free from spherical aberration, while the condition that all rays from A unite in A'

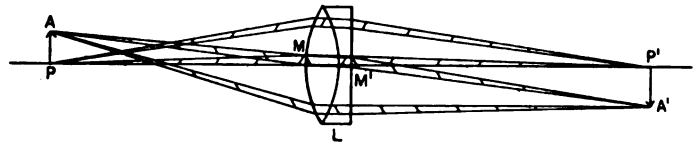


FIG. 19.

requires that the system be free from spherical aberration for the secondary axis AMA'. The mathematical expression for this condition has been deduced in a number of different ways, the simplest being possibly that of Hockin.* In Fig. 20, let PA and $P'A'$ be small conjugate central surface elements; from P and A let two parallel rays emerge which intersect at E

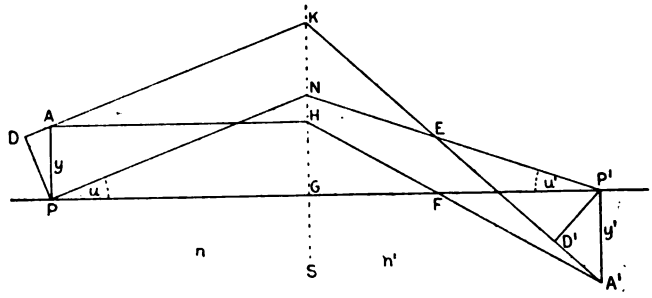


FIG. 20.

in the image space; let also F be the point of intersection in the image space of the axis with the ray through A parallel to the axis; let DP be normal to the incident ray A and $D'P'$ to the refracted ray KA'. If the spherical wave disturbance emerging from P is to be focussed at P', then P' must be the center of curvature of the spherical wave surface in the image space, and the

*Jour. Roy. Micros. Soc. (2), 4, 337, 1884.

time required for the wave disturbance to travel from P to P' must be the same for all radial directions emerging from P; the same holds true for the conjugate points A and A'. Accordingly, the optical distances

$$PNEP' = PGFP' \text{ and } AKEA' = AHFA'$$

therefore

$$AKEA' - PNEP' = AHFA' - PGFP' = AHF + FA' - PGF - FP'$$

but a plane wave-front PA becomes a spherical wave, after refraction, with F as center; therefore $PGF = AHF$; in like manner the plane wave-front DP becomes spherical after refraction with E as center, and $DKE = PNE$; as the distance PA is indefinitely small we may consider as a first approximation the length $EP' = ED'$, in which case $AKEA' = PNEP' = DKED'$ and therefore $D'A' = DA$, an equation which referred to vacuum becomes

$$n' . P'A' . \sin(-u') = n . PA \sin u$$

But

$$\frac{PA}{P'A'} = \frac{y}{-y'} = -\frac{1}{\beta};$$

accordingly

$$\frac{n' \sin u'}{n \sin u} = \frac{1}{\beta} \qquad (a)$$

or

$$\frac{\sin u'}{\sin u} = \frac{n}{n'} \frac{1}{\beta} \qquad (b)$$

This formula states that the ratio of the sine of the axial angles u and u' of any two conjugate points in plane through the aplanatic points normal to the axis is constant. If the Abbe sine condition be properly met, the chief and marginal rays from the point A will converge to the point A', as illustrated in Fig. 19; but in actual practice the point A represents any point in the field, and we have, therefore, under these conditions, correction of spherical aberration in the plane passing through the object point and the optical axis (*tangential or meridional plane*). If the sine condition be fulfilled, the images of the object produced by the central and marginal zones are identical in size and position. But the sine condition may also show zones, just as spherical aberration proper; if the marginal zone fulfills the sine condition, the intermediate zones do not generally do so. Systems that are spherically corrected and fulfill the sine condition are, by Abbe's definition, called *aplanatic*. Aplanatism is possible only for one pair of conjugate points.

In the actual construction of microscope objectives, these corrections can be effected only for three zones simultaneously (central, marginal, and an intermediate zone); these are then so chosen that the deviations of the remaining zones are small and exert only slight influence on the resulting image. In immersion objectives the aplanatic points are of prime importance, as the front lens of such systems is simply a small uncorrected glass hemisphere. The object is placed in the near aplanatic point and the emergent wave is strictly spherical and of lower aperture than the original pencil. In dry objectives such ideal conditions are not possible.

ASTIGMATISM.

In the case of an oblique bundle of rays impinging on the lens, as in Fig. 21, from a point not on the optical axis, the lens does not present a symmetrical front to the incident rays; as indicated in the figure, the lens appears foreshortened in the plane of incidence (containing optical axis PP' and point A; usually called *tangential* or *meridional* plane). The effect of the lens is to produce an emergent wave of less radius of curvature for

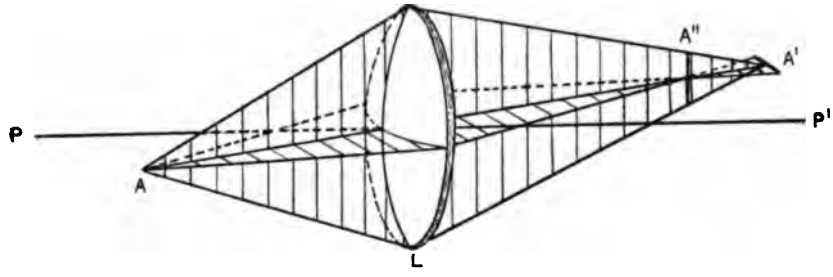


FIG. 21.

the tangential plane than for the plane normal to it (*sagittal plane*) (Figs. 21, 22*). The image formed in the tangential plane will then fall short of the image formed in the sagittal plane. The distance between the image points in the tangential and sagittal planes is the *astigmatic difference* and the aberration is called *astigmatism*. The total effect of astigmatism on an incident spherical wave is to produce an emergent wave of spherical front whose radius of curvature in the tangential plane is less or greater than that in

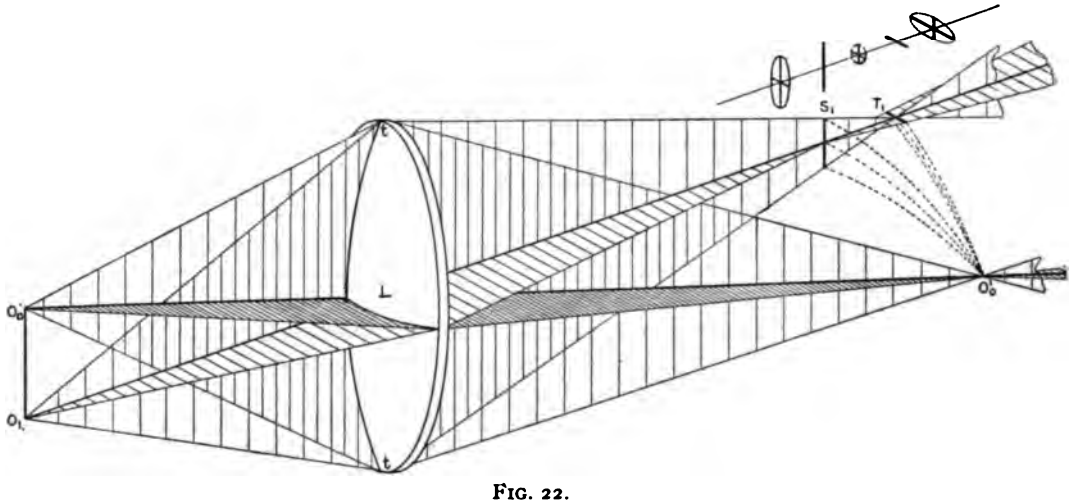


FIG. 22.

the sagittal plane; the practical consequence of this is that astigmatism produces from a plane object an image which consists of two coaxial warped and irregular surfaces. In correcting for astigmatism, the effort is made to make these two surfaces coincide for the useful field of the objective. Astigmatism is not, in general, very noticeable in microscopic objectives, because the inclination of the chief rays is comparatively small. It usually

*Fig. 22 was suggested to the writer by a drawing by Dr. H. Kellner, illustrating astigmatism.

is more serious in low-power than high-power objectives. Eye-pieces and pocket magnifiers should be corrected for astigmatism and for curvature of field as well.

CURVATURE OF THE FIELD (PETZVAL CONDITION).

After astigmatism has been corrected, the lens produces a point-shaped image of any point in the object field; these image points are not contained, however, in a plane normal to the principal axis, but on a curved surface generally concave toward the objective. In order that the image be flat, the radii of curvature and the refractive indices of the component elements must be such that the sum of their reciprocal products is zero (Petzval condition). The flattening of the curved surface of astigmatism is called astigmatic flattening of the image. An apparent lack of flatness of field in the microscope is usually due not to curvature but rather to imperfect correction of the other spherical aberrations (especially sine condition and chromatic differences of magnification).

DISTORTION (TANGENT CONDITION).

Even after the image is flat, it may not be similar to the object if there is a variation in the magnification of the lens for points away from the optical axis. The image of a rectilinear grating (Fig. 23, *a*) from a lens showing

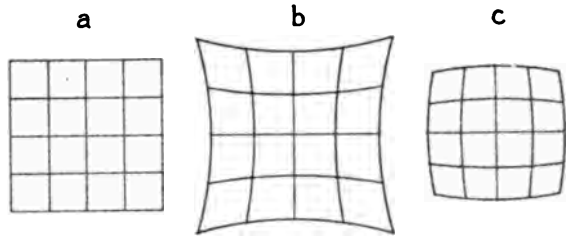


FIG. 23.

distortion is no longer rectilinear but curved and cushion-shaped or barrel-shaped (Fig. 23, *b*, *c*). In microscope objectives the construction is such that distortion is not serious in the image. If distortion be present, it is usually due to the eye-piece.

In a lens system it is not possible to correct all the above spherical aberrations at once with equal perfection, and certain corrections are sacrificed for others, depending on the purpose for which the lens is to be used. In microscope objectives of high aperture, where wide pencils of light with small inclination of the chief ray to the axis are used, the spherical correction of three zones of the aperture and the observance of the sine condition are generally sufficient; in the photographic lens, on the other hand, where the aperture is smaller and the field larger, astigmatism and distortion have to be corrected, while the spherical correction need not be carried so far as in the microscope or telescope where the image is highly magnified. From the eye-piece narrow pencils of light, highly inclined to the axis, enter the eye, the function of the eye-piece being that of a magnifier with large field. It is accordingly corrected, especially for astigmatism, curvature, and distortion, but not necessarily for spherical aberration, which, in the axis, is not serious, since the diameter of the pencils is small—equal to or less than the diameter of the pupil of the eye.

CHROMATIC ABERRATIONS.

In describing the above third-order spherical corrections, we have assumed that the light source is monochromatic. If now white light be used, as is ordinarily the case, a given lens, although satisfactorily corrected for one particular wave-length, may be seriously defective for rays of another color, and *chromatic* aberrations may result. Any glass from which a lens may be ground shows noticeable dispersion, the refractive index for blue light being higher than that for red. The effect of any collective lens is,

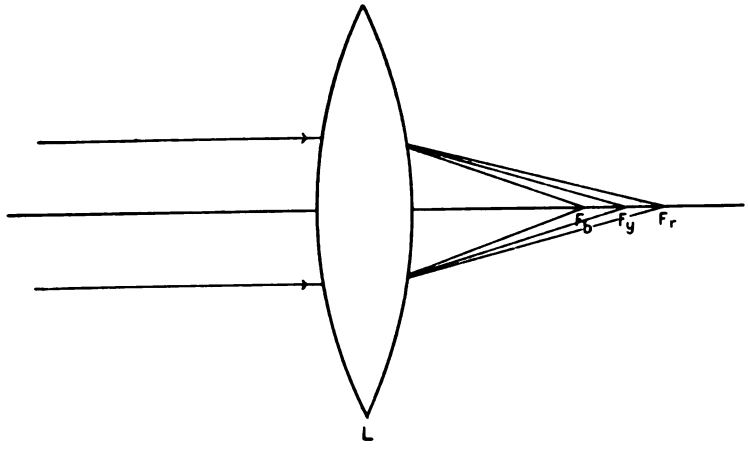


FIG. 24.

therefore, to converge the blue rays to a point nearer the lens than that for the red rays (Fig. 24); the equivalent focal length of the lens varies accordingly with the wave-length. A simple positive lens shows *chromatic undercorrection* for both location and size of image (Fig. 25), while a negative element shows *chromatic overcorrection*. The chromatic aberration of a simple collective lens can be readily detected by observing the image which

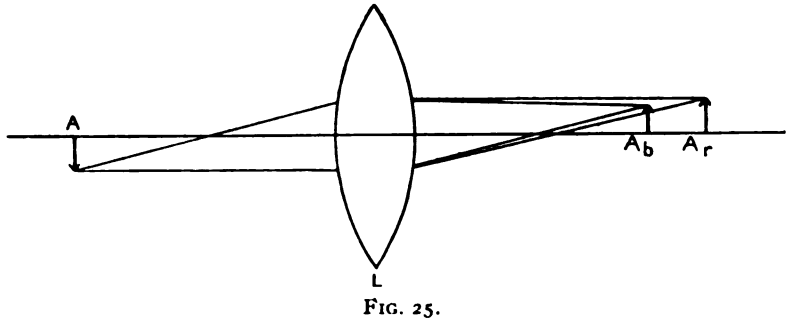


FIG. 25.

it forms of a distant luminous point. At F (Fig. 24) the image is a blue disk with a marginal red fringe, while at F_r the center is red and the margin blue. By combining two lenses, a positive of weak dispersion and a negative of strong dispersion, it is possible to make both the image distance and also the equivalent focal length of the lens equal for two colors of the spectrum. This applies to the paraxial rays. The lens is then achromatic for

two colors, but only for two, red and blue being selected generally for visual purposes. The part of the spectrum lying between the red and blue is brought to focal points nearer the lens, while the colors beyond red and blue focus at points farther from the lens. The spectrum is folded, as it were, on itself, the red and blue parts being brought to coincidence, while the center or yellow-green part is focussed at points near the lens. This remnant of chromatic aberration is called *secondary spectrum* and results from the fact that the dispersion of crown and flint glasses is not proportional throughout the spectrum. The secondary spectrum may be diminished by a suitable choice of glasses, but with the material at present available at least three lenses are required, for a reasonably high aperture, to bring to the same focus three different colors and thus to eliminate the secondary spectrum.

Even after this has been done and there is practically no chromatic difference in image distance, the magnification of the different colored images at the image point may be different (Fig. 26). The E. F. of the lens for blue

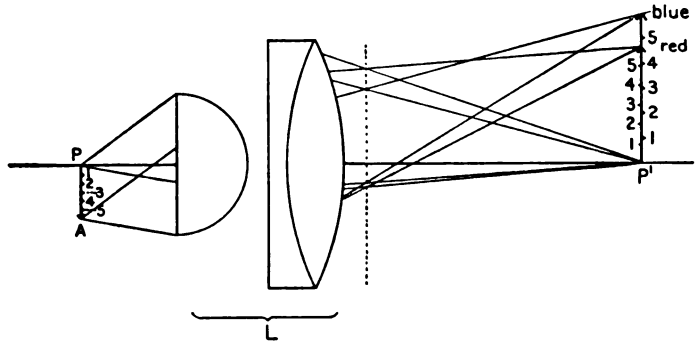


FIG. 26.

is less than that for red and consequently the blue image is larger than the red image. There is a chromatic difference in magnification of the images, which increases as we recede from the axis. Achromasy in the axis and freedom from chromatic difference of magnification can not be obtained in an objective of high aperture; microscope objectives (and in fact all objectives of high aperture) show this defect near the margin of the field.

Magnifiers, on the other hand (in which the aperture of the image forming pencils is limited by the pupil of the eye and hence small), are not sensitive to chromatic aberration in the axis, but are seriously affected by chromatic differences of magnification, especially near the margin of the field. They are corrected, accordingly, not for axial achromatism (as is the objective), but to give equal focal lengths for two colors. This correction is attained in ordinary achromatic eye-pieces with two lenses of the same kind of glass by mounting them at a distance $d=\frac{f_1+f_2}{2}$ apart. The images may then be viewed at infinity; the angles which they subtend at the eye are identical for the two colors, and they appear, therefore, to be of the same size. By changing this distance d it is possible to give the eye-piece chromatic overcorrection or undercorrection, thus making the image for red larger or smaller than that for blue. By proper chromatic overcorrection of the eye-

piece it is thus possible to compensate the undercorrection in the objective so that the eye-piece has just as much greater magnification for red as the objective has for blue, and the result is an image practically free from color, even to the margin of the axis. This method was adopted by Abbe in his compensating oculars.

In lenses of wide aperture the chromatic aberration varies from zone to zone. Thus in Fig. 27 the paraxial rays for blue and red unite in one point, P', while the marginal blue ray intersects the axis at P_b and the marginal red ray at P_r. In lenses of the same shape, the higher the refractive index the smaller the spherical aberration; consequently, if the spherical aberration is corrected for yellow rays, the blue rays show spherical overcorrection. The lenses may accordingly be considered as chromatically corrected in the axis and as having increasing chromatic overcorrection toward the margin. In simpler objectives no real correction is attempted. The zone of best chromatic correction is placed between the axis and the margin, so that the "circle of confusion" caused by the overcorrected marginal zone is equal to that caused by the undercorrected axial zone. All ordinary chromatic objectives are corrected by this method. Abbe was the first to point out

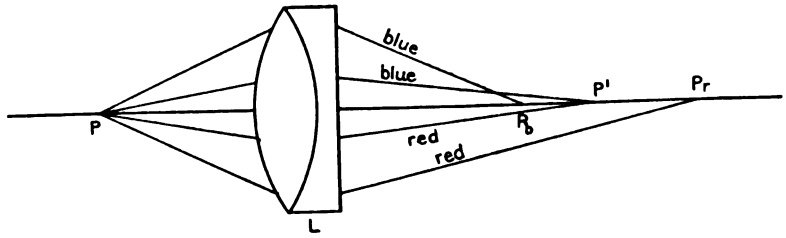


FIG. 27.

the importance of this *chromatic difference of spherical aberration* and its detrimental influence on the microscopic image. He showed how a microscope objective can be spherically corrected for two colors, or, in other words, chromatically corrected for two zones. Such an objective has the same quality of chromatic correction over the entire opening, the remnants, which are left uncorrected outside of the two zones, being too insignificant to cause trouble.

The sine condition also varies for different colors, but is usually corrected only for the yellow-green. In the apochromatic objectives the sine condition is corrected for two colors. These objectives are the most highly corrected type yet devised. They are aplanatic for two colors (show no chromatic difference of spherical aberration and fulfill the sine condition for two colors) and are free from the secondary spectrum.

As noted above, the corrections in the optical system are made only for a single, definite position of the object and image (tube length given) and for a definite thickness of cover-slip. For this reason it is essential for the best results that the tube length be correct and also the cover-glass thickness, otherwise the quality of the image may be seriously impaired. High-power dry-objectives are usually fitted with a correction collar to compensate for the change in spherical overcorrection resulting from the use of cover-glasses of different thicknesses. This defect may also be compensated for by changing the tube length, but less expeditiously.

DIAFRAMS IN THE MICROSCOPE.

In tracing the course of a wave impulse from any point in the object through the lens system of the microscope, the effect of the centered lens mounts and other diaframs, including the pupil of the eye, in limiting the wave-front is an important factor to be considered; these stops not only restrict the aperture of the effective wave-front, but also determine the extent of the object reproduced in the image.

Let Fig. 28 represent a lens system which consists of two component lenses L_1 and L_2, and is corrected for spherical and chromatic aberrations and the sine condition; let AB be an object and $A''B''$ the final image produced by the lens system; let CD be a centered stop and $C'D'$ and $C''D''$ its conjugate images formed in the object and image spaces respectively by the lenses L_1 and L_2. These conjugate apertures $C'D'$ and $C''D''$ define the apertures which the wave-front must have in the object and image space respectively in order to pass through the stop CD or iris of the instrument. In the case of several different diaframs in the object space, that diafram for which the angle $C'MD'$ in the object region is the least is called the *entrance*

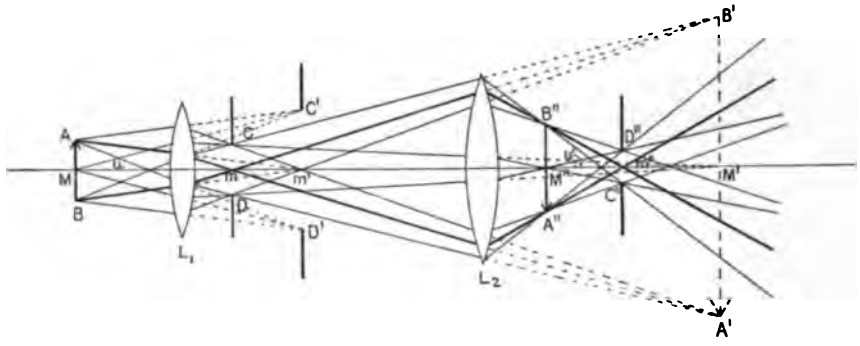


FIG. 28.

pupil of the instrument and the angle $2m'MC' = 2u$ the *angular aperture* of the instrument; that diafram $E'F'$ (Fig. 29) for which the angle MmA in the object region is the least, determines the *angular field of view* and is called the entrance *field of view diafram* or "*entrance port*"* of instrument. Similarly the conjugate image $C''D''$ (Fig. 28) in the image space is called the *exit or emergence pupil* and the corresponding conjugate image $E''F''$ the *exit or emergence port.* The angle $D''M''C''$ is the *projection angle* or angular aperture in the image space; the angle $B''m''A''$ the *image angle* or angular field of view in the image space. The *principal rays* are by definition those which pass through m, the center of the entrance pupil. From Fig. 29 it is evident that, if the *entrance port* $E'F'$ or *object field of view stop* does not coincide with the object, it cuts off rays from points near margin of the object and consequently produces unequal illumination and imperfect correction for the margin of the field. In the microscope it is essential, therefore, that the *entrance port* coincide with the object, and its conjugate plane or the *exit port* with the image plane, in which case the field is sharply bounded and equally illuminated throughout.

*Suggested by Southall as translation of The German "Eintrittsluke."

In optical systems it frequently happens that the conjugate image of an object serves as starting-point or object for a second set of emergent light-waves which are again brought to focus by a second part of the optical system, and a new conjugate point is established. In this last point we find imaged not only the details of the original object but also any other details which the first image plane may have contributed. In the microscope this phenomenon of superimposed images or concatenation of conjugate planes is not uncommon and is of importance in determining the positions of the diaframs to be used.

In Fig. 30 the path of the rays through the microscope lens system is sketched. Pencils of parallel rays are brought to focus by the condenser on the object AB; the rays emerging from points of the object are brought to focus by the objective alone in the image plane $A'B'$; before reaching this

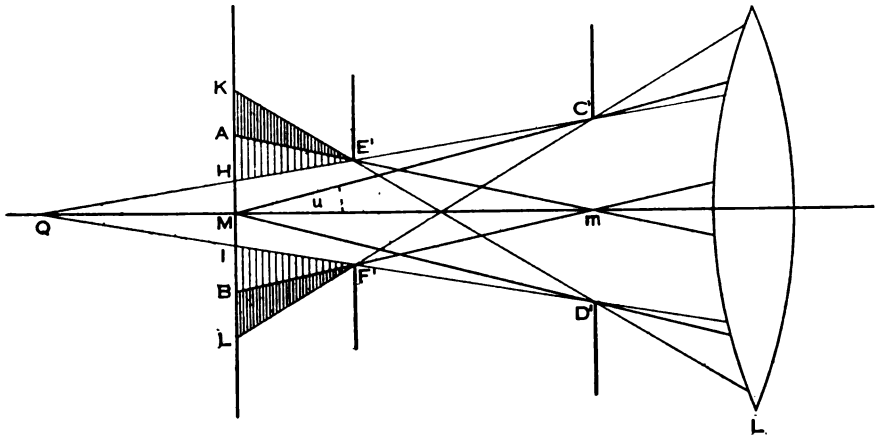


FIG. 29.

plane, however, the rays are intercepted by the field lens of the ocular and conveyed to the image $A''B''$; with respect to this lens the image $A'B'$ serves as object and $A''B''$ is its conjugate image; $A''B''$ serves in turn as object for the eye lens of the ocular; its conjugate image is the virtual image $A'''B'''$ which is viewed directly by the eye placed at the eye-circle or Ramsden disk of the ocular $C'''D'''$. The apertural plane of the incident wave-front is obviously the stop CD at the lower focal point of the condenser. Its image $C'D'$, which is located at infinity, serves in turn as object for the objective and is imaged in the upper focal plane of the objective at $C''D''$. Owing to the construction of the objective, this is not imaged in a plane but approximately in a curved surface as indicated by the figure. It is not possible optically to correct an objective so that both the image of the object and the rear focal plane of the objective are plane. The image $C''D''$ or exit aperture diafram of the objective is imaged in turn at $C'''D'''$ by the ocular. At this point all principal rays of the system cross. The substage diafram CD is accordingly the *entrance pupil* of the microscope and the *eye-circle* (or eye-point or Ramsden disk) $C'''D'''$ is the *exit pupil*.

From Fig. 30 it is apparent that in C''' D''' or the eye-circle of the ocular we have imaged the substage diafram and the rear aperture diafram of the

objective. By examination of this disk C''' D''' with a lens we are able to determine at once whether the substage diafram and the condenser lens are centered and also whether the upper objective lens is encumbered with dust. To be satisfactory the diameter of this eye-circle should not be greater than that of the pupil of the eye. In this case the eye receives all the light

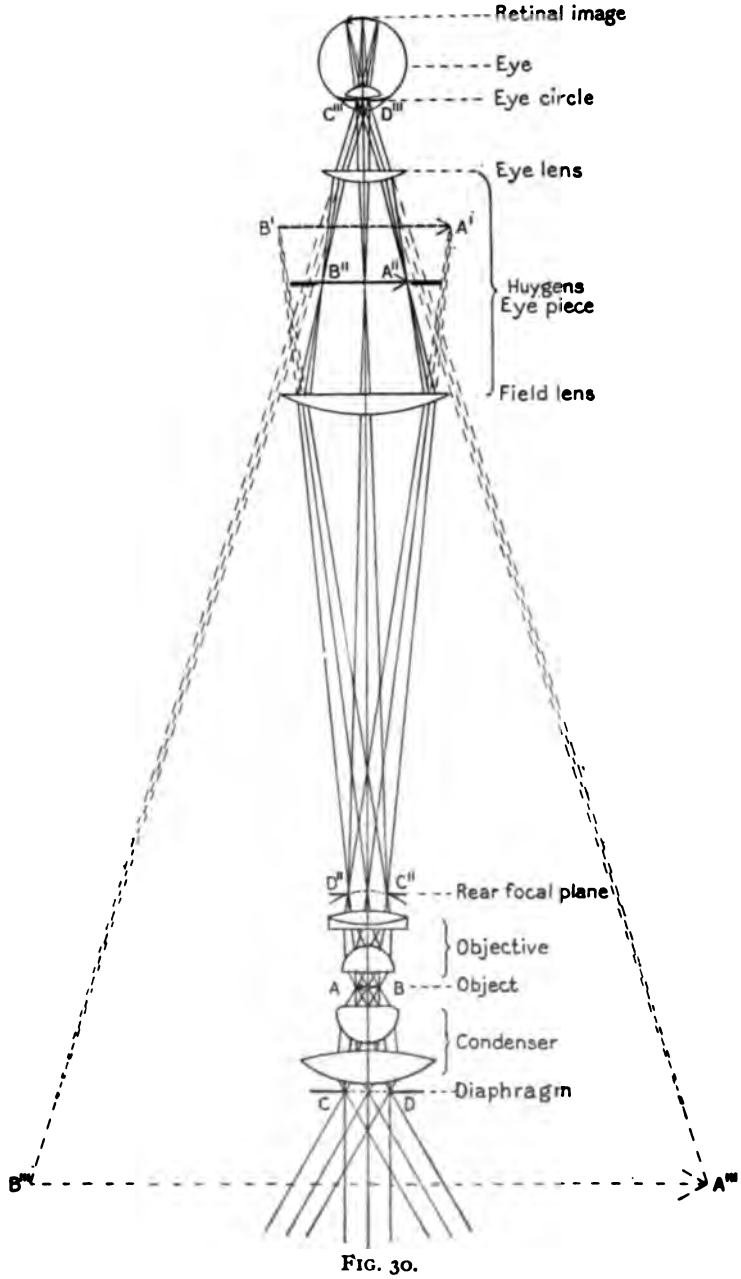


FIG. 30.

from the object and the illumination is good. From the figure it is obvious that as the eye-circle is the image of the rear aperture diafram of the objective, its diameter is directly dependent, for any ocular, on the size of this rear aperture objective diafram, which in turn is a function of the angular aperture. This function was first investigated by Abbe, and called by him the numerical aperture. To find its value, let $B'A'$ (Fig. 31) be the image of the object BA as formed by the aplanatic microscope objective L. As indicated in the figure, the incident bundle of rays from any point in the object includes a large angle u, while the emergent rays converge at a small angle u' (less than 3°) to the corresponding image point. The larger the angle u' the greater the quantity of light which reaches any given point in the image; the solid angle u' may be considered, therefore, a measure of the relative quantity of light effective in forming an image point. The sine condition for which the objective is corrected was found above (page 28) to be $\frac{\sin u'}{\sin u} = \frac{n}{n'\beta}$.

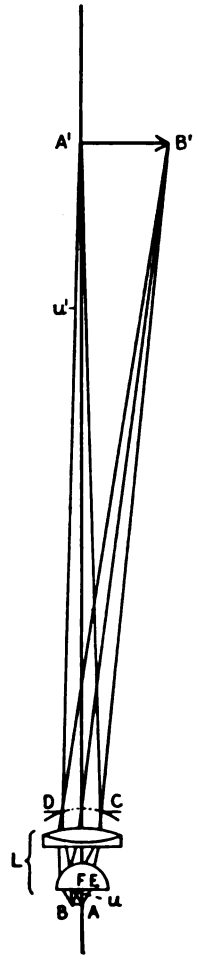


FIG. 31.

In applying this formula to the objective of Fig. 31 we may substitute u' for $\sin u'$ (the angle being small); furthermore, the image is formed in air and hence $n' = 1$; β is also constant, as the system is aplanatic. The equation reduces accordingly to

$$u' = \frac{n \sin u}{\beta} \text{ or } u'^2 = \frac{n^2 \sin^2 u}{\beta^2}$$

But as noted above, the solid angle (u'^2) is a measure of the quantity of light which emerges from the objective and on this the illumination of the image is dependent. Conversely, the quantity of light which passes through the optical system is proportional to the square of the product, $n \sin u$ (n being the refractive index of the object space) or to the *numerical aperture* of the system. Abbe found that not only does the brightness of the image increase, for any given magnification, with the square of the *numerical aperture*, but that the resolving or imaging power of the objective varies directly with the numerical aperture. Underlying this expression and intimately associated with it is the theory of the formation of the image in the microscope, which will be presented in outline later (page 42).

The equation for the sine condition may be written $\frac{n \sin u}{n' \sin u'} = \beta$, which states that the ratio of the numerical apertures of the incident and emergent pencils is the lateral magnification β due to the objective.

The numerical aperture $n \sin u$ is dependent not only on the angular aperture of the incident rays but on the refractive index of the medium

surrounding the objective. The higher the refractive index, the higher the numerical aperture of the objective for the same apertural angle of the incident rays. In case the objective be a dry objective (object space is air and consequently $n = 1$), the numerical aperture is directly the sine of the angle included by the incident cone of rays. Its highest possible value is accordingly 1. In case the objective be immersed in oil, correspondingly higher numerical apertures are obtainable and at the same time better spherical corrections, because of the advantage taken of the aplanatic points of the front lens in the construction of such systems.

The importance of the numerical aperture is also apparent from Fig. 30. As the distance between the eye-piece and objective is practically prescribed, the shorter the focal length of the ocular the smaller the eye-circle and the closer it is to the eye-lens of the ocular. If the diameter of the eye-circle be reduced below 0.5 mm. experience has shown that the effects of diffraction become serious and practically destroy the definition. This fact sets at once a limit to the amount of magnification possible in ordinary microscopes. With a given objective of definite numerical aperture, the highest power ocular which it is possible to use is one which furnishes an eye-point at least 1 mm. in diameter. Of two objectives of the same focal length, but of different numerical apertures, the objective with the higher numerical aperture (larger rear aperture diafram) will furnish the larger eye-circle and thus permit the use of a higher power ocular. When the diameter of the eye-circle becomes much less than 1 mm. shadows from dust particles on the eye and irregularities in the eye-lens become prominent and obstruct clear vision. For satisfactory work it is essential, therefore, that oculars of not too high power be used; the diameter of the eye-circle should not be larger than that of the pupil of the eye and not smaller than about 1 mm.; it should be located at some distance back of the eye-lens of the ocular in order that the eye-lashes may not brush against the ocular.

The diameter of the eye-circle can be readily calculated by the equations given above. The eye-circle $C'''\ D'''$ is the conjugate image of the aperture $C''D''$ (Fig. 30). As the focal length of the objective is small compared with the tube length of the microscope, the distance of aperture $C''D''$ from the lower focal point of the ocular is approximately D of equation (11); accordingly

$$\frac{\frac{1}{2}C''D''}{\frac{1}{2}C'''D'''} = \frac{D}{f_2}$$

But the ratio of half the rear aperture objective diafram to the focal length equals approximately the numerical aperture a or $\frac{\frac{1}{2}C''D''}{f_1} = a$. Therefore

$$\frac{C'''D'''}{2} = \frac{f_1 f_2}{D} \cdot a = f \cdot a = \frac{a}{V} \qquad (20)$$

where V is the magnifying power and f the focal length of the microscope. This equation states that the radius of the Ramsden disk varies directly with the numerical aperture and inversely with the magnification. If the numerical aperture of the objective be known, the total magnification of the microscope can be obtained directly by measuring the diameter of the eye-circle.

As all of the light which reaches the eye must pass through the eye-circle, the illumination for eye-circles less than the pupil of the eye in diameter will vary with the area of the eye-circle or as

$$\left(\frac{C'''D'''}{2}\right)^2 = \frac{a^2}{V^2} = \frac{l^2a^2}{N^2}$$

in which l is the distance of normal vision, a the numerical aperture and N the magnification as defined on page 41. For constant numerical aperture the illumination decreases inversely with the square of the magnification. From the above it is evident that the larger the numerical aperture the brighter the image, the higher the resolving power, and the less danger of shadows from intervening dust particles.

The larger the pupil of the eye the stronger the illumination. If the eye observe a light surface at the end of a darkened tube, the surface appears considerably brighter than when examined without the tube. The tube shields the eye from extraneous light and allows the pupil to dilate, wherefore the importance of a shield or shade in front of the microscope to protect the eye from foreign light and thus to increase the illumination.*

For observations in convergent polarized light, the arrangement of the lenses is indicated in Fig. 32. The in-

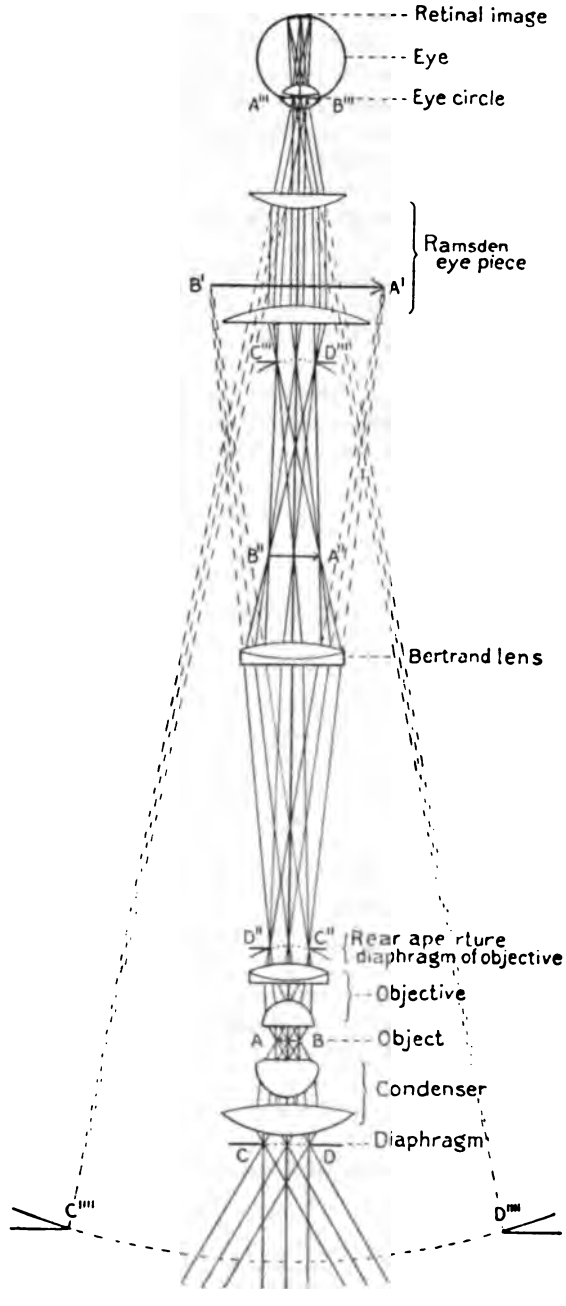


FIG. 32.

*A convenient shield is a piece of velvet 50 × 100 cm. suspended from a wire stretched across the center of the microscope table and about 75 cm. above the table. This shade can be pushed aside when not desired and serves as a cover for the microscope when it is not in use.

terference figure may be observed directly as it is formed in the rear focus of the objective at $C''D''$ or it may be viewed with the Bertrand lens and ocular, which together constitute a magnifying microscope focussed on $C''D''$. The conjugate image $C'''D'''$ is viewed directly through the positive ocular by the eye at $A'''B'''$ and appears in the position $C''''D''''$. The interference figure can also be viewed directly, as it forms in the eye-circle of the ocular in Fig. 31. For the measurement of optic axial angles a scale can be introduced at any one of the conjugate images CD or $C''D''$ or $C'''D'''$ and by its use the angular direction corresponding to any point in the field determined.

In the arrangement of Fig. 32 the image $A'''B'''$ is located at the eye-circle of the ocular and can be seen with the aid of a hand lens at that point. From Figs. 30 and 32, the reciprocal relation between the object or entrance port and the entrance pupil is clearly shown. In case the entrance pupil or one of its conjugate images serves as object, the original object or entrance port serves as entrance pupil, and *vice versa*. The recognition of the functions of the apertural planes in a lens system and the reciprocal relation between such planes and the object or its image planes is of great assistance in tracing the actual path of the rays through the lens system in the microscope. The importance of diaframs to the designer of the optical system in restricting the aperture of the transmitted beams of light and thus improving the quality of the image will not be considered in this paper.

MAGNIFICATION.

In lens systems used for projection (objective magnification) the lateral magnification is directly the ratio of the linear size of the image to that of the object; with a given lens system it is possible to obtain any desired magnification if mere magnification is sought. In instruments used to aid vision (subjective magnification) the eye is an essential part of the optical system and the magnification is determined by the size of the image on the retina rather than by its actual size, the purpose of the lens system of the instrument being to increase the size of the retinal image. The retinal image depends primarily on the angle subtended by the principal rays at the eye-circle of the instrument (the angle u', Fig. 30). This angle of view u' can be expressed in terms of the relative positions of the pupils and images. In Fig. 33 let PM be an object viewed through the lens by the eye O'; let $C'D'$ be the diameter of the pupil of the eye and CD its conjugate image, which is the entrance pupil of the system; let $P'M'$ be the virtual conjugate image of the object PM; F and F' the principal foci of L, and the distances $OM=\xi$, $O'M'=\xi'$, $FM=x$, $F'M'=x'$, $FO=X$, $F'O'=X'$. Then

$$\tan u'=\frac{y'}{\xi'} \text{ or } \frac{\tan u'}{y}=\frac{y'}{y}\cdot\frac{1}{\xi'}=\frac{\beta}{\xi'}$$

On the assumption that the system is free from distortion we find β or the lateral magnification $=\frac{x'}{f'}$; but $\xi'=x'-X'$ and therefore

$$\frac{\tan u'}{y}=\frac{1}{f'}\left(1+\frac{X'}{\xi'}\right)=V. \qquad (21)$$

This expression, the ratio of tangent of the angle of view to the actual size of the object, is a measure of the magnification V as determined by the lens system alone. Since X' is usually very small compared with ξ', the magnifying power of the instrument is measured approximately by the reciprocal of the focal length, $\frac{1}{f'}$.

In case the eye is considered part of the lens system the magnification is defined by the ratio of the apparent size of the object at the distance of distinct vision l (250 mm. or 10 inches) to that of the image at the same distance, or

$$N=\left(\frac{y}{y}\right)_l=\frac{y'}{l}:\frac{y}{l}=\tan u':\frac{y}{l}=\frac{\tan u'}{y}\cdot l=l\cdot V \qquad (22)$$

The distance of distinct vision is considered to be the nearest distance to which the normal eye can accommodate itself and distinguish details without appreciable fatigue; for the normal eye which is at rest when focussed at infinity, this distance l is 250 mm. The magnification is therefore

$$N=\frac{l}{f'}\left(1+\frac{X'}{\xi'}\right) \text{ or approximately } N=\frac{l}{f'}$$

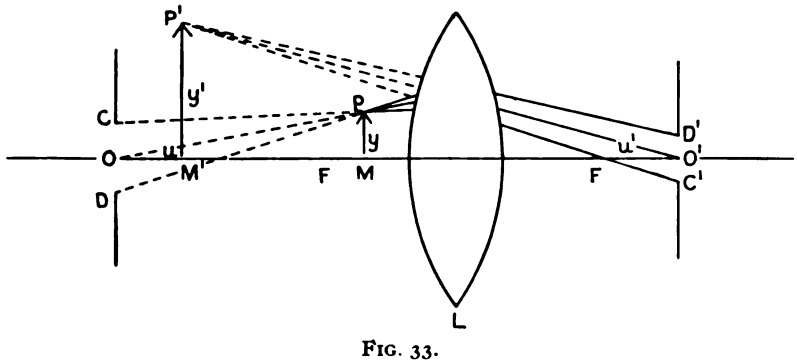


FIG. 33.

An approximate measure for the magnification is accordingly the ratio of the distance of distinct vision to the focal length. In the case of the microscope the focal length f' is determined by the focal length f'_1 of the objective, by that of the ocular f'_2, and by the distance D, the optical interval between upper focal point of the objective and the lower focal point of the ocular as expressed in equation (11) $f'=\frac{f'_1 f'_2}{D}$. The magnification for the distance of distinct vision l is accordingly

$$N=\frac{l}{f'}=\frac{D\cdot l}{f'_1 f'_2} \qquad (23)$$

The first part of this formula, $\frac{D}{f'_1}$, can be considered as the magnification due to the objective of the power $\frac{1}{f'_1}$ for the distance D and the second part, $\frac{l}{f'_2}$, as that due to the ocular acting as a magnifier.

From equation (23) it is evident that the magnification increases with D, the optical tube length. This fact furnishes a convenient method for changing the magnification in low-power objectives; in medium to high-power objectives, however, the corrections are such that the quality of the image is more sensitive to change in the tube-length and suffers noticeably if great changes are made. The slight remnants of spherical aberration resulting from the use of cover-slips of different thicknesses from that for which the objective is corrected may be remedied by altering the tube length. (The mechanical tube length, *i. e.*, the distance from top to bottom of the draw-tube, is altered and with it the optical tube length defined above.)

THE FIELD OF THE MICROSCOPE.

The apparent field of the microscope is determined by the apparent diameter $A'''B'''$ (Fig. 30) of the diafram $A''B''$ in the eye-piece as viewed from the eye-circle. The image thus seen subtends usually an angle of 20° to 36° at the eye-point. As indicated by the diagram, the ocular diafram frames the image from the objective and sharply limits the apparent field. The diameter of the apparent field is measured by use of a camera lucida. The real field, on the other hand, is measured by means of a micrometer on the stage. It varies with the focal length of the objective and with the optical tube length. The greater the magnification of the microscope the smaller the field. The diameter of the field is not affected by the numerical aperture of the objective, as is evident from Fig. 30, and can also be shown by observing the field in a microscope and then reducing the aperture by means of the condenser diafram.

THE PHYSICAL SIGNIFICANCE OF THE NUMERICAL APERTURE.

In the above paragraphs the optical system of the microscope has been treated from the standpoint of geometrical optics. From each point of the object a bundle of light rays has been considered to emanate, each individual ray being supposed capable of separate treatment, irrespective of the others. Such separation and individual treatment of the rays is, however, physically not justifiable beyond a certain limit. The rays are simply the energy flow lines or the paths along which the wave impulse is transmitted. In case the object to be imaged is not self-luminous, but is lighted by transmitted light as in the microscope, the light on emerging from the object suffers diffraction (Fig. 34); a diffraction pattern is formed in the rear focal plane of the objective, and the image in turn is derived from this diffraction pattern. Abbe was the first to recognize the secondary character of the image observed in the microscope and its dependence on diffraction. He proved that not only is the degree of similarity of the image to the object directly proportional to the number of diffraction spectra entering the objective, but that the limit of resolution is reached when the first diffraction spectrum no longer enters the objective. Different objects produce different diffraction patterns in the focal plane of the objective; but if the aperture of the objective be so small that only the central element of the diffraction pattern of each object is transmitted, the result is an equally illuminated field

incapable of producing details in the secondary image plane.* The exact mathematical analysis of these diffraction phenomena is exceedingly complex and beyond the scope of the present paper. It was first given in detail by Abbe,† to whom we are also indebted for the following approximate solution of the problem.

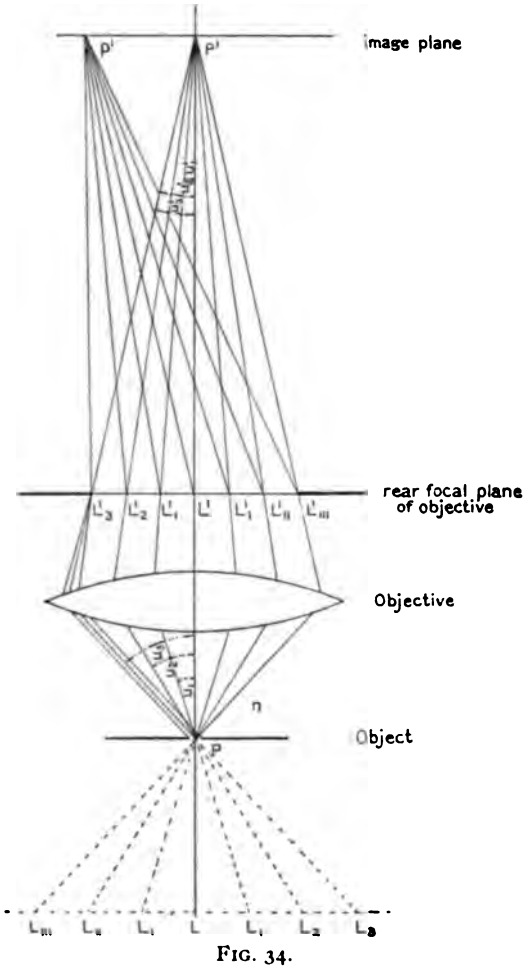


FIG. 34.

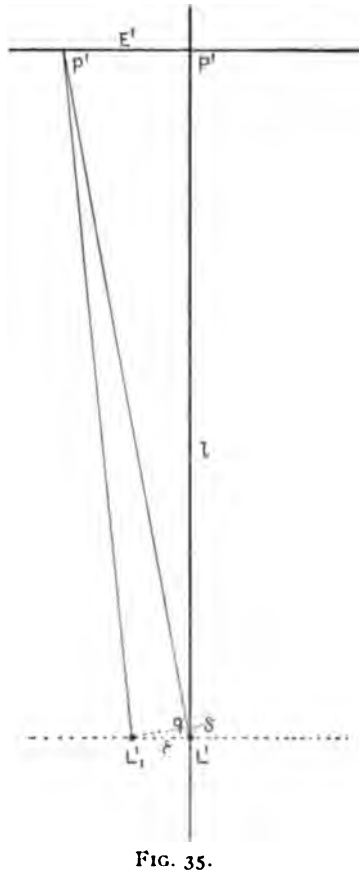


FIG. 35.

In microscope objectives the focal length is relatively so short that the source of illumination may be considered a point infinitely distant. In this case the light emerging from the object suffers diffraction, as indicated in Fig. 34. To take the simplest case, let the object consist of a series of equidistant light and dark lines (a diffraction grating) and let the distance between the center of any two successive light bands be e. Let λ be the wave length of light used. If the refractive index of the medium above P (Fig. 34) be n, then the angle u_m, which the direction of the m^{th} diffrac-

*Failure to appreciate the secondary character of the image formed in the microscope and to distinguish between its mode of formation and that of a primary image (*e. g.*, the image produced in the telescope from distant luminous points or stars) has led to much of the criticism which has been made of the Abbe theory of image formation in the microscope.

†Die Lehre von der Bildentstehung im Mikroskop, von E. Abbe, bearbeitet von Otto Lummer und Fritz Reiche, 1910.

tion maximum includes with axis LP, is readily found from the standard equation

$$\sin u_m = \frac{m\lambda}{n \cdot e} \tag{1}$$

the derivation of which is given in text-books on optics. As the interval e of the grating is small, the emergent diffraction beams are practically parallel and the effect of the objective is to bring them to focus in its rear focal plane, as indicated in Fig. 34. At L'_1, L'_2, etc., the light appears to come from the conjugate distant points L_1, L_2, etc., which apparently illuminate the point P. The points L'_1, L'_2, etc., serve in turn as radiant points from which the secondary points at P', p' in the image plane are derived.

Assuming now the objective to be aplanatic, the sine condition (page 28) is valid

$$\frac{n' \sin u'}{n \sin u} = \frac{1}{\beta} \tag{2}$$

where u is the slope angle of any of the diffracted beams in the object space and u' the slope angle of the resultant beam in the image space; β is the linear magnification. In the microscope the angle u' is always small and may be used in place of sin u' without appreciable error; the refractive index n' of the image space is unity. Equation (2) may accordingly be written

$$u' = \frac{n \sin u}{\beta} = \frac{a}{\beta} \tag{2a}$$

where $a = n \sin u$, the numerical aperture. In Fig. 34 let $P'L' = l$ and the distance $L'L'_1 = \epsilon_1$, $L'L'_2 = \epsilon_2$, etc. Then

$$\epsilon_m = lu'_m \tag{3}$$

From (2a) and (3) we find

$$\epsilon_m = \frac{ln \sin u_m}{\beta}$$

But from equations (3) page 15, and (9) page 18, $\beta = \frac{y'}{y} = \frac{x'}{f'}$, where f' is the rear E.F. of the objective and $x' = P'L = l$ or the distance of the image point P' from the focal point L'. Accordingly,

$$\epsilon_m = f' . n . \sin u_m \tag{4}$$

On combining equations (1) and (4) we have

$$\epsilon_m = \frac{mf'\lambda}{e} \tag{5}$$

The interference fringes L', L'_1, L'_2, etc., are accordingly equidistant, the distance increasing directly with f' and λ and inversely with e. As the points L, L_1, L_2, etc., are derived from the single luminous point L, the wave impulses which emanate from them are coherent and capable of interference. This would not be the case were they independent radiant points or non-coherent, in which case no image at P' could be formed. The different wave impulses from the different points L', L'_1, L'_2, impinge on the different points of the image plane and by their mutual interference produce a diffraction pattern. To show this more clearly, let L' (in Fig. 35) be the central

diffraction zone, L'_1 the first zone to the left of the axis; then $L'L'_1 = \epsilon$; to find the path difference δ at an image point p' of the two beams $L'p'$ and L'_1p' let $P'L'$ be so large in comparison to $L'L'_1$ that the beams L'_1P' and $L'P'$ are practically parallel, and $p'L'$ is practically equal to $P'L'$ or l. Then the path difference $\delta = L'q$; and the proportion is valid

$$\frac{P'p'}{P'L'} = \frac{L'q}{L'_1q}$$

or approximately

$$\frac{E'}{l} = \frac{\delta}{\epsilon} \tag{6}$$

p' will accordingly show a maximum or minimum of intensity when δ is an even or uneven multiple of $\frac{\lambda}{2}$, and the image will consist of a series of dark and light fringes and thus be similar in a measure to the object. If the distance between any two successive maxima in the fringes be e', then from (6) we have

$$e' = \frac{l\lambda}{\epsilon}$$

an equation which, combined with (5), reduces to

$$e' = le/f_1' = N.e \tag{7}$$

The distance e' between the successive interference fringes in the image is accordingly N times that of the grating interval e. This distance is, moreover, independent of the wave-length of light used, and the diffraction pattern developed in the image plane from the colored diffraction spectra due to the object is colorless. This diffraction pattern is the only image formed of the object. In case some of the radiant points, as L'_1, L_3', etc. are cut out by a suitable stop, the interval becomes $L'L'_2 = 2\epsilon$ and accordingly $e' = N\frac{e}{2}$. The same effect can obviously be obtained by suppressing the points L', L'_2, L'_4 . . . and allowing L'_1, L'_3, etc., to act. If $L'_1L'_2$, $L'_4L'_5$, etc., be stopped, then the interval becomes $3e$ and $e' = Ne/3$. The similarity of the diffraction pattern in the image is accordingly directly dependent on the number of diffracted points L', L'_1, L'_2 . . . , which contribute to the formation of the image. In case the aperture of the objectives be so small that only the central element L' is transmitted, no diffraction pattern is possible in the image and no detail will be revealed. One diffraction maximum at least (corresponding to L'_1) must pass the objective if resolution is to be attained. Accordingly, from equation (1)

$$e = \frac{\lambda}{n \sin u} = \frac{\lambda}{a} \tag{8}$$

where e is the distance between the two object points to be resolved, λ the wave-length of light used, and a the numerical aperture of the objective. The distance e varies accordingly with the wave-length and inversely with the numerical aperture. Having given the wave-length of light, the numerical aperture is therefore a measure of the resolving power of any given objective.

By using oblique light, it is often possible to include in an objective the first diffracted beam (as indicated in Fig. 36, where A is the source of light and PL'_1 the first diffracted beam which would be transmitted were the incident beam the pencil LP) and thus to render visible details half as large as that indicated by equation (8); wherefore the value of a condenser of large aperture, equal at least to that of the objective. The smallest possible detail resolvable is accordingly

$$e=\frac{\lambda}{2a} \tag{9}$$

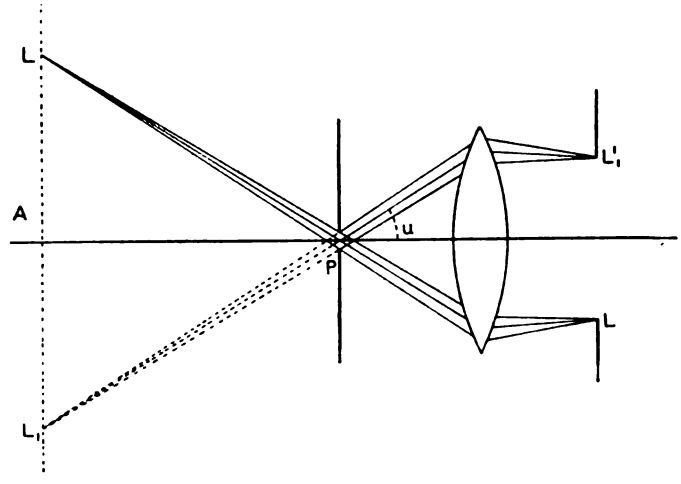


FIG. 36.

Any increase in the numerical aperture of an objective enhances its resolving power and also the brightness of the image. The numerical aperture ($n \sin u$) may be increased either by increasing u or by raising the refractive index of the medium between the objective and the object. Thus if with a dry objective of numerical aperture 0.85 ($\sin u = 0.85$; $u = 58°13'$) an immersion liquid (cedar oil) of refractive index 1.51 be used, the numerical aperture of the combination is no longer 0.85 but n times 0.85 = 1.28 approximately. As noted above (page 37), the relative brightness of an image is directly proportional to the square of the numerical aperture of the objective; the image observed through the immersion liquid appears therefore $\frac{1.28^2}{0.85^2} = 2.28$ as bright as when observed without the liquid, even with the same magnification. With the dry objective the smallest resolvable detail in sodium light of wave-length 0.000589 mm. is

$$e=\frac{\lambda}{2a}=\frac{0.000589}{2\cdot 0.85}=0.00035 \text{ mm.}$$

while with the oil immersion

$$e=\frac{\lambda}{2n \sin u}=\frac{0.000589}{2\cdot 1.51\cdot 0.85}=0.00025 \text{ mm.}$$

USEFUL AND EMPTY MAGNIFICATION IN THE MICROSCOPE.

Experience has shown that the normal eye is able to resolve details in an object which are separated by slightly less than $1'$ of arc. If, therefore, the angular distance between two points in the image is $2'$ or more, they are clearly distinguishable; if this distance be much less than $2'$ they do not appear distinctly separate and the limit of useful efficiency of the microscope has been reached. The angular size δ of the image details depends directly on the magnification by the microscope. If an object e be viewed through the microscope, its apparent size δ is then

$$\delta = e.V = \frac{e.N}{l} \tag{10}$$

where V is the magnifying power of the instrument and N the magnification of the instrument for the distance of normal vision l as defined on page 41. Combining equations (9) and (10) we find

$$\delta = \frac{V\lambda}{2a} = \frac{N\lambda}{2al} \tag{11}$$

or

$$V = \frac{2a\delta}{\lambda} \text{ and } N = \frac{2al\delta}{\lambda} \tag{12}$$

From equation (12) it is evident that the limit of normal magnification for the microscope varies with the numerical aperture a of the objective and inversely with the wave-length of light used. In table 1* the size e of the details distinguishable for the different numerical apertures and the limits of useful magnification are listed on the assumption that $\delta = 2'$ and $\lambda = 0.00055$ mm. $= 0.55\mu$; f' is the equivalent focal length (E. F.) of the microscope.

TABLE 1.

$a = n \sin u$	e	N	f'
	μ		$mm.$
0.10	2.75	53	4.72
0.30	0.92	159	1.58
0.60	0.46	317	0.79
0.90	0.31	476	0.52
1.20	0.23	635	0.39
1.40	0.19	741	0 34
1.60	0.17	847	0.30

The higher the magnifying power used, therefore, the greater must be the numerical aperture of the objective for satisfactory definition. Low-power objectives have low numerical apertures, but high-power objectives must have correspondingly higher numerical apertures. As a rule it may be stated that with each increase of 50 in magnifying power the numerical aperture of the objective should be increased about 0.1.

As indicated on page 41, equation (23), the magnification of the microscope for the distance of distinct vision may be considered to be due to two

*S. Czapski, Theorie d. Opt. Inst., 2d ed., 355. 1904.

factors: (1) The magnification due to the objective, which produces an image of the object and (2) that resulting from the eye-piece, which functions as a magnifier through which the image is examined by the eye. If the eye-piece magnification be too low, the eye is unable to perceive all the details, which are then so minute as to be beyond the limit of visibility, and the resolving power of the objective is rendered worthless because of the insufficient magnification. The imaging or resolving power of the objective, on the other hand, depends on its numerical aperture, and a reduction in its E. F. (increase in magnifying power) without corresponding increase in its numerical aperture does not reveal any new detail in the image. The best magnification is obviously the lowest magnification which presents to the eye all the details imaged by the objective. This is called the *useful magnification*. Any increase over this magnification is useless, as there are no further details in the image to be resolved. With greater magnification the image increases in size but becomes fainter and less distinct; the field and illumination are reduced and diffraction phenomena disturb the image seriously. Mere magnification can not be carried on indefinitely, as with increase in magnification the residual traces of aberrations, which can not be removed completely, become prominent and destroy the sharpness of outline in the image. Apochromats which are more highly corrected will bear higher magnification than achromats. Magnification beyond the useful magnification is called *empty magnification*. With useful magnification an objective attains its highest efficiency; all the details in the image are seen; and the illumination, the size of field, and the depth of sharpness are at a maximum.

THE OPTICAL ELEMENTS OF THE MICROSCOPE LENS SYSTEM.

The primary purpose of the lens system is to present to the eye an enlarged image of an object and to render details visible which would otherwise not be detected. If the lens system were perfect the image would be similar to the object in every respect; all light-waves emerging from any given point in the object and entering the system would converge to the corresponding point in the image, a condition which would be fulfilled for any and all colors of visible light; the image points would be either strictly points or areas so small as to appear to the eye to be points; and the image would be a true representation of the original. These conditions involve not only correction for the spherical aberrations, for the chromatic aberrations, and the chromatic differences in spherical aberration, but require at the same time that the angular aperture of the ray pencil emerging from each point in the object be large, in order that the resolution and illumination be satisfactory. If only low-angular apertures were to be used, the effects of diffraction with high powers would be so serious as practically to destroy the definition in the image. To meet all these requirements simultaneously is unfortunately a physical impossibility, since they are in part mutually exclusive; in the actual construction, a compromise is therefore made and those aberrations are decreased which are the most serious under the stipulated conditions of observation.

It is the task of the optician to produce with the means at his disposal (1) glasses and minerals of different refractive indices and dispersion, all definitely known, (2) differences in the shapes of the component spherical lenses, (3) their distances apart—a lens system which shall meet these requirements in the best manner possible with the serious aberrations reduced to within negligible limits, so that the circles of confusion or "blur circles" in the image from all causes combined subtend an angle of less than 2′ at the eye of the observer and appear, in consequence, as points. In designing the different parts of the microscope care is taken to assign to each particular part that portion of the burden it is best adapted to carry.

THE OBJECTIVE.

In the optical system of the microscope the major corrections are effected in the objective itself; for satisfactory work it is essential that the lens system of the objective be corrected spherically and satisfy the sine condition, *i.e.*, be aplanatic; also that it be corrected for chromatic aberration and the chromatic difference of spherical aberration. The object field in the microscope is limited in practice to small areas near the axis and the aberrations (as astigmatism, curvature of field, and distortion, which result from oblique rays from points far removed from the axis) are practically negligible in the construction of high-power objectives, which are corrected primarily for rays entering under wide aperture from points on or very near the axis.

Objectives are classified, according to the degree of their correction for aberrations, as *achromats*, *semi-apochromats*, and *apochromats*. In the ordinary *achromatic objectives* the spherical corrections are made chiefly for the yellow-green, which, visually, is the brightest part of the spectrum. In such

objectives the spherical aberration is usually undercorrected for the red and overcorrected for the blue rays; they exhibit, in other words, chromatic undercorrection for the axial zone and overcorrection for the marginal zone. The equivalent focal length of the objective varies with the color, being shortest for the yellow-green and increasing for both ends of the spectrum, thus giving rise to the "*secondary spectrum.*" *Semi-apochromatic objectives* are spherically corrected for two colors; the chromatic difference of spherical aberration is eliminated, so that there is practically even chromatic correction for the whole aperture of the objective. Semi-apochromats do not differ, however, from achromats in respect to the secondary spectrum. In *apochromatic objectives* the secondary spectrum is absent. They are aplanatic for two colors (*i. e.*, are spherically corrected and fulfill the sine condition for two colors). The chromatic difference in magnification (the E. F. of the objective is shorter for blue rays than for red) which remains in the apochromat, as in all objectives with non-achromatic front lens, can be eliminated by the use of compensating eye-pieces which work well with high-aperture achromats and semi-apochromats, but to best advantage with apochromats where the chromatic difference of magnification is the same for all zones of the objective. Low-power achromats show practically no chromatic difference of magnification and exhibit, in consequence, colors near the margin of the field when used with overcorrected compensating eye-pieces. In low-power apochromats, which might also be made free from chromatic difference of magnification, the amount of this aberration which is present in the high-power objectives is introduced purposely in order that the compensating eye-pieces can be used with them as well as with the high-power objectives. In the apochromatic objectives, which were first devised by Abbe, practically all the corrections are carried to greater refinement than in the ordinary achromatic objectives, especially for the central part of the field. The definition which is the result of the designer's close figuring and the operator's workmanship is consequently better in an apochromat than in any other objective.

In very low power objectives, the object field is proportionately larger and the aberrations (astigmatism, etc.) due to oblique rays from points distant from the axis become more prominent and have to be taken into account by the lens designer.

Objectives are also classified as dry and immersion object glasses respectively, according as air or a liquid is to serve as medium between the cover-glass and the front lens of the mount. If the liquid and front objective lens are of the same refractive index the immersion is said to be homogeneous. The immersion lenses, although not often used in petrographic microscopic work, are optically superior to the dry series of objectives because of their higher numerical aperture, better optical corrections, longer working distance, and freedom from aberrations due to variability in thickness of cover-glass and freedom from loss of light by reflection at the surface of the cover-slip and the front lens of objective. Notwithstanding these advantages the oil-immersion lenses are rarely used in petrographic work, because of the relatively low magnifications there required.*

*It may be noted in passing that xylol or benzene and not alcohol should be used in cleaning the liquid from an immersion objective.

The following summary of the corrections required in the different forms of objectives is given by Spitta* in his recent book on microscopy, in which the subject of the optical system of the microscope is treated in an unusually clear and non-technical manner:

"(*a*) An ordinary low-power achromatic must be corrected for—
(1) Primary spherical aberration.
(2) Elimination of coma.
(3) Primary color.
(*b*) An ordinary high-power achromatic, in addition, must be approximately corrected for—
(4) Secondary spherical aberration (spherical zones) for the preferred color.
(*c*) Semi-apochromatics, besides fulfilling the above four conditions in a highly perfect manner, must be made free from—
(5) Primary spherical aberration for a second color (and approximately for all colors); and
(6) Should be computed so as to give equal magnifications for all colors when used with compensating eye-pieces; while
(*d*) The full apochromatic must further show—
(7) Freedom from the secondary spectrum."

On high-power dry objectives correction collars are used to correct for variability in thickness of the cover-glass. The use of such a correction collar requires some practice, as the correct thickness is judged directly by the appearance of the image. To facilitate such tests Abbe devised a test plate which consists of a thin silver grating mounted between an object-glass and a wedge-shaped cover-slip, the thickness of which for different points is indicated by a scale reading to hundredths of a millimeter. In making the test the silver strips are brought to focus and the images in the central part of the field of view examined as to their quality and the effect of central and oblique illumination on their definition. If the objective be properly adjusted to cover-glass thickness and be correctly constructed, the edges of the silver strips should appear perfectly sharp without hazy outline, both with central and with oblique illumination. If this be not the case the spherical correction is more or less imperfect. The degree of chromatic correction is estimated by the character of the color fringes produced by oblique illumination. Good achromatic objectives should display in the center of the field only narrow bands in the complementary colors of the secondary spectrum, yellow-green on one side and violet to rose color on the other. With apochromatic objectives these colors should not be present. After some practice the eye recognizes at once the defects produced in the image by improper thickness of cover-glass; these are then to be eliminated either by use of the correction collar of the objective or by lengthening or shortening the draw-tube of the microscope.

The numerical aperture can be determined by a number of different methods. A simple and accurate method is by means of the Abbe apertometer, which is a thick polished glass semicircle terminated on its straight-edge side by a surface at 45° to the normal, which reflects light rays, entering along the circular rim of the disk up into the microscope. By means of

*E. J. Spitta, Microscopy, London, 1909.

sliding stops and a scale engraved on the disk, a ray of any desired angle of inclination can be had emerging from the glass disk into air or into a refractive liquid and thus the aperture of the objective ascertained.

Objectives may also be tested by use of test objects, as microscopic diatoms or mounted bacilli, but for petrographic work the high magnifications necessary for the examination of such preparations are not required and need not be considered further here.

THE OCULAR.

The ocular, which functions primarily as a magnifier, is corrected as such for the aberrations of narrow pencils under great inclination to the axis, astigmatism, curvature of field, distortion and chromatic differences of magnification. In the apochromatic systems the compensating oculars are made to bear part of the burden in the chromatic aberration correction. Two types of oculars are used in petrographic microscopic work: (1) The ordinary Huygens or negative type, whose field lens intercepts the rays before they come to focus in the image (Fig. 30) and produces a reduced image in the focal plane of the eye lens, thus increasing the size of the field. This type is used in ordinary microscopic work. (2) For micrometer oculars the positive or Ramsden ocular (Fig. 32) is preferable because its lower focal point, and consequently the micrometer scale, fall outside of the lens combination. Changes in magnification and micrometer value are therefore proportional to changes in tube length, which is not the case with the Huygens eye-piece. The Ramsden ocular in its original form is impracticable and the Ramsden eye-pieces commonly used for micrometer eye-pieces are only approximations to the original. They are not achromatized as well as the Huygens eye-piece, and show color fringes opposite to those exhibited by compensating oculars. A practical positive eye-piece can only be made satisfactorily achromatic by the use of at least one cemented doublet in the combination.

In both oculars the cross-hairs or micrometer scale are usually so placed that the emergent rays are parallel. Under these conditions the normal eye is focussed for distant objects and can be used with the least fatigue. The drawings, Figs. 30 and 32, represent the object at the distance of normal distinct vision, but for practical work with the normal eye these images should be infinitely distant if the eye is to experience the least fatigue in making long series of observations.

THE CONDENSER.

The condenser is designed primarily to furnish a wide cone of incident light, its purpose being to send light through the object under the largest angle which may be intercepted by the objective. In critical work with high powers it is necessary to focus the pencil of light accurately on the object point (critical illumination) and the condenser is corrected with this end in view. In work with interference figures it is also important that the condenser system be aplanatic as well as achromatic. The rays should all pass through the same object point if they are to be properly focussed in the rear focal plane of the objective, where the interference figure is formed.

The numerical aperture of the condenser should be high. If it be too high for the objective in use and the field is flooded with light, the aperture can be decreased and with it the intensity of illumination diminished, either by closing the substage diafram (Fig. 37, *b*) or by lowering the entire condenser (Fig. 37, *a*).

In a thin section illuminated by a narrow beam, the differences in refractive index of the minerals present are emphasized, the rims of total reflection are wide and pronounced, and much of the section appears to stand out in relief. The Becke lines are sharply defined. The best outline pictures are obtained by the use of dark-ground illumination, but this method of illumination will not be discussed here, as it has not been applied to any extent in petrographic microscopic work. In several of the methods

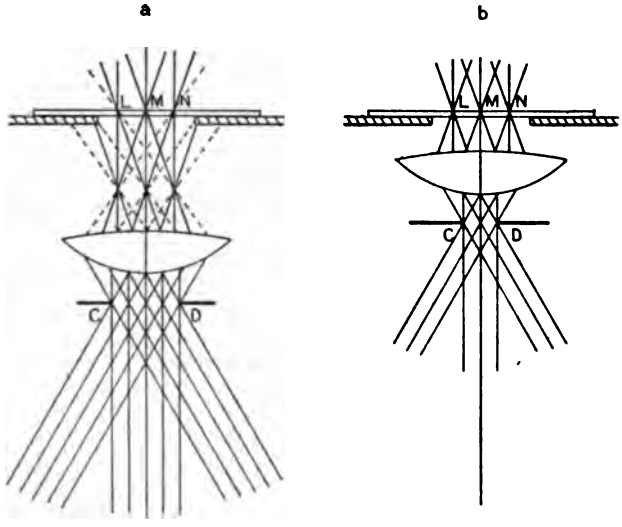


FIG. 37.

noted below oblique illumination is employed with low powers and is obtained most readily by placing the finger between the reflector and the condenser and cutting off part of the light (Fig. 38.). With this arrangement a shadow is cast over half the field but the light beams which emerge are all confined to one side of the axis. In critical work with high powers this method is less satisfactory and is replaced by the methods of dark-ground illumination noted above.

There are still certain factors in the petrographic microscope which tend to decrease the quality of the image obtained by the corrected optical system and which are rarely taken into account by the practical optician. The presence of the analyzer in the path of the rays not only increases the optical length of the tube, but seriously disturbs the quality of the image. Pronounced distortion results, even when the best type of analyzer is used (Thompson prism cemented with thickened linseed oil). In ordinary work, where the stage is rotated, this distortion is not detected by the eye, but if the analyzer be rotated while the stage remains stationary, the different

quadrants in the image move perceptibly—the movement in the one quadrant being away from the center while in the adjacent quadrants it is toward the center. On further rotation of the analyzer these directions of motion are reversed, the effect in each quadrant, during a complete rotation of the nicols, being a pendulum-like movement, the points swinging out as the principal plane of the analyzer passes from a position normal to the diagonal of the quadrant to one parallel with it, and swinging in as it returns to its original position. This periodic contraction and expansion of the different quadrants in the image field means a difference in objective magnification for the two principal positions of the analyzer, as is indicated by the series of measurements of Table 2. These were made with the microscope (Plate 1, Fig. 3) by using a 0.1 mm. micrometer scale as object and a similar 0.1 mm. scale in the lower focal plane of a Ramsden ocular as micrometer scale

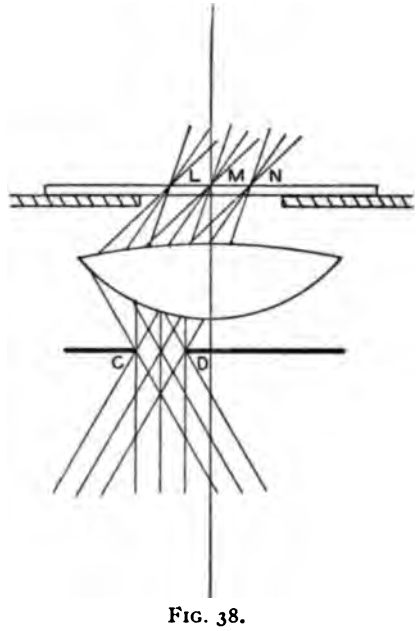


FIG. 38.

for the image. Tests were made with three different objectives (Fuess No. O and Zeiss apochromats 16 mm. and 4 mm. respectively), and with two different analyzers (Thompson type, the one 27 mm. long and 10 mm. wide; the other 19 mm. long and 7 mm. wide). After careful adjustment of the microscope, the micrometer scales were placed parallel with the vertical cross-hair and centered; and readings taken, on the upper micrometer scale, of the number of divisions from the center of the field to a given division in the image of the lower object scale. Similar readings were then taken on the same points after the analyzer had been rotated through 90°, in which position its principal section coincided with the graduated line of the micrometer scale. Thus with the large analyzer and the Zeiss apochromat 16 mm., the distance from the center of the field to the 4th division of the object micrometer scale measured 47.75 divisions on the eye-piece microm-

eter; after rotating the analyzer to its second position the distance between the same two points measured 48.9 divisions, which means an increase in magnification of 2.4 per cent. From Table 2 it is evident that the distortion increases (1) with the distance from the center of the field; (2) with the size of the analyzer, and (3) with the focal length of the objective used, low-power objectives suffering most from this defect. With large analyzers, the general disturbance of the optical correction of the lens system is so serious that only a small part of the field can be brought to sharp focus at a time. It is advisable, therefore, to use small, accurately adjusted analyzing prisms in the microscope. Some distortion is always present, but under these conditions it is reduced to a minimum.

TABLE 2.

Objective.	Distance of object point from center in image in terms of image divisions.	Distance between center and object point as measured by micrometer eye-piece.		Percentage increase in magnification.
		Principal plane of analyzer parallel with micrometer line.	Principal plane of analyzer normal to micrometer line.	
Large analyzer:				
Fuess O	11	50.4	52.	3.18
	6	27.6	28.3	2.44
Zeiss Apochromat 16	4	47.75	48.9	2.40
	2	23.9	24.45	2.30
Zeiss Apochromat 4	1	47.	47.75	1.60
Small analyzer:				
Fuess O	11	52.	53.	1.92
	6	28.3	28.8	1.77
Zeiss Apochromat 16	4	48.75	49.65	1.85
	2	24.37	24.75	1.56
Zeiss Apochromat 4	1	48.25	48.9	1.35

The distortion is evidently due to two factors: (1) Extraordinary light waves contained in the plane normal to the principal section of the nicol (in the Thompson type of prism the optic axis is parallel with the line of intersection of the end surface with the oblique plane along which the prism is cut) have the same refractive index ϵ for all angles of inclination; while for extraordinary waves transmitted in the principal section of the analyzer, the refractive index increases with the angle of incidence. For small angles of incidence, the difference in angles of refraction for waves incident at the same angle i, but in azimuths 90° apart, is not great (for $i = 5°$, $r = 3°21'42''.6$ (azimuth 90° from principal analyzer plane), $r' = 3°21'38''.5$ (azimuth 0° from principal analyzer plane); similarly, for $i = 10°$, $r = 6°42'34''.4$, $r' = 6°42'1''.8$, for $i = 15°$, $r = 10°1'53''.5$, $r' = 9°59'55''.1$) but it is perceptible, and increasingly so as the margin of the field is approached (i increases) and as the length of the prism increases. The second factor is the cementing film in the prism which has not precisely the same refractive index ϵ or ϵ' as the calcite, and produces, therefore, in the transmitted light beams, a slight displacement which becomes more serious the larger the angle of incidence.

Object points in a plane parallel with the principal plane of the analyzer will appear, in consequence, nearer together in the image than corresponding, equally distant object points in the plane normal to the principal section of the analyzer—a conclusion which is corroborated by the measurements of Table 2 above.

It would be better, for many reasons, to use only a short cap nicol above the ocular, but this arrangement cuts off part of the field and quickly tires the eye. There seems to be no satisfactory method at present for overcoming this defect and the most convenient place for the analyzer is in the draw-tube above the objective. The lengthening of the optical path by the upper nicol is often counteracted by the insertion of a weakly magnifying lens just above the nicol; this is, however, ordinarily a defect, since it only compensates approximately and does not restore the quality of the optical image in proportion; unless centered carefully it causes the field to shift and this is a far more disturbing feature than a slight change in focus, which is immediately and almost unconsciously adjusted for by means of the fine focussing screw. The same defects appear on the insertion of a quartz wedge or plate just below the analyzer, but these can be obviated by inserting the wedge or plate below the substage condenser lens and just above the polarizer, as is the case in some English microscopes.

THE NICOL PRISMS.

The Nicol prisms of petrographic microscopes are selected with reference to their optical quality and their size. The optical quality of a prism depends on several factors: (1) the character of the materials of which it is made; (2) its design; (3) the accuracy of its construction. The Iceland spar used in the construction should be clear and free from all imperfections. Many designs have been suggested for Nicol prisms, and if cost be not considered that design is best which polarizes the transmitted light most perfectly and gives uniformly illuminated field with minimum loss of light and whose angular aperture is the largest. Plane polarized light-waves emerging from inclined surfaces (even of isotropic substances) suffer a slight rotation of their plane of polarization, and for this reason a Nicol prism with oblique terminal faces polarizes the transmitted waves less perfectly than the square-end Thompson or Ahrens or Glan type of prism. For accurate work the square-end type of prism (as the Glan or Thompson) is therefore superior to the Nicol prism ordinarily furnished in microscopes. The width of the polarizer should be such that it does not act as a diafram and decrease the aperture of the condenser. In many of the best modern microscopes the polarizer is too narrow and cuts down the available aperture of the condenser to such an extent that in the observation of interference figures only a narrow field is available. The disturbing diafram effect of the polarizing prism can often be counteracted by lowering the condenser lens, but only to a certain extent. If a condenser system of large aperture be used it is essential that the width of the polarizer be such that it does not limit too seriously the aperture of the condenser. The analyzer should also be of the square-end type and of sufficient size so that it does not cut out the relatively narrow beam of light from the objective; it should not be too large, however, as the effects of distortion due to its action increase with its length.

THE PETROGRAPHIC MICROSCOPE AS AN ACCURATE MEASURING DEVICE.

In the preceding pages attention has been directed to the importance of the optical system of the microscope and to some of the details which the optician has to observe in designing and constructing his system. The second function of the petrographic microscope, that of an optical measuring device, introduces a number of new factors into its construction which have to be considered and which vary to some extent with the purpose which the microscope is to serve. The importance of accurate construction and of adjustment is obvious. In quantitative work it is essential that all instrumental errors be reduced to a minimum and that their effect on the final result be definitely known. The demand for accurate numerical data in modern petrography is increasing and is the natural outcome of the growth of the science. Notwithstanding this tendency the quantitative element is still absent in many petrographic descriptions and the reason for it is evidently to be sought in the cumbersome methods and appliances now available for the determination of the optical constants of minerals under the microscope. The observer has not the time to carry out such measurements properly and in consequence does not attempt them at all. If, however, the appliances could be simplified so that the different measurements could be made easily and rapidly and the correction factors eradicated these objections would be removed and petrography might profit accordingly.

In the microscope pictured in Plate 2, Fig. 1, the attempt has been made to devise an instrument which is simple and yet with which most of the optical constants can be determined directly and accurately without the use of complicated accessory apparatus. The microscope of Plate 1, Fig. 3, is intended for precise work and contains several features which have not been included in that of Plate 2, Fig. 1, and this in turn has some features not included in Plate 1, Fig. 3. Several of these features are in part new and merit a brief word of description.

(1) For the simultaneous rotation of the nicols a rigid-bar connection has been adopted, which is free from lost motion and the inaccuracies of gear-wheel devices which are ordinarily used for this purpose. The rigid-bar connection between the two nicols was first used by Mr. Allan B. Dick* in 1888. In his arrangement the connection was made between the polarizer and a cap nicol above the ocular and was intended as an inexpensive substitute for the pin and ratchet movement with gear wheels, also first devised by him. The connecting bar was broken at five points, was long, and could hardly furnish results of a high order of accuracy, and was evidently not intended to do so, as the small degree circles without verniers testify.

In 1904 E. Sommerfeldt described a microscope especially adapted for work with high temperatures; in it the two nicols were connected by a rigid bar as in the Dick microscope; the angle of rotation of the polarizer and cap nicol were read off directly on the stage of the microscope. In 1905

*Miner. Mag., 8, 660, 1888: Notes on a new form of polarizing microscope, 1890. Some additional notes on the petrographic microscope. Published by James Swift and Son, London, 1894.

Sommerfeldt* adapted this scheme to the polarizer in the draw-tube and emphasized the advantages of rotating the nicols rather than the stage in the examination of interference figures. In 1910 the present writer,† unaware of Sommerfeldt's paper, described practically the same method for rotating the nicols, giving particular attention in the construction to precision. Recently Souza Brandäo‡ has adapted the same device and apparently also without knowledge of the previous work of Dick and others.

In the arrangement of Plate 2, Fig. 1, the nicol in the draw-tube is rotated and not a cap nicol, thus relieving the difficulty occasioned by the cap nicol in the eye-point of the ocular, which decreases the field of view and tires the eye of the observer; the rigid-bar connection is built moreover of few parts and of heavy, rigid material, in order that the adjustment may not be disturbed on rotation of the nicols by a bending or looseness of the device. The upper nicol can be inserted or withdrawn from the tube at will; also rotated by itself, as can also the lower nicol. In Plate 1, Fig. 3, the lower nicol is not attached to the condenser and can be inserted or withdrawn at will. In Plate 2, Fig. 1, this is not the case and the design is defective in that respect. The angle of rotation of the nicols can be read off either on the degree circle just above the upper nicol or on the circle of the stage. This arrangement does not suffer from lost motion and is especially useful in the examination of fine-grained rock or artificial silicate preparations where accurate centering is not easy, especially if the individual grains are mounted in a liquid and tend to shift their position with the slightest motion of the microscope stage. In the measurement of the optic axial angle by means of the cross-grating ocular the simultaneous rotation of the nicols is necessary. Though this particular arrangement has been devised several different times by different observers without the knowledge of Mr. Dick's arrangement, the credit for first suggesting and first using a rigid-bar connection for the simultaneous rotation of the nicols belongs unquestionably to Mr. Dick, to whom petrologists are indebted for many of the improvements adopted in modern petrographic microscopes.

(2) For the accurate measurement of the birefringence, the optic axial angle and the extinction angle of mineral plates or grains, the upper part of the draw-tube has been modified as indicated in Plate 2, Fig. 1. A permanent attachment or holder has been added, into which different wedges and plates, mounted in metal carriages, can be inserted. A corresponding opening is made in the Huygens ocular to receive such plates and has been so designed that the upper surface of any particular plate practically coincides with the plane of the cross-hairs, so that a scale engraved on the inserted plate can be viewed together with the cross-hairs and without appreciable parallax. The plates and wedges used for this purpose will be described in detail later, together with the methods of their application.

(3) The sensitive-tint plate is introduced at Q (Plate 1, Fig. 3) just below the condenser. It is supported by the carriage F attached to the lower rim of the substage condenser lens support and can be rotated about the axis of the microscope. This arrangement often facilitates the determination of the ellipsoidal axis of a particular section because it allows the observer

*Zeitschr. wissen. Mikrosk., **21**, 181–185, 1904; **22**, 356–362, 1905.
†Amer. Jour. Sci. (4), **29**, 407–426, 1910; see also C. Leiss, Zeitschr. Kryst., **47**, 377, 1909.
‡Zeitschr. Kryst., **49**, 193, 1911.

to pass more readily from the quadrant in which the colors rise to that in which they fall than is the case with the slower-moving stage. The introduction of the sensitive plate below the condenser rather than above the objective is an advantage, since the optical system is not thereby disturbed and the field shifted and rendered less distinct, even to the extent that refocussing is necessary.

(4) The Bertrand lens *E*, Plate 1, Fig. 3, is mounted on a sliding arrangement which, in connection with the sliding ocular tube, permits of different magnifications of the interference figure from 6 to 15 fold. An iris diafram is introduced directly below the Bertrand lens and slides up and down simultaneously with it. To be of service in this connection this diafram should be located precisely in the image plane of the object as formed by the objective, for in that plane alone can light be excluded from adjacent minerals in the thin section, as was first emphasized by S. Czapski.* To accomplish this readily a small lens *L*, Fig. 1, 19 mm. focal length, has been introduced in the microscope (Plate 1, Fig. 3) above the Bertrand lens; in conjunction with the ocular, the lens serves the purpose of bringing to sharp focus the image picture in the plane of the diafram in accord with the principle noted above. In place of the small, auxiliary lens *L*, the writer has heretofore used a lens of long focal length and viewed the diafram and image directly from the top of the tube. The new arrangement is more convenient, however, and obviates the necessity of removing the ocular for the purpose of bringing the image to coincide with the plane of the diafram by raising the microscope tube. The lens *L* swings on an axis and can be instantly thrown out of the field. A small spring with pointer automatically indicates the correct position of the lens when thrown into the field.

From Fig. 32 it appears that, if the diafram were placed in the image plane $A''B''$, light could be excluded from any part of the field more efficiently than with the first arrangement; the image is more perfect and the objective is not disturbed. The objections to this arrangement, however, are serious. The Bertrand lens does not come to exactly the same point, except in very accurate construction, and the same point in the image is in consequence not always brought to focus in the center of the diafram. The image can not be examined directly in the position $A''B''$ by the ocular alone without the aid of an auxiliary lens. The position $A''B''$ obtains only for a fixed position of the Bertrand lens, the objective and the object. As Czapski has shown, the best place for the diafram is in the upper focal plane of the Bertrand lens, provided the system is telecentric; but the disadvantages cited above have proved sufficient to discourage the use of the diafram in this position.

For the examination of interference figures directly as they are formed in the rear focal plane of the objective, the small cap stop (Plate 1, Fig. 1) with two sets of slides (S_1 and S_2) at right angles to each other has been found useful. This cap fits into the microscope tube and is inserted in place of the ocular. By means of the eye-lens *a*, the image is focussed in the plane of the slides and any particle singled out for examination. Because of diffraction phenomena the aperture should not be made less than 0.5 mm. in diameter, but even with this restriction and with ordinary objectives of

*Neues Jahrbuch, Beilage Band, 7, 506, 1891.

3 or 4 mm. focal length, grains 0.01 to 0.02 mm. in diameter and either in the powder or the ordinary thin section, furnish good interference figures which ordinarily would be completely overshadowed and not discernible if the adjacent light were not excluded.

(5) A large substage condenser is used together with a large Thompson or Ahrens prism in place of the usual nicol and condenser with removable upper lens. This arrangement, which was first introduced by Leiss on the Fuess microscope, is a marked improvement over the usual arrangement, as it does away with the more or less complicated devices for removing the upper condenser lens from the optic axis of the microscope.

(6) The objective clamps and supporting rings are made of casehardened steel, and not of brass, which is soft and too easily indented to hold its surfaces true for any length of time.

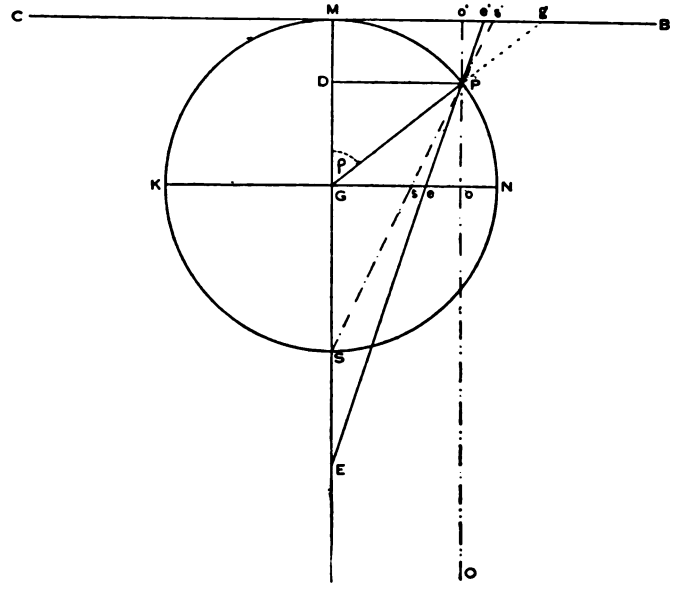


FIG. 39.

(7) The mechanical stage is new in design (Plate 1, Fig. 3), is practically dust proof, has a play of 24 mm., and is mechanically simple in construction. By means of the screws H_1 and H_2, of half millimeter pitch and with graduated heads, movements of 0.01 mm. can be read off directly.

(8) The distance between the stage and the arm of the draw-tube support is sufficient to allow the use of the universal stage.

(9) In this instrument the axis of both the draw-tube and the substage condenser support coincide precisely with that of the rotating stage. Since the ocular and the condenser remain automatically centered with respect to the rotating stage, while the objective changes its position slightly on each insertion, the usual centering screws (a_1 a_2, Plate 1, Fig. 3) for the objective itself have been introduced. The direction of motion of such centering screws should be parallel with the cross-hairs of the ocular, as the

eye estimates coordinate directions much more readily than diagonal directions. The practice of placing the adjustment screws on the rotating stage instead of above the objective is wrong. The part of the optical system which is not in adjustment is the objective and not the stage. The axis of rotation of the stage should form the starting-point for the adjustment of the whole instrument and should always remain fixed in its position. To this axis the ocular, condenser, and objective should be adjusted, and since the ocular and condenser remain practically stationary while the objectives are changed constantly the only logical point of adjustment is above the objective.

(10) Verniers to read the stage circles are attached both to the rotating connection between the nicols and to the body of the microscope; they read directly to 3′ of arc.

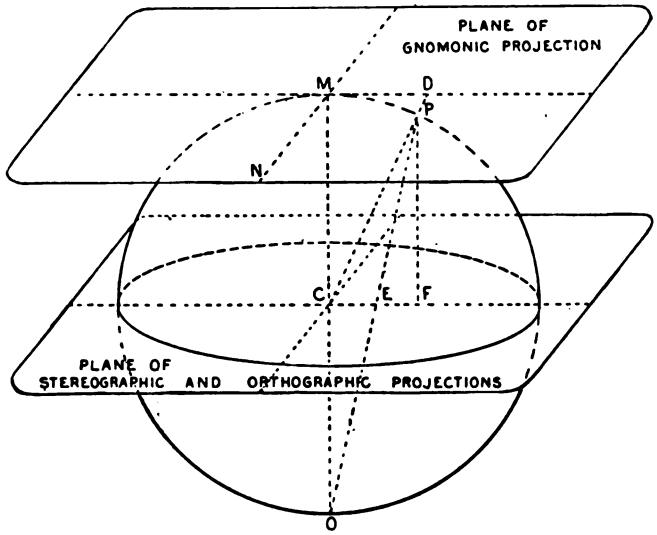


FIG. 40.—In this figure, the point D is the gnomonic projection point of the point P on the sphere or of the direction CP in the crystal. The distance MD is evidently $r \tan \rho$, r being the radius of the sphere and ρ the angle MCD. Similarly, the distance CE in the sterographic projection (Fig. 41) is $r \tan \frac{\rho}{2}$ and CF in the orthographic projection (Fig. 42) is $r \sin \rho$.

THE ADJUSTMENT OF THE PETROGRAPHIC MICROSCOPE.

The properly adjusted petrographic microscope should satisfy the following requirements: (1) The optical system should be centered and its axis should contain the center of rotation of the stage. (2) The nicols should be accurately crossed. (3) The cross-hairs of the ocular should be parallel with the principal planes of the nicols.

(1) The first condition involves (*a*) centering of the objective by the usual method; (*b*) centering of the condenser by observing its image in the eye-point of the ocular. In this adjustment it is assumed that the draw-tube and the substage are in alinement. This can be tested by focussing

the objective on the upper surface of the condenser and noting that the field does not shift appreciably in the different positions. A slight shift is of little consequence in its effect on results of measurement and can, therefore, be neglected. Its correction is not an easy task and is fortunately rarely necessary.

(2) Although many methods are available for the accurate crossing of the nicols, the following method is perhaps the simplest and most accurate: Remove from the microscope all lenses—ocular, objective, and condenser—and point it directly at the sun, whose rays are parallel and so intense that a rotation of less than 1′ of one of the nicols from the position of total extinction is readily discernible. If the lower nicol be of the usual type, with oblique end surfaces, the rotatory effect of these faces on the plane of polarization of transmitted light-waves is such that it is advisable to close the diaframs of both the substage and the draw-tube so that only central

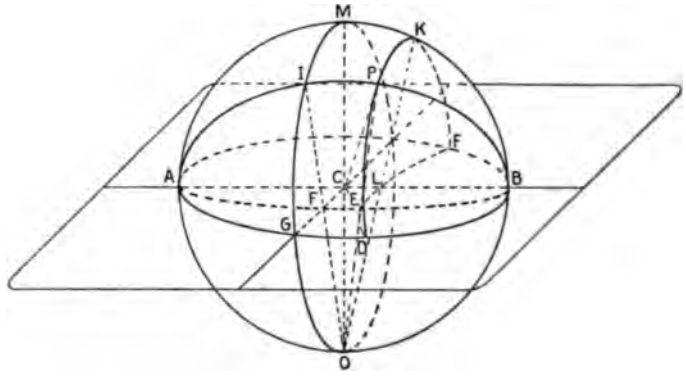


FIG. 41.—In this figure the point *P* of the sphere, corresponding to the direction *CP* within the crystal and located in this instance by the great circle *AIP* and the small circle *DPK*, becomes *E* in the stereographic projection plat and is there located at the intersection of the great circle *AEB*, the stereographic projection at *AIP*, and by the small circle *DEL*, the stereographic projection of *DPK*. *E* is also the point of intersection of the line *OP* with the horizontal diametral plane *CGB*.

rays pass through the prism. The square-end type of prism, either Glan-Thompson or Ahrens, does not rotate the plane of polarization of transmitted light waves to the extent of the ordinary nicol and is therefore to be preferred in accurate work. For satisfactory work it is essential that the analyzer (Thompson prism with square ends) be mounted with its end surfaces normal to the axis of the microscope, otherwise the field is shifted on insertion of the analyzer.

(3) For the adjustment of the cross-hairs of the ocular and the principal planes of the nicols, a mounted cleavage plate of some mineral showing parallel extinction, as anhydrite or anthophyllite or a crystallite of quartz with sharp prism edges, is observed under the microscope fitted with the objective and ocular but not with condenser. The microscope is pointed directly at the sun and the position of total extinction of the plate determined readily to within 1′ of arc. The cross-hairs of the ocular are then adjusted to parallelism with the cleavage edge of the plate in its position of total extinction. For still finer adjustment the rotatory effect of the interven-

ing lens surfaces of the objective on the plane of polarization of the transmitted light-waves can be eliminated by using the Bertrand lens, which together with the ocular forms a low-power microscope for observing the crystal plate. With this arrangement there are no lens surfaces between the two nicols and the disturbing rotatory effects of the oblique glass surfaces on the plane of polarization of transmitted light-waves are not present.

On cloudy days, when the sun is not available, the adjustment of the nicols can be made with a fair degree of accuracy by means of the Bertrand ocular or the bi-quartz wedge plate*, while for the adjustment of the cross-hairs a twinned plate of selenite† or an artificially twinned quartz plate or wedge‡ renders good service.

THE STEREOGRAPHIC, ORTHOGRAPHIC, GNOMONIC, AND ANGLE PROJECTIONS.

In the study of crystal optics, as well as in the description and application of petrographic microscopic methods, the phenomena considered often involve special relations and require the concepts of solid geometry for their solution. In order to represent these adequately on a plane, different projections have been devised and aid the observer materially in forming correct conceptions of the morphological and optical relations in crystals, which are frequently complicated and difficult to describe accurately. Actual models might be used to represent these special phenomena, but usually it is neither convenient nor feasible to make such models and the observer is forced to use some form of projection. In all types of projection the relation of the object to its projection is one of definite construction and is dependent on the method of projection adopted. In each projection the directions (optical or crystallographic) within the crystal are considered to pass through the center of a sphere of unit radius. In space any radius can be represented accurately by its point of intersection with the surface of the unit sphere and, like any point on the earth's surface, its position can be definitely fixed by two angles equivalent to those of latitude and longitude. In the projection, such points on the unit sphere are pictured in a fixed plane and represent definite directions within the crystal.

The kinds of projection§ required in optical work are different from those which serve in ordinary map projections where the effort is made to indicate on a plane surface the relative positions of points and lines on the earth's surface. Such representation can only be approximately correct, as it is geometrically impossible to develop a spherical surface on a plane; and different projections have been devised to meet the different requirements which may arise. Map projections which represent the different areas on the globe in correct relations are called *equal surface* or *equivalent projections*, while those which preserve the angular relations are called *orthomorphic projections*.

*Compare F. E. Wright, Amer. Jour. Sci. (4), **26**, 386, 1908.
†E. Sommerfeldt, Zeitschr. für wissenschaftliche Mikroskopie, **24**, 24–25, 1907.
‡F. E. Wright, Amer. Jour. Sci. (4), **26**, 387, 1908.
§For comparative studies on projection see A. A. C. Germain, Traité des Projections, Paris, 1865; H. F. Gretschel, Lehrbuch der Kartenprojektion, Weimar, 1873; C. A. Schott, Appendix 15, Ann. Report 1880, U. S. Coast and Geodetic Survey; Thomas Craig, A Treatise on Projections, U. S. Coast and Geodetic Survey, 1882; V. Goldschmidt, Ueber Projektion und graphische Kristallberechnung, Berlin, 1887; Ueber Winkelprojektion, Z. Kryst. **30**, 388–402, 1902.

In a third class of projections, the *perspective projections*, the eye of the observer is considered at some definite point in space from which the points on the surface of the sphere are viewed. The points of intersection of the lines of sight with the fixed plane of projection are then the desired projection points. The projection thus obtained is dependent both on the positions of the eye-point and on that of the plane of projection.

In Fig. 39 let BMC be a plane tangent at M to the sphere MPS; let GP be a direction in space which includes an angle ρ with the vertical axis. Then if the eye be at E, the intersection, e', of the line EP with the plane BC is the projection point of GP. From the figure it is evident that

$$DP : Me' = (EG+GD) : (EG+GM)$$

On substituting in this equation $DP = \sin\rho$, $DG = \cos\rho$, $GE = c$, and $Me' = x$, we find

$$\sin\rho : x = (c+\cos\rho) : (c+1)$$

or

$$x = \frac{(1+c)\sin\rho}{c+\cos\rho} \qquad (1)$$

If the plane of projection be the central equatorial plane KGL, then

$$x = \frac{c\sin\rho}{c+\cos\rho} \qquad (1a)$$

Equations (1) and (1*a*) represent the general form of zenithal perspective projections on a horizontal plane and from them the different special types can be readily derived.

If the eye-point be at G ($c=0$; $x=\frac{\sin\rho}{\cos\rho}=\tan\rho$) and the projection plane the horizontal tangent plane (Fig. 40), the projection is the *gnomonic projection*. In this projection all great circles are represented by straight lines and the small vertical circles by hyperbolas. Plate 10 is a *gnomonic meridian projection* (radius of projection sphere = 5 cm.), the interval between the successive great circles (straight lines) and also the small circles (hyperbolas) being 2°. The gnomonic projection is best suited to crystallographic work, since by its use all crystal faces are reduced to points and all zones to straight lines.

If the eye be located at S ($c=1$) (Fig. 39), and the plane of projection is the equator, the projection is the *stereographic**

$$\left(x = \frac{\sin\rho}{1+\cos\rho} = \tan\frac{\rho}{2}\right).$$

(Fig. 41.) The stereographic projection is unique in that all circles, whether great or small, appear in the projection as circles instead of ellipses, as might be supposed at first thought. Moreover, the angle which two great circles make with each other is preserved unaltered in the projection. The projection is thus angle-true. In Plate 3† the portions of great circles of the

*S. L. Penfield, this Journal (4) **11**, 1, 115; E. Fedorow, Zeitschr. Kryst., **26**, **27**, **29**, and G. Wulff, Zeitschr. Kryst., **21**, 249, 1893; **36**, 14–15, 1902, have given complete descriptions of the stereographic projection; also H. E. Boeke, Die Anwendung der stereographischen Projektion bei krystallographischen Untersuchungen, 1911.

†Photolithographic reproduction of the meridian stereographic projection plat by Prof. G. Wulff, Zeitschr. Krystall. **36**, 14, 1902.

upper half of the sphere are represented by the circular arcs, of which the horizontal radius is the limiting case, and the small circles are represented by the arcs of which the vertical radius is the limiting case (radius of projection sphere = 10 cm.)

In the *orthographic* (also called orthogonal or parallel or ocular) projection the eye of the observer is supposed to be at an infinite distance above the plane of projection and to look directly down upon the sphere ($c = \infty$ and $x = \sin \rho$). The lines of sight are then parallel and the points on the sphere are vertically above their projection points on the central diametral plane (Fig. 42 and Plate 4). In this projection great circles appear as ellipses and vertical small circles appear as straight lines. This projection is especially important, since all interference phenomena observed in convergent polarized light under the microscope appear to the eye of the observer as they

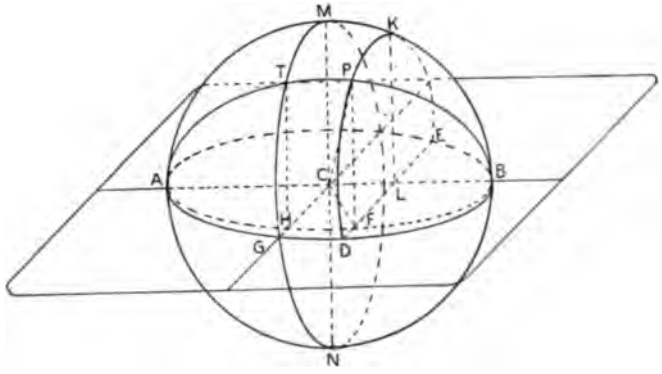


Fig. 42.—In this figure the point P of the sphere, located in this case by the intersection of the great circle ATP and the small circle DPK, becomes in the orthographic projection F and is there located by the ellipse AHF, the orthographic projection of ATP, and the straight line DFL, the projection of DPK. F is also the point of intersection of the diametral plane CGB with the line PF, normal through P to that plane.

would were the actual interference phenomena plotted in this projection. The serious drawback to its general application in optical work lies in the rapid decrease of its sensitiveness to differences in angular distances near its outer margin. In Plate 4 (meridian orthographic projection) the ellipses represent great circles with a common diameter of intersection and drawn at intervals of 2°, while the straight lines are the projections of vertical small circles, also 2° apart, and correspond to small circles of latitude. As on the sphere itself, the angular distance between any two points in projection can be found by passing through the two points, the common great circle (ellipse in projection) and counting directly the distance in degrees by means of the small circles. The actual *modus operandi* of this and the stereographic projection will appear more clearly later when actual data of observation are plotted.

These three methods of projection are commonly used in crystallograhpic and optical work. Each type possesses certain favorable and certain unfavorable features. The gnomonic projection shows excessive distortion for large polar angles and can not be used to advantage for polar angles much above 75°. The stereographic projection shows less distortion, but

even in it the length of a degree (both for radial polar angles and for tangential azimuthal angles) near the horizon (margin of the projection) is nearly twice that of a degree near the zenith (center of the projection). In the orthographic projection the distortion is in the opposite sense, the degrees near the margin being too closely crowded. The result of such distortion is a variation in the relative accuracy of different parts of the projection plat, a millimeter polar distance representing 1° in one part of the projection plat and only ½° in another part, while on the sphere 1° has the same length of arc throughout. It would be obviously better for many purposes if this distortion could be reduced as far as possible. This has been attempted in the so-called *equidistant projections* by placing the eye at a point c (Fig. 39) intermediate between the stereographic point of view S and the infinitely distant orthographic point O. In de la Hire's projection of 1701, this point is located at 1.7071 $(=1+\sqrt{\frac{1}{2}})$ times the radius below the center of the sphere, the result being that the polar distance for 45° is exactly half the radius of the projection plat. In Lowry's projection of 1825 the eye-point is 1.69 times the radius below the center of the sphere. The best average value for c may be obtained by transforming equation (1*a*) above to read

$$c = \frac{x \cos \rho}{\sin \rho - x} \qquad (2)$$

In this equation it is desired that the polar distance x in the projection be proportional to the polar angle ρ or

$$x : 1 = \rho : 90°$$

substituting this value in (2), we have

$$c = \frac{\frac{\rho}{90} \cos \rho}{\sin \rho - \frac{\rho}{90}} \qquad (3)$$

from which the mean value of c can be found by determining by integration the area included by the curve from $\rho = 0°$ to $\rho = 90°$; but this integration is so complex that a method of mechanical quadrature is simpler. From equation (3) the accompanying values for c were calculated, from which the mean value of c was found by Euler's method of mechanical quadrature to be $c = 1.6917$. With this value c the distances x for the different angles ρ were calculated and are listed in column 5, Table 3.

ρ	c	ρ	c
0°	1.7519	50°	1.6966
5	1.7514	55	1.6849
10	1.7498	60	1.6720
15	1.7470	65	1.6580
20	1.7431	70	1.6429
25	1.7381	75	1.6266
30	1.7320	80	1.6092
35	1.7248	85	1.5906
40	1.7165	90	1.5708
45	1.7071		

For the sake of comparison the corresponding values x for the same polar angles ρ are listed (Table 3) for the *gnomonic*, the *stereographic*, the *orthographic*, and the *angle* or *equidistant zenithal projections*. In this last pro-

jection, which was used by Postel in 1581, the polar distances x are directly proportional to the circular arc ρ, and are plotted as $x=\frac{\rho}{90}$. In the *equator angle projection* the concentric polar degree circles are equidistant (as on the sphere) while the azimuth great circles are the radii of the circular projection plat. In the *meridian angle projection* (Plate 11) both great and small circles become oval curves resembling hyperbolas. This particular projection, which apparently has not been used before in optical work, is preferable to

TABLE 3.

ρ	Gnomonic: $x=\text{tg}\,\rho$	Stereographic: $x=\text{tg}\,\frac{\rho}{2}$	Orthographic: $x=\sin\rho$	Equidistant x	Angle $x=\frac{\rho}{90}$
0°	0.00000	0.00000	0.00000	0.00000	0.00000
5°	.08749	.04366	.08716	.05485	.05556
10°	.17633	.08749	.17365	.10976	.11111
15°	.26795	.13165	.25882	.16475	.16667
20°	.36397	.17633	.34202	.21986	.22222
25°	.46631	.22169	.42262	.27519	.27778
30°	.57735	.26795	.50000	.33071	.33333
35°	.70021	.31530	.57358	.38645	.38889
40°	.83910	.36397	.64279	.44244	.44444
45°	1.0000	.41421	.70711	.49868	.50000
50°	1.1918	.46631	.76604	.55513	.55556
55°	1.4281	.52057	.81915	.61175	.61111
60°	1.7321	.57735	.86603	.66847	.66667
65°	2.1445	.63707	.90631	.72516	.72222
70°	2.7475	.70021	.93969	.78167	.77778
75°	3.7321	.76733	.96593	.83777	.83333
80°	5.6713	.83910	.98481	.89314	.88889
85°	11.430	.91633	.99619	.94740	.94444
90°	∞	1.0	1.0	1.0	1.0

the others because its polar distances are not distorted at the margin. The relative accuracy of the different parts of the projection is therefore more nearly uniform and approaches that of the sphere. Some distortion is of course present as is evident from the fact that both the quarter circle and the radius represent 90°, the length of the former being $\frac{\pi}{2}$ greater than the latter. The distortion of the polar angles is nil throughout while that of the tangential azimuthal angles increases from angle true projection at the center to $\frac{\pi}{2}$ to 1 at the margin. But this distortion is less than that in the other perspective projections and the *angle projection* is accordingly better adapted for use in optical work than any of the preceding projections. The terms *equator* and *meridian angle projection* have been adopted for this projection in preference to the longer terms *equator* and *meridian equidistant zenithal* or *Postel* projections.

BIOT'S OR FRESNEL'S RULE.

In several of the optical methods to be described frequent use is made of a rule first formulated by Biot,* by which the directions of extinction for any section of a birefracting mineral can be found. Some ten years before Biot announced this general rule, Malus† had found that the light-waves emerging from a calcite rhomb were plane polarized and that for any given section of calcite the lines of extinction were parallel with and at right angles to the trace of the plane containing the optic axis and the normal to the section; in other words, the orthogonal projection of the optic axis on any given section of a uniaxial mineral determines its lines of extinction, which are parallel with and normal to this projection line. By modifying the wording of this rule slightly, it is possible, as Biot proved experimentally and Fresnel‡ demonstrated theoretically, to make it of general application to all birefracting substances; thus, the directions of extinction of a biaxial mineral¶ cut after any section are parallel to the traces (on that section) of the planes bisecting the angles between the two planes containing the normal to the section and the optic axes (optic binormals); in other words, the lines bisecting the angles between the lines of orthogonal projection of the optic binormals on any given section of a biaxial mineral are the directions of extinction for that section for the particular color of light employed. It should be noted that this rule applies to optic phenomena within the crystal itself, and that for oblique incidence of light, as in convergent polarized light, the angles of incidence must be reduced to the angles which the refracted waves include with the normal to the plate. This is usually accomplished by means of the formula $\sin V = \frac{\sin E}{\beta}$, E being the observed angle, V the required angle, and β the intermediate refractive index of the mineral. Strictly speaking, it is not correct to use the refractive index β in this equation, but in minerals of weak to medium birefringence the difference between β and the correct index is so slight that the error introduced by using β is practically negligible. The effect of the boundary surfaces in rotating the plane of polarization of transmitted waves is also, in general, slight for small angles of incidence and is usually disregarded in practical work.

*Biot, J. B., Mém. de l'Acad. de l'Inst. de France, 3, 228, 1820.

†Malus, E. L., Théorie de la double réfraction de la lumière dans les substances cristallisées. Mém. prés. à l'Inst. Sc. math. et. phys., 2, 303, 1811.

‡Fresnel, Second Mémoire sur la double réfraction, Pogg. Am., 23, 542, 1831.

¶The uniaxial minerals may for the moment be considered the limiting case of biaxial minerals for which $2V = 0$.

CHAPTER I.

As noted in the introduction, the optical and crystallographical features, on which mineral diagnosis under the microscope is based, may be grouped into two classes: those of the first class (*color, pleochroism and absorption, crystal habit, optical character of elongation, optical character of the mineral, dispersion of the optic axes, dispersion of the bisectrices*) are ascertained ordinarily by direct observation without measurement, while for the second class (*refractive indices, birefringence, extinction angles, optic axial angles, cleavage angles*) the numerical results of actual measurement are required. This distinction is drawn somewhat arbitrarily and is not meant to imply that the properties of the first group are strictly qualitative in their nature, but that they are treated at the present time in ordinary petrographic microscopic work as qualities of an object rather than quantities which must be definitely measured. With greater refinement in the methods of determination, some of these properties of the first class will undoubtedly be included in the second, essentially quantitative group. In this chapter the first or qualitative group of characters will be considered briefly and with special reference to their determination in fine-grained preparations.

In the descriptions below, a working knowledge is assumed of the ordinary petrographic-microscopic methods, as treated in detail in the standard text-books on microscopical petrology. This assumption has been found necessary in order to save space and to avoid needless repetition of well-known methods.

Microchemical methods will not be considered in the present paper. They are described in detail in the standard text-books, especially in the treatises by H. Behrens and by Klement & Renard.

COLOR.*

In the examination of thin sections of rocks one of the first features to attract the observer is the color of the different mineral components; yet, until recently, there has been no satisfactory method for determining color. Radde's color scale† has been used for the purpose, but is not satisfactory for several reasons; the comparison itself is exceedingly rough and the colors themselves, being largely of aniline or anthracene dyes, fade with time. Recently Mr. Frederic E. Ives‡ placed the Ives standard colorimeter on the market and through his courtesy the writer has had opportunity to test and to use one of these instruments. Although the colorimeter in its present form is not intended primarily for use with the microscope, yet with the aid of a small prism both the field of the microscope and the colorimeter can be viewed side by side and the color of any mineral section matched in the colorimeter. The Ives colorimeter consists of three standard ray filters, red, green, and blue, so chosen that if the light from the three filters be observed simultaneously the effect is that of white light. In the con-

*For a discussion of color scales, see P. G. Nutting, Bull. U. S. Bureau of Standards, 6, 89–93, 1909.
†H. Fischer, Neues Jahrbuch, 1879, 854–857; Rosenbusch-Wülfing, (I), 1, 348, 1904.
‡The Universal Colorimeter, The Ives Inventions, New York, N. Y.

struction of this instrument advantage has been taken of the fact that the eye is unable to analyze a series of impressions which follow in such rapid succession that each impression lasts less than about one-tenth of a second. The optical "mixer," as it is called, in this instrument consists of a rotating wheel, to the rim of which are attached, at equal intervals, lenses of uniform focal length, which, together with the low-power eye lens and a cylindrical lens, project an image of the colored apertures in the eye-point. A small electric motor drives the wheel at uniform speed, so that the three colored apertures pass the eye repeatedly in such quick succession that resolution is impossible and the color resulting from the mixture of the three colors appears uniform. By means of properly adjusted shutters, the requisite amount of light of each standard color can be made to enter the system, thus obtaining any desired color. As the scale on each shutter reads from 0 to 100, any given color can be expressed by the percentage amounts of the standard colors used to reproduce the color: thus for a particular color, red 82, green 28, blue 63 was used. The color is thus expressed in definite terms and is reproducible. In the colorimeter one-half of the cylindrical lens is covered by an acute-angled prism which refracts the light from a fourth aperture through which the substance whose color is to be determined is examined. The field of the colorimeter appears thus divided in two; half of it receives light from the colored substance while the other half is illumniated by the standard color plates. In using the instrument the field is first adjusted to a pure white and then the apertures of the standard color plates are opened until their resultant color matches that of the substance as it appears in the second half of the field. The standard color plates have been chosen with reference to their purity and their durability.

Another recent device for the determination of colors is the chromoscope of L. Arons* which is based on the interference colors produced by quartz plates of known thickness on rotating the analyzer. A great range of tints and shades of practically every color can thus be obtained and definitely described by noting the thickness of the quartz plate and the angle included between the nicols, provided the same source of light be used for all determinations.

Still another arrangement is being successfully developed by P. G. Nutting† for this purpose. In his apparatus any given color is matched by adding to the proper spectral hue the quantity of white light necessary to produce the desired shade or tint. As both these quantities can be definitely determined, the new method promises to furnish the most reliable values yet obtained.

In petrographic microscopic work the lack of a suitable method for designating colors properly has been keenly felt; but even now, after half a century of microscopic work, colors are designated in the same general terms that prevailed at the beginning. The color of one and the same mineral often varies in the thin section noticeably, and there is no doubt that, with a convenient and accurate method available, detailed studies of such color variations will lead to interesting conclusions regarding the effects of certain elements as pigments in crystal solutions.

*Ann. Physik (4), 33, 799–832 (1910). See also the Schistoscop of Brücke, Pogg. Ann., 74, 582 (1849).
†Outline of Applied Optics, Philadelphia (in press).

PLEOCHROISM AND ABSORPTION.

With the exception of the Ives colorimeter method for designating colors, the writer has had no opportunity to use other than the standard methods for the determination of these two properties.* At present the methods for investigating absorption phenomena in minerals are complicated and not in general use by petrologists. Such phenomena, however, are essentially quantitative in nature and in time will undoubtedly be included in the list of properties to be determined by exact methods.

CLEAVAGE AND CRYSTAL HABIT.

The methods for ascertaining these characteristics of minerals in the thin section are described at length in the standard text-books on petrology and need not be repeated here.

CHARACTER OF THE PRINCIPAL ZONE.

The determination of the position and character of the ellipsoidal axes $\mathfrak{a}$ and $\mathfrak{c}$ in any mineral section is one of the most common problems in thin-section work and is satisfactorily accomplished by standard methods involving the use either of a sensitive tint plate cut from selenite or from quartz parallel or normal to the axis, or of a quartz wedge or a quarter undulation mica plate. The relative value of the ellipsoidal axes is ascertained by noting, on insertion of the wedge or plate, the rise or fall of the interference color in the crystal section under examination. Many minerals, however, are deeply colored and the natural color of the mineral is so intense that it veils seriously the interference color, so that it is often difficult to recognize the true succession of the interference colors. The same holds true for thick sections of strongly birefracting minerals. For these cases, in particular, the second standard method which was used by Fouqué at least 20 years ago) renders good service. This method is based on the direction of motion of the interference bands along the wedge-shaped edges of the mineral plate or grain, whether away from or toward its center (Fig. 43), on insertion of the quartz wedge. If like ellipsoidal axes in wedge and mineral plate coincide, the path difference between the emergent waves is increased on insertion of the wedge and the interference bands appear to have moved from the center of the plate toward the margin; if like ellipsoidal axes do not coincide, the reverse phenomenon is observed and the interference bands move from the margin of the plate or grain toward its center, as indicated in Fig. 43.

THE COMBINATION WEDGE.

In actual work with any one of the plates or wedges noted above, the observer contends with the disadvantage that the interference color of the mineral section in the slide rises or falls abruptly to some other interference color on insertion of the plate or wedge. The thin edge of the quartz

*H. Rosenbusch, Mikroskop. Physiogr. 1st. ed., 105, 1873; P. Glan, Ann. de. Phys. N. F., 1, 351, 1877; Th. Liebisch, Phys. Krystallogr., 520, 1891; Abbe-Zeiss Ocularspectroscope in Rosenbusch-Wülfing (1), 1, 349, 1904; J. Koenigsberger, Zeitsch. f. Instrumentenkunde, 21, 129, 1901.

wedge is not very thin and usually shows gray of the first order between crossed nicols, while the sensitive tint and quarter undulation plates are still thicker and produce on insertion a sudden increase or decrease in path difference between the waves emerging from the crystal section, thus giving rise to the abrupt change or discontinuity observed in the interference color, which is often sufficient to render the determination uncertain. In

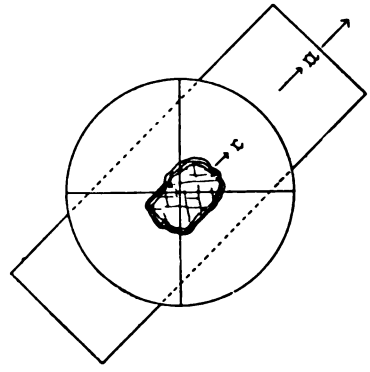


FIG. 43.

deeply colored minerals this is especially noticeable, as the natural color of the mineral veils the interference color to a great extent; on inserting the plate or wedge one observes a change in the interference color of the mineral, but is often unable to distinguish whether the color has risen or fallen.

This defect has been remedied in the combination wedge* which consists of (1) a quartz (or selenite) wedge showing I–III order interference colors and elongated parallel with the greatest ellipsoidal axis (a), and super-

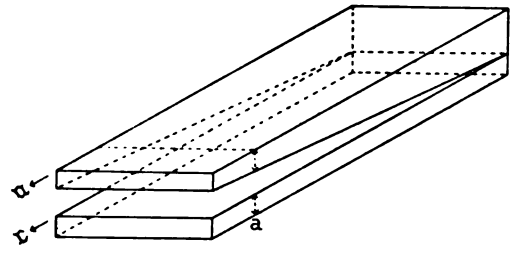


FIG. 44.

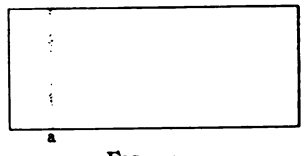


FIG. 45.

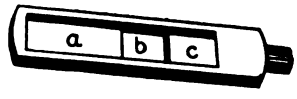


FIG. 46.

imposed on (2) a quartz (or selenite) plate showing an interference color of the first order and elongated parallel with the least ellipsoidal axis, c, or principal axis of quartz, as indicated in Fig. 44. With this arrangement the point *a* of the combination wedge, for which both wedge and plate are of the same thickness, appears dark under crossed nicols, the effect of the quartz wedge on the transmitted light-waves being exactly compensated

*F. E. Wright, Tschermak's Miner. Petro. Mitteil., 20, 275–276, 1901; Journ. Geology, 10, 33–35, 1902.

by the quartz plate (Fig. 45). On both sides of the line of exact compensation the interference colors rise gradually without abrupt change at any point. This device is equivalent to a wedge which has been ground down to an infinitely thin edge; at the line of exact compensation it has no noticeable effect on the light-waves passing through, with the result that the interference color of a mineral seen through the wedge at this point appears the same as though no wedge intervened. If the wedge be slowly withdrawn from this position, the interference color of the mineral is observed to rise or fall gradually without an abrupt change at the start.

Near the center of the wedge is a point for which the path difference between the two emergent waves is $\frac{1}{4}\lambda$. This part of the wedge can therefore be used in place of the $\frac{1}{4}$ undulation mica plate.

To render this combination wedge as useful as possible, it is fitted in a metal frame (Fig. 46) of the same outer dimensions as the ordinary wedge and with it in the same frame a short quartz (or selenite) sensitive-tint plate is placed at one end. A space *b* is left free and is thrown into the field when the wedge is not in use. To steady the motion of the wedge and also to mark the position of the open space *b* a small steel spring with small rounded tip is fitted to the draw tube of the microscope and presses against the metal frame of the wedge. This arrangement combines, therefore, the three attributes ordinarily used, obviates one of their defects, and is attached permanently to the microscope.

As noted in the Introduction, the practice of inserting wedges and plates above the objective does not improve the optical system, but tends rather to decrease materially the definition in the image (especially if high powers be used) as well as to disturb the focus; these annoying features can be remedied by inserting the sensitive plate or wedge between the polarizer and condenser lens. (Plate 1, Fig. 3.) The sensitive plate mounted in a metal frame fits in a collar which can be rotated about the axis of the microscope, thus permitting the plate to be turned rapidly from one quadrant to the second and allowing the rise and fall of the interference colors in a crystal section or an interference figure to be observed in quick succession without disturbing the section itself, as is ordinarily done by rotating the stage.* If a wedge be inserted in place of the sensitive tint plate, the substage diafram must be stopped down as far as possible or a slit aperture introduced in place of the diafram in order that the interference effects due to a single thickness of the wedge be obtained.

As the relative value of the ellipsoidal axes in the wedge or plates is definitely known, the relative value of the ellipsoidal axes in a given crystal section is obtained by direct comparison with the standard plate or wedge, thus determining the optical character of the principal zone or the relative value of the ellipsoidal axis parallel to the direction of elongation of the crystal.

*In case the mineral section is exceedingly minute, and the interference color, dark gray of the first order and hardly perceptible, the brilliantly illuminated field, produced on insertion of the wedge in the diagonal position, may be sufficient to veil the delicate color phenomena which appear on the mineral grain. In such instances the wedge below the condenser should be rotated until its ellipsoidal axes almost coincide with the principal nicol planes. The illumination resulting from the wedge is then hardly perceptible, while the change in interference tint from red to blue is clearly marked in the minute mineral plate or lath in the diagonal position.

THE OPTICAL CHARACTER OF BIREFRACTING MINERALS.

The optical character of a mineral, whether positive or negative, depends by definition solely on the value of the bisector of the acute angle between the optic axes (optic binormals); it is, therefore, independent of the crystal system and pertains to all birefracting minerals; its determination, moreover, under the microscope is relatively simple and does not require elaborate apparatus. For these reasons the optical character is one of the most useful traits in the practical determination of minerals under the microscope. The crystal sections of birefracting minerals from which decisive interference figures can be obtained are those cut (1) exactly or nearly perpendicular to one of the bisectrices, (2) perpendicular to one of the optic axes, and (3) parallel to the plane of the optic axes. These sections and the methods applicable to them can be discussed for all birefracting substances if the uniaxial minerals be considered a limiting case of biaxial minerals for which $2V=O$. The methods for the determination of the optical character are based in large measure on phenomena observed in convergent polarized light. For the simple observation of interference figures without measurement the method adopted by Lasaulx* of viewing the image $C''D''$ directly in the rear focal plane of the objective (Fig. 32) is usually preferable to that requiring the use of the Bertrand lens and ocular; the first image $C''D''$ is brighter and more sharply defined, although smaller than the image $C''''D''''$ observed in the second arrangement. In the formation of interference figures by this method the following factors have an important bearing and will be considered in some detail, as they are generally overlooked and entirely neglected.

The first factor is the rotation of the plane of polarization of a wave transmitted through the lens system. Although the lenses are made of isotropic material throughout, still their surfaces exert, in general, a rotatory effect on the plane of vibration of any transmitted plane polarized light-wave. Fresnel was the first to develop the theory of refraction and reflection for isotropic plates, and from his formulas the amount of rotation due to the influence of the boundary surfaces of a given isotropic plate on the plane of polarization of a transmitted wave impulse can be calculated. The formulas are standard and are developed in text-books on physical optics; the theory on which they are based is, however, usually presented in mathematical form and is difficult and involved, requiring close study to master its details.†

In the following derivation, which is apparently novel and which is to be considered only as a convenient method of explanation for this particular case, the Huygens theory of wave propagation is assumed and the formulas are obtained directly by noting that in an isotropic medium, as air or glass, the forces are distributed in such a manner that a given light-wave may vibrate in any azimuth; also that the planes of vibration for each direction

*Neues Jahrb., 377–380, 1878; an ingenious method for obtaining interference figures from very small grains was described by J. L. C. Schroeder v. d. Kolk (Zeitsch. wissen. Mikrosk., 8, 459–461, 1892). He covered the section with a viscous liquid (glycerin) which contained a number of small air-bubbles and these served in place of the higher power objective.

†In a recent number of the American Journal of Science [(4),31, 157–211, 1911] the writer has reviewed the literature on the particular part of this subject which has to do primarily with the effects of the surfaces of crystal plates on transmitted light-waves and has presented the general theory from the standpoint of the electromagnetic theory of light. The results deduced from theory were found to agree fairly well with those of experiment and prove the importance of this factor.

of propagation are not prescribed, as is the case in anistropic crystals. A light-wave, vibrating in any given azimuth ϵ in an isotropic medium, will continue to vibrate in that azimuth, as there is no force active in the medium which tends to alter its plane of vibration. Similarly a plane polarized light-wave in passing from one isotropic medium to a second encounters no forces, due to the structure of the adjacent media, which tend to divert the line of vibration from its original plane. Thus the line of vibration of a plane polarized light-wave in passing from air into a glass plate suffers change in direction, but it is still contained in the original plane of vibration, as there is no force present in the system which prescribes that it shall vibrate in another plane; the direction of propagation of the wave (and with it the wave-front) is, however, changed on entering the glass plate; the line of vibration of the refracted wave must be contained, therefore, in both the original plane of vibration and the wave-front of the

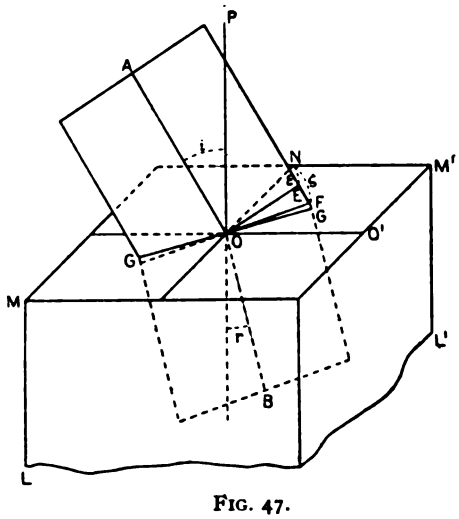


FIG. 47.

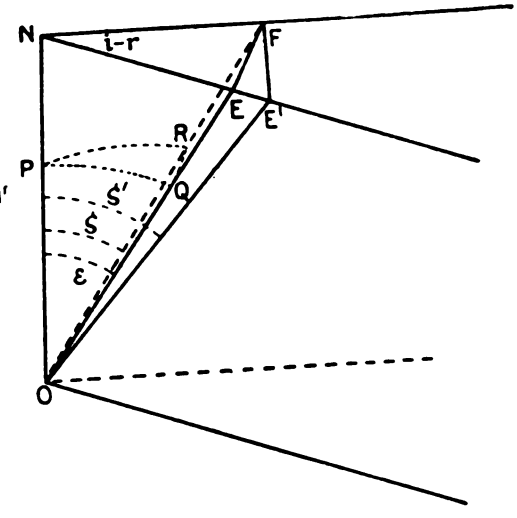


FIG. 48.

refracted wave; its direction accordingly is definitely fixed by the line of intersection of these two planes. In Fig. 47 let MM' be the boundary surface between air and the glass plate $MM'LL'$; let a wave AO be incident at O and let the plane EON represent the wave-front of the incident beam; let also EO be the line of vibration and AOE the plane of vibration; similarly let OB and FON be the direction of propagation and the wave-front respectively of the refracted ray. Then the azimuth ϵ (angle EON) of the line of vibration of the incident wave is definitely fixed by stating the angle which it includes with the line normal to the plane of incidence. To present the same phenomena from another viewpoint, let EON in Fig. 48 be the wave-front of the incident wave and FON the wave-front of the refracted wave; let EO be a line of vibration in the incident wave-front EON. The problem of determining the line of vibration in the refracted wave-front consists simply in finding the line of intersection of the plane FON (the wave-front of the refracted ray) with the plane of vibration passing

through *EO* and the line of propagation of the incident wave or, in short, of finding the point of intersection *F* of the line of propagation *EF* with the refracted wave-front *FON*. On emergence from the plate the same process is repeated and the direction *OF* is projected back to the incident wave-front by the line FE' normal to the refracted wave-front. From the spherical triangle *PQR*, (Fig. 48) the relation is readily obtained

$$\cos(i-r) = \cot\zeta \cdot \tan\epsilon, \quad \text{or} \quad \cot\zeta = \cos(i-r)\cot\epsilon$$

Similarly, for the azimuth of the line of vibration of the emergent wave we have

$$\cot\zeta' = \cos(i-r) \cdot \cot\zeta = \cos^2(i-r) \cdot \cot\epsilon$$

This is the fundamental formula of Fresnel for waves transmitted through an isotropic plate. The angle ζ may be taken either between the line of vibration and the normal to the plane of incidence or between the plane of polarization (normal to the plane of vibration) and the plane of incidence.

The above equation indicates that the amount of rotation ($\zeta' = \epsilon$) of the plane of polarization increases with the difference $(i-r)$ or with the inclination of the incident ray and also with the azimuth ϵ. For $\epsilon = 0$ or $90°$, $\zeta = 0$ and no rotation occurs; for a given angle of incidence the amount of rotation is greatest for $\epsilon = 45°$ in which case

$$\cot\zeta' = \cos^2(i-r)$$

From the above it is evident that at each lens surface the planes of polarization of all transmitted waves are in general rotated slightly, the amount of rotation increasing with the angle of incidence and ranging up to 5° and more for steeply inclined rays. This rotation occurs at all surfaces of the condenser lenses, of the glass mount and of the objective lenses, and gives rise to the faint dark cross observed when viewing the image $C''D''$ (Fig. 30) in the rear focus of the objective, even when no mineral plate is under the objective.* This cross is accentuated if the ordinary type of nicol prism be used with oblique end surfaces, in which case the beam of polarized waves emerging from the polarizer itself is rotated slightly. If, moreover, the lenses of the condenser or objective systems are not carefully mounted, their brass supports may produce strain, which in turn affects the polarized waves and illuminates the field to a greater or less extent.

The crystal plate, from which the interference figure is obtained, also has a rotatory effect on the plane of polarization of the transmitted rays and tends to decrease the accuracy of any measurement which depends on the degree of curvature or the exact location of points on the black bars or zero isogyres of the interference figure. A second factor which tends to diminish the accuracy of measurements of interference figures is the objective itself. An oblique parallel beam of light is unfortunately not brought to focus at a point as indicated in conventional diagrams of interference figures, but to two different lines, the one vertical and the second horizontal; as these foci are not located in the same plane, the result is that the horizontal lines

*The first correct explanation of this dark cross was given apparently by F. Rinne, Centralblatt für Miner., 1900, 88–89; G. Césaro, Bull. de l'Acad. roy. de Belgique (Classe des Sciences, 1906, 459) has also described the phenomenon and directed attention to the fact that from Fresnel's formula for refraction it is evident that the plane of vibration is not changed on the passage of a plane polarized light-wave from one isotropic medium into a second. Compare also F. E. Wright, Amer. Jour. Sci. (4), 31, 187, 1911.

are not in focus at the same time with the vertical, while oblique lines are never strictly in focus. This phenomenon of astigmatism is due to the fact that the radius of curvature of the lens on which the oblique rays impinge is shorter in the horizontal plane than in the vertical; the lens appears foreshortened in the horizontal plane. (Figs. 21, 22.)

A still further defect is the fact that it is not possible to correct a lens for the sine condition for more than one image plane; the rear focal plane of the objective is not plane but curved; it consists in fact of two coaxial surfaces which are more or less spherical or paraboloidal or irregular and wavy in shape, even when monochromatic light is used. To bring into accurate focus all the points on these two surfaces at one and the same time is obviously impossible, and a compromise is made by using a small stop in the eye-point of the ocular if the observations be made with a Bertrand lens, thus reducing the parallax and the effect of astigmatism as much as possible; points midway between the center and margin of the field are brought to the sharpest possible focus, in which case the focus over the entire field is fair. If the interference figure be examined without the Bertrand lens, a stop should be placed in the image plane and all but the central, axial portion of the object cut out, thus reducing the astigmatic errors. For the same reason a stop should be introduced either in the upper focal plane of the Bertrand lens, or in the conjugate image plane of the upper focal plane of the objective as formed by the Bertrand lens, and the image narrowed down to the axial portion. A third and more important factor is the distortion due to the analyzer, which tends to disturb the symmetry of the interference figure and thus to render the measurements less exact.

Another factor in the interference figure is the chromatic error of the objective. Although in designing an objective the optician strives to reduce the chromatic aberrations to negligible limits in the image plane, this is not the case for the focal plane of the objective and the image there formed is not achromatic. As a result, the colors in the interference figure are not strictly pure, but are more or less modified and veiled by the chromatic aberration colors of the objective itself. In testing for dispersion of the optic axes the chromatic errors of the objective should be taken into account.

These factors indicate clearly that an interference figure in the microscope is so encumbered with disturbing elements that measurements of a high degree of precision are not possible under any conditions. The approximate results obtained, however, are sufficiently correct for practical purposes, data of a higher order of accuracy being rarely required.*

*The following method of artificial illumination has proved satisfactory in petrographic microscopic work. The source of light (acetylene or Nernst light) is placed in the principal focal plane of a large condenser lens from which parallel rays then emerge and pass to the substage reflector and into the microscope. A pale blue glass disk with one side finely ground is placed in the lower focal plane of the condenser approximately, and serves two purposes: (1) to reduce the intensity of the red and yellow from the artificial light and thus to render the light more nearly like daylight; (2) to furnish by means of the finely ground, lower surface of the blue glass disk a series of radiant points in the lower focal plane of the condenser from which light rays are refracted in all directions. This insures uniform illumination over the field and for ordinary purposes, especially observation of interference figures, has proved more satisfactory than the usual method of placing the ground blue glass between the substage reflector and the lower nicol. The effects of depolarization at the finely ground surface are not serious and in general not noticeable. By using a concave mirror (attached to the arm of the microscope) in conjunction with the artificial light source, excellent illumination for the study of minerals in reflected light can be obtained. By means of colored glass screens it is, moreover, possible in certain instances to illuminate the surface of the preparation with light of such a hue that the difference in aspect between two minerals, as hematite and magnetite, which resemble each other closely and are not always easy to distinguish under the microscope in ordinary light, is accentuated in a measure and their determination facilitated to that extent.

The importance of excluding all light except that from the mineral section or grain under examination was emphasized in the Introduction and a small device with adjustable aperture was described for the purpose (Plate 1, Fig. 1). This device was constructed primarily to replace the sets of circular stops which were formerly furnished with the petrographic microscope, but which allowed the observer to stop off only a particular portion of the field; in consequence of their lack of adaptability they have been gradually discarded. The ordinary iris diafram in the draw-tube of the microscope is located too far distant from the eye to be of much service in the direct observation of interference figures with the unaided eye, because of the diffraction phenomena which arise when small apertures are used and which destroy the definition. If the objective be properly corrected, the stop in the image plane is effective and interference figures from grains 0.01 to 0.02 mm. in diameter and of medium birefringence can readily be obtained.*

PLATE PERPENDICULAR TO THE ACUTE BISECTRIX.

For birefracting minerals in which $2E$ is less than 140° the methods for ascertaining the optical character ordinarily described in text-books are applicable and satisfactory. Both optic axes appear then in the field and the optical character can be ascertained either in convergent or in plane polarized light by the usual methods. If the optic axial angle is so large that the optic axes do not appear in the field of vision, the aperture of the objective can be increased by using homogeneous immersion, and if this be not sufficient an approximate idea of the value of $2E$ can be obtained by Michel-Lévy's method† of noting the angle of rotation of the stage from the position where the zero isogyres of the interference figure form a cross to that at which they are tangent to a given circle (usually margin of the field of the microscope). From this angle $2E$ can be determined and from it in turn, the correct optic axial angle ($2V$), if the medium refractive index of the mineral plate be known. This method, however, is not in general use because of the inaccuracy of the results obtained and of the calculations involved, the microscopist preferring usually to find another section on which the optical character can be ascertained directly and with greater accuracy.

PLATE PERPENDICULAR TO AN OPTIC AXIS.

The interference figure obtained from this plate consists ordinarily of a black achromatic bar which, on moving the stage, rotates in a direction opposite to that of the stage. This bar is in general a straight line only when it is parallel to the planes of polarization of the nicols. In the intermediate positions it is more or less convex, depending on the angle between the optic axes. If $2V$, however, is equal to 90°, the curve is a straight line

*In weakly birefracting grains (uniaxial or biaxial) the field between crossed nicols in convergent polarized light is so dimly lighted that, on the insertion of the sensitive tint plate, it is flooded with light and the delicate differences in tint caused by the crystal plate are often difficult to recognize and may render the determination uncertain. In such instances, the sensitive tint plate below the condenser (*Q*, Plate 1, Fig. 3) should be rotated until its axes are almost parallel with the principal nicol planes, in which position the intensity of illumination from the sensitive tint plate itself is very slight, while the path differences remain unchanged and the differently colored quadrants or areas stand out in sharp contrast.

†Michel-Lévy, Les Mineraux des Roches, 94–95, 1888. Compare also F. E. Wright, Amer. Jour. Sci. (4), 20, 287, 1905; H. Tertsch, Tscherm. Min. Petr. Mitteil. 27, 589–594, 1908.

in all positions for the usual microscopic field of view. Mallard* was apparently the first to direct attention to the fact that the convex side of this dark axial bar always points toward the acute bisectrix. Since Mallard's time this method has been used and described by a number of writers, including F. Becke in 1904,† the writer in 1905,‡ and C. Césaro in 1906.|| From the descriptions it is apparent that each of these investigators discovered this method anew and independently of the others.

Having once determined the convex side of the axial bar in the section normal to an optic axis the optical character of the mineral is then most readily ascertained by use of the sensitive-tint plate. If the achromatic bar be placed in the position of Fig. 49 with its convex side pointing to the northwest, and the arrow of the sensitive-tint plate (= $\mathfrak{c}$, direction of least ellipsoidal axis) be in the same direction, the convex side of the curve will show a blue interference color if the mineral is optically negative; if it be optically positive, the blue spot or border will appear on the concave side of the axial bar.

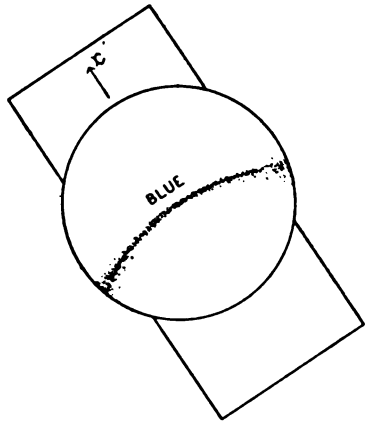


FIG. 49.

This method can always be applied if the convexity of the curve is well marked. In certain plagioclase feldspars the limiting case of $2V = 90°$ is encountered occasionally and there the bar is in fact a straight line. If the section be not cut precisely normal to the optic axis, the optical character can still be safely determined if the curvature of the axial bar is well marked and the optic axis is not too near the margin of the field.¶

PLATE PARALLEL TO THE PLANE OF THE OPTIC AXES.

In uniaxial minerals this plate corresponds to any section parallel with the optic axis.

The interference figure from such a plate is readily recognized by noting that on rotating the stage of the microscope the field becomes suddenly dark, remains so for an instant, only to become light again on further rotation of the stage through a few degrees. In the position of darkness the greatest and least ellipsoidal axes, $\mathfrak{a}$ and $\mathfrak{c}$, are parallel with the principal sections of the nicols. No distinct cross is seen, as in the interference figures of plates normal to the bisectrices. The entire field appears dark, with perhaps a weak fringe of light along the outer diagonals of the quadrants.

If the field be placed in the dark position and turned slightly, faint, dark hyperbolas can be seen to open and leave the center of the field, similar to

*E. Mallard, Traité de crystallographie, Tome II, 229, 1884.
†Rosenbusch-Wülfing, Microskopische Physiographie I, I, 335, 1904.
‡Amer. Jour. Sci. (4), **20**, 292, 1905.
||Bull. de l'Acad. roy. de Belgique (Classe des Sciences), 323–324, 1906.
¶In 1907 the writer endeavored to determine graphically the degree of curvature of the axial bars for different positions of the optic axis in the field by the direct application of Fresnel's rule. In the curves thus prepared (Amer. Jour. Sci. (4), **24**, 338–340, 1907) the rotation of the crystal plate and intervening glass plates and lens on the plane of polarization of the transmitted waves was disregarded as a negligible quantity and the curves are inaccurate to that extent. They prove, however, that on sections showing an optic axis in the field the optical character can be determined by this method, provided the axial bar is noticeably convex.

the dark hyperbolas of the biaxial interference figures perpendicular to the bisectrices, the chief difference between the two being one of intensity and rapidity of motion. The hyperbolas are very weak and require close observation to be noticed at all.

Since the ordinary approximate methods for calculating the positions of the zero isogyres for different angles of rotation of this section are involved and difficult, a graphical method with the stereographic projection plat as base can be used to advantage. Having given the positions of the optic axes, the directions of vibration of waves propagated along any given direction can be found directly by use of the Biot-Fresnel rule. In this method the slight rotatory effects of the boundary surfaces of the crystal plate on the plane of polarization of the transmitted waves are disregarded and the average refractive index of the mineral is used in place of the correct indices of refraction for the different directions. These assumptions render the results obtained by use of the graphical method slightly inaccurate, but the general relations are not seriously affected thereby. To find the lines of vibration for any given direction in the stereographic plat, the entire projection plat is rotated first on its horizontal and then on its vertical axis until the given direction coincides with the pole or center of the projection plat. The wave-front is then the plane of the plat and the directions of vibration bisect the angles of the projections of the optic axes on the wave-front. Fig. 50 was constructed in this manner and shows that the recession of the dark achromatic lines from the center of the field of view for the optic axial angles $2V=0°$, 10°, 80° and 90°, after a rotation of the stage through 1°, is very marked and that except in the limiting case of $2V=90°$, the dark hyperbolas pass out of the field most slowly in the direction of the acute bisectrix. For $2V=90°$, the hyperbolas in all quadrants recede from the center with equal rapidity. In Fig. 50 the lines between the outer and inner marginal circles represent the actual positions of the bisectrices and the optic axes under the conditions stated.

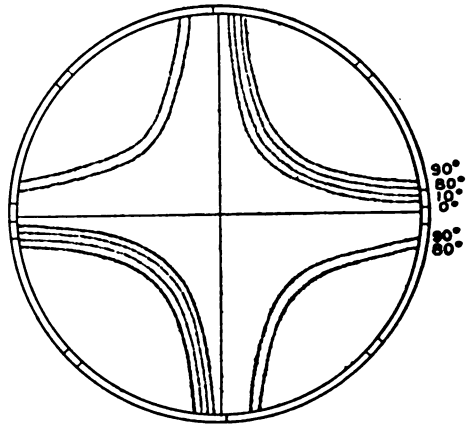


FIG. 50.

Owing to the fact that for this section the angles of extinction are very low for all waves whose angle of incidence is small, the intensity of the waves adjacent to those of the achromatic curves is also low, since it varies with the square of the sine of the angle ρ between the planes of polarization of the nicols and that of the section according to the standard formula

$$I=\sin^2 2\rho \sin^2 \frac{\pi\Delta}{\lambda}$$

The black curves are, therefore, indistinct and require careful scrutiny to be observed at all.

After the direction of the acute bisectrix has been found by this method its value, $\mathfrak{c}$ or $\mathfrak{a}$, can be readily ascertained by ordinary methods either in parallel or in convergent polarized light.*

The isochromatic interference curves which appear in the interference figure from a plate parallel with the plane of the optic axes can also be used to locate the direction of the acute bisectrix, as was first shown by F. Becke.† It can be proved in several different ways that the acute bisectrix is generally a direction of less birefringence than the obtuse bisectrix. The birefringence of any section can be calculated approximately by means of the usual formula

$$\gamma' - \alpha' = (\gamma - \alpha) \sin \theta_1 \sin \theta_2$$

where γ' and α' denote the maximum and minimum refractive indices of the given section, γ and α those of the mineral, θ_1 and θ_2 the angles between the normal to the section or direction of wave propagation and the two optic axes respectively (Plate 5 is a graphical solution of this equation). This standard formula indicates clearly that except in the limiting case of $2V = 90°$, the birefringence for sections in the alternate quadrants containing the acute bisectrix is less than that for corresponding sections in the two remaining quadrants. The rule resulting from this fact is that the interference colors for points in the quadrants containing the acute bisectrix are lower than those for corresponding points in the direction of the obtuse bisectrix.‡ Having once determined the direction of the acute bisectrix by this method, either by observing the interference figure directly or by using a sensitive tint plate or quartz wedge, the relative value of the acute bisectrix, whether $\mathfrak{c}$ or $\mathfrak{a}$, is determined most readily in parallel polarized light by the ordinary methods.

G. Césaro§ has described several methods by which the optical character of the mineral can be determined directly on such sections in convergent polarized light by observing, on the insertion of a quartz wedge, (1) that the black curves of exact compensation appear first in the quadrants containing the acute bisectrix; (2) that the isochromatic hyperbolas cross in the lower or in the upper part of the field, depending on the optical character of the mineral and the direction of elongation of the wedge; (3) that if the point at which the hyperbolas cross be not visible in the field, the direction of the point toward which they converge or from which they diverge on inserting the wedge can be seen and thus the character determined.

These methods of Césaro are implied by Becke in his paper on the Skiodroms,|| wherein a very useful rule for determining the sign of reaction of a section is given. More recently J. W. Evans,¶ in discussing this rule,

*This method was first described in 1904 by the writer (Amer. Jour. Sci. (4), **17**, 388, 1904). It has since been described by G. Césaro (Bull. de l'Acad. roy. de Belgique (classe des Sciences), pp. 290, 392, 459, 493, 1906; also p. 161, 1907) and evidently without knowledge of the description by the present writer.

†Tschermak's Miner. Petro. Mitteil., **16**, 181, 1897.

‡F. Becke, T. M. P. M. **16**, 181, 1897; also in Rosenbusch-Wülfing, Mikroskopische Physiog. (I), **1**, 315 and 335. Compare also F. E. Wright, Amer. Jour. Sci. (4), **17**, 389–390, 1904; (4), **20**, 295, 1905.

§Bull. de l'Acad. roy. de Belgique (Classe des Sciences) 290, 392, 459, 493, 1906; 159–161, 1907.

||T. M. P. M. **24**, 1–34, 1905. In this paper Becke (see also T. M. P. M. **27**, 177, 1908) directs attention to the difference in behavior of the zero isogyres of uniaxial and biaxial crystals. The black brushes in the interference figures from uniaxial crystals are essentially straight lines and are always parallel with the polarization plane of the nicols. In biaxial crystals these dark achromatic bars turn as the stage is rotated and may include any angle with the principal planes of the nicols. This fact serves to distinguish a uniaxial crystal from a biaxial crystal, even in sections in which no optic axis is visible in the interference figure.

¶Miner. Mag., **14**, 231–234, 276–281, 1907.

has described methods which are similar to those of Césaro and by means of which the direction of the acute bisectrix can be ascertained in the interference figure from certain sections.

DISPERSION OF THE OPTIC AXES.

The methods for determining this feature (both the relative dispersion and the kind of dispersion) of biaxial minerals are described in detail in the standard text-books* and will not be discussed further than to note that the chromatic aberrations of the objective contribute a certain amount of color to the interference figure and tend to modify it slightly, a fact which should be considered in critical work.

DISPERSION OF THE BISECTRICES.

This feature can be determined occasionally in the interference figure, but parallel polarized light should be used for accurate work and the positions of extinction on a section parallel with the plane of the optic axes should be determined accurately for different colors.

*See also E. v. Fedorow, Zeitschr. Kryst., 37, 143, 1902.

CHAPTER II.

REFRACTIVE INDICES.

Many methods have been suggested for measuring directly the refractive indices of minerals, but few of these have proved applicable to fine-grained minerals or minerals in the covered thin section.* For the accurate measurement of refractive indices it is, at present, necessary that at least one surface of the mineral plate be uncovered. In the covered thin section the petrologist is able, in general, to determine only the relative refractive indices of adjacent mineral plates and not their absolute values. With practice he becomes able to estimate the approximate refractive index of a mineral plate from its relations to adjacent plates or the surrounding medium (usually Canada balsam), and this is ordinarily sufficient for actual determinative work. As the methods for ascertaining the relative refractive indices between mineral plates are primarily microscopical methods they will receive first consideration in the following paragraphs, while the methods for the direct measurement of the refractive indices of an isolated plate or prism will be referred to only briefly because they are described at length in the standard text-books.

Underlying all methods which have been suggested for determining the relative refractive indices of mineral grains or plates in the thin section is the effect of different kinds of illumination of the object on the development of the image. This is a fundamental factor and will be considered in detail before attempting the descriptions of the different methods.

If a wide cone of rays be incident on the object each element of the object receives light from all sides with the result that the different details appear about equally bright and can not be readily distinguished unless they differ in color or in the amount of light they absorb. This kind of illumination is, therefore, best suited for work with fine-grained particles embedded in a medium of about the same refractive index, as stained biological and bacteriological specimens. With a wide cone of light the field is often "flooded with light" and the phenomena of refraction and reflection due to differences in refractive indices between adjacent elements in the object are in consequence veiled and lost to view. To determine the relative refractive indices of two adjoining sections, the aperture of the condenser should be decreased, either by lowering it or by closing the substage iris diafram or by both. Beams of parallel or weakly convergent polarized light are thus obtained and differences in the refractive indices are clearly marked. These differences are based on the phenomena of refraction, of dispersion, and of reflection.

In Fig. 51, *a*, let a mineral grain, more or less round or lenticular in shape, be immersed in a liquid of higher refractive index and illuminated by a narrow cone of light from the condenser. The paths of the rays impinging

*The method of the Duc de Chaulnes (Mém. Acad. R, 423, 1767,) and its many modifications which have since been suggested, do not furnish accurate results and are rarely used by petrologists. In describing a new method for measuring the refractive index of a prism under the microscope C. Miculescu (Bull. d. Soc. Sci., Bucarest, 14, 280–288, 1905; 16, 8–14, 1906) discusses the Duc de Chaulnes method briefly.

on the mineral grain are then deflected by it after the manner of a divergent lens, as shown in the figure; the aperture of the transmitted cone is increased. If now the objective be raised and focussed on successive parts of the grain, the distribution of the intensity of illumination varies noticeably in the different planes *a*, *b*, *c*, as illustrated in the figure, the effect of the mineral

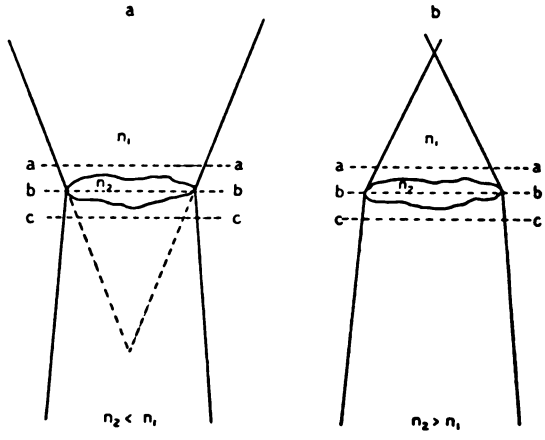


FIG. 51.

grain under these conditions being that of a divergent lens on the incident rays. If, on the other hand, the mineral grain has a higher refractive index than the liquid (Fig. 51, *b*), it will tend to converge the rays and to bring them to focus. From Fig. 52, *a*, *b*, it is also apparent that total reflection may occur and tend likewise to modify the aperture of the incident cone.

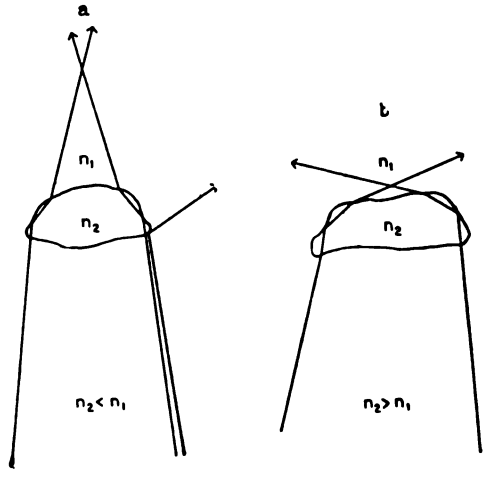


FIG. 52.

On viewing the grain of Fig. 51, *b*, under the microscope and raising the objective from the position of exact focus on the plane indicated by the dotted line *b*-*b* to that on the plane *a*-*a* the intensity of light will appear to

increase toward the center of the grains; in short, a bright fringe appears in this case to wander toward the center of the grain on raising the tube, while for Fig. 51, *a*, the reverse takes place on raising the objective and the fringe of light recedes from the grain in all directions as a halo. On lowering the objective, the reverse phenomena are observed in both instances.*

If pencils of obliquely incident light be used, the conditions represented in Fig. 53, *a*, *b*, will obtain.† In Fig. 53, *a*, the mineral has a lower refractive index than the liquid and the rays are concentrated along the left margin of the grain. When viewed through the objective focussed on the grain, this margin appears, therefore, much brighter than the opposite edge. If the mineral has the higher refractive index the rays are deflected as shown in Fig. 53, *b*. Obliquely incident light beams enable the observer to determine at once whether the mineral grain has a higher or a lower refractive index than the liquid or medium which envelops it.

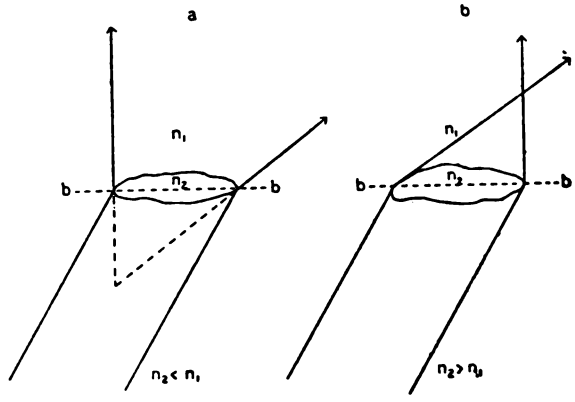


FIG. 53.

The best methods for obtaining obliquely incident rays are (1) dark-ground illumination from a part of one quadrant only, (2) by use of the movable substage iris or stop diafram, (3) shading half the field by inserting the forefinger between the reflector and polarizer (Fig. 38), and thus cutting out half the incident cone of rays. This last method is extremely simple in its application and is satisfactory in practical determinative work.

In case the mineral grain and liquid have the same refractive index for some color, as yellow or yellow-green, interesting effects are produced which enable the observer to determine the refractive index of the grain with considerable accuracy, provided that of the liquid be known. As a general rule liquids have a greater dispersion than minerals. This is shown graphically in Fig. 54, in which the refractive index curves‡ for ω and ϵ of quartz are plotted and also those for the refractive liquids benzoylchloride, bitter almond oil, apiol, ethylenebromide. Under these conditions the liquid will have a higher refractive index for the waves at the blue end of the spectrum and a

*O. Maschke, Pogg. Ann. Phys. Chem. **145**, 568, 1872; Wied. Ann. Phys. Chem., **11**, 723, 1880; A. Brun, Arch. Soc. Phys. and Math., Génève, **32**, 3, 1894.

†Compare Schroeder von der Kolk, Zeitsch. Wissen. Mikros, **8**, 456–458, 1892; F. Becke, S. W. A., I. Abt. **102**, 358, 1892.

‡Refractive indices taken from the tables of Landolt-Börnstein.

lower index for the red rays than the mineral. On raising the objective, the red rays wander toward the center of the grain, as in Fig. 51, *b*, while the blue rays recede from the center toward and beyond the margin, as in Fig. 51, *a*, with the result that the mineral grain is tinged pale orange red and appears set in a blue background. On lowering the objective the reverse phenomena take place and the grain is colored pale blue and is surrounded by a pale reddish or orange-colored halo. By properly adjusting the refractive index of the liquid, it is possible to obtain clear color reactions of this kind and thus to determine definitely when the mineral and liquid have the same refractive index for a color as yellowish-green midway between red

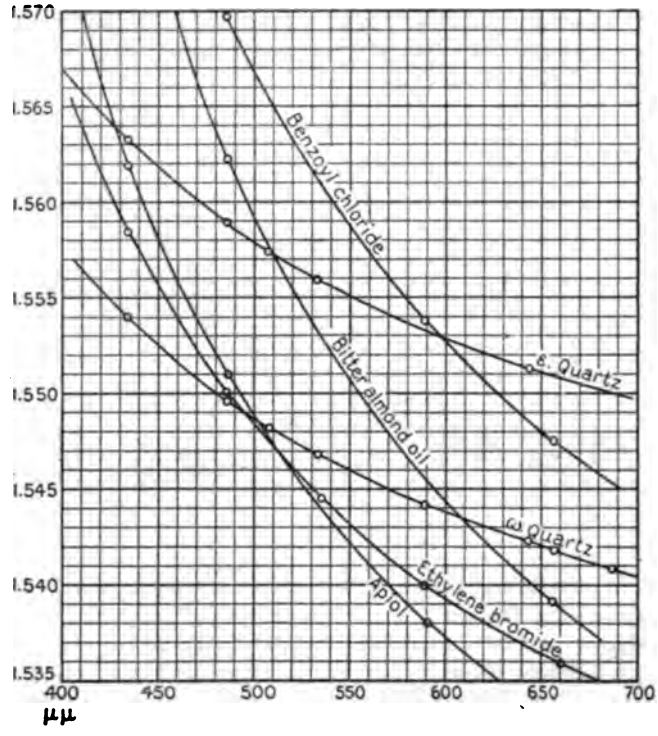


FIG. 54.

and blue or about the center of the visible spectrum. The greater the relative dispersion of the liquid the more pronounced the colors observed, but also the less sensitive the determination. If the dispersion were the same in both liquid and mineral, no such color phenomena would result, because the rays would then pass through the mineral undisturbed and no fringes, colored or colorless, would be observed along its edges on raising or lowering the objective. For satisfactory work it is essential, therefore, as Maschke was the first to emphasize, to choose liquids whose dispersion is sufficiently strong to produce distinct color phenomena in mineral grains, but still low enough so that these colors do not appear on mineral plates whose refractive indices are noticeably different.

The center of the visible spectrum of ordinary daylight is in the yellowish-green and if care be taken in these experiments to obtain the same intensity of light in the red band as in the blue band, by lowering or raising the refractive index of the liquid, it is possible to determine the refractive index of the mineral plate for the color about 555 $\mu\mu$ with a probable error of 2 or 3 in the third decimal place.

If oblique illumination be used the phenomena are even more distinct, as the colored red and blue fringes appear on opposite sides of the grain. From Fig. 53, *a*, *b*, it is evident that in case both liquid and mineral grain have the same refractive index for yellow, the red rays are directed toward the right side of the grain (the mineral having the higher refractive index for the red rays), while the blue rays appear on the left side of the grain.*

Greater accuracy than ± 0.002 can not be obtained by methods based on relative dispersions, because of certain disturbing factors which it is difficult to control satisfactorily. The point of average intensity in the visible daylight spectrum is not constant, but shifts its position noticeably on different days and under different conditions of weather.† If, therefore, the refractive index of the grain be determined by matching the intensities of the red and blue bands on the opposite sides of an immersed grain, the value obtained will be slightly different on different days. Still another factor is the sensitiveness of the eye, which varies with the intensity of illumination, the maximum being at about 510 $\mu\mu$ for weak illumination and increasing toward the yellow up to at least 540 $\mu\mu$.‡ If the observations be made, therefore, with raised condenser and brightly lighted field, the sensitiveness of the eye will not be the same as it is for a weakly lighted field (condenser lowered and aperture decreased) and slightly different results would be obtained under the different conditions. Several of these factors are subjective in character and not capable of precise measurement by simple methods, and prescribe, therefore, a practical limit to the accuracy of the determination by such relative dispersion methods. The center of the visible spectrum can be shifted by the use of suitable ray filters which absorb part of the light from either the blue or red end of the spectrum, as was first suggested by Maschke,§ and the refractive index for a different part of the spectrum thus ascertained with about the same probable error, 0.002. The chromatic errors of the lens system are a third factor which exerts a disturbing influence on the color phenomena observed in oblique illumination.

For more accurate work, the color phenomena with their subjective elements can not be used and recourse must be had to strong monochromatic light. Having once determined the approximate refractive index of a mineral grain by one of the relative dispersion methods, the more exact measurements are made in intense monochromatic light either from a monochromatic illuminator or Geissler tube or monochromatic Bunsen flame from sodium, lithium, thallium, or other suitable salt. Either the Becke line method or

*It should be noted that, in case the oblique rays are obtained by shading half the condenser with the forefinger, the phenomena can be reversed by raising the condenser beyond the point where the finger edge is focussed on the object.

†E. L. Nichols, Phys. Rev., **26**, 497, 1908; **28**, 122, 1909; Trans. Ill. Soc., **3**, 301.

‡A detailed consideration of the subject of the Luminous Equivalent of Radiation has recently been given by P. G. Nutting, Bull. U. S. Bureau of Standards, **5**, 261–308, 1908.

§Wied. Ann. Phys. Chem., **11**, 772–734, 1880; ray filters for the same purpose have also been used by Dr. Merwin of the Geophysical Laboratory.

that of oblique illumination can then be used and the refractive index determined with an accuracy of ±0.001.

The following method for producing an intense monochromatic flame has been found convenient and satisfactory in practice.* A 25 cc. platinum crucible, filled with a mixture of equal parts of sodium chloride and sodium carbonate and held in a special mounting of thick platinum wire, is heated over a Bunsen burner, as indicated in Fig. 55. A wick of fine platinum wires carries the molten salts from the base of the crucible out into a strong and constant blast-lamp flame, the high temperature of which produces an intense sodium flame which lasts for days until the salts in the crucible are exhausted. The fumes from the salts are carried off under a hood. An oxyhydrogen blast may be used in place of the blast lamp. It gives a much more intense flame, but requires careful regulation; otherwise it may melt down the platinum wick.

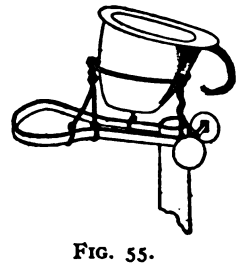


FIG. 55.

In case only one side of a mineral grain or plate be observed, the phenomena observed vary slightly with the character of the boundary surface, as indicated by the following figures and calculations:

In Fig. 56† let n_1 be a mineral plate in the thin section and n_2 a second plate adjoining n_1, the refractive indices of the two plates being n_1 and n_2 with $n_1 < n_2$; let the junction plane be vertical. A ray of light B_1C_1 entering n_1 will be deflected to D, where again it is deflected to F_2 and finally to H_2 in air. The relation of the direction H_2F_2 to B_1C_1 or the angle ϑ_2 to a_1 is readily found by use of the sine relations

$$\sin a_1 = n_1 \sin \beta_1 \tag{1}$$

But $\quad \measuredangle I_1DC_1 = 90° - \beta_1$ and $\measuredangle I_2DF_2 = 90° - \gamma_2$

$$\therefore \quad n_2 \sin (90 - \gamma_2) = n_1 \sin (90 - \beta_1) = n_1 \cos \beta_1 \tag{2}$$

Squaring both sides of (1) and (2) and adding, we find

$$\sin^2 a_1 + n^2_2 \sin^2 (90 - \gamma_2) = n^2_1 \tag{3}$$

We have also:

$$\sin \vartheta_2 = n_2 \sin \gamma_2 \tag{4}$$

$$\therefore \quad \sin^2 \vartheta_2 = \sin^2 a_1 + n^2_2 - n^2_1 \tag{5}$$

$$\frac{1 - \cos 2\vartheta_2}{2} = \frac{1 - \cos 2a_1}{2} + n^2_2 - n^2_1 \quad \therefore \quad \cos 2\vartheta_2 = \cos 2a_1 - 2(n^2_2 - n^2_1) \tag{6}$$

In like manner it can be shown that

$$\cos 2\vartheta_1 = \cos 2a_2 + 2(n^2_2 - n^2_1) \tag{7}$$

For the limiting angle a_2 at which total reflection just occurs at the junction between n_1 and n_2, (heavy line, Fig. 57) $\vartheta_1 = 0$ accordingly

$$\cos 2\vartheta_1 = 1 \text{ and } \cos 2a_2 = 1 - 2(n^2_2 - n^2_1) \text{ or } \sin^2 a_2 = n^2_2 - n^2_1$$

*Amer. Jour. Sci. (4), 27, 195, 1909; 31, 185, 1911.

†In Figs. 56 and 57 the rays from the condenser lens are represented as coming to focus at the point D. This has been done purposely in order to render the figures as simple as possible. In actual work, this condition, however, is only approximately attained, the foci for the rays in the higher refracting substance n_2 being farther away from the incident surface a_1a_2 than the foci of those in n_1. In both figures the directions are angle true for the refractive indices given.

It is evident, therefore, that at the junction between the two minerals, refraction and total reflection take place and if the objective be first focussed on a plane $I_1D_1I_2$, Fig. 56, a bright point D will be observed at the junction line, all the rays from the condenser being brought approximately to focus at this point. On raising the objective until it is in focus with the top of the section, MN, differences in intensity of illumination will be observed, the distance ML, Fig. 57, being lighted by the cone of rays included between rays 6 and 7, while LN receives the rays between A and f and also the rays between A and 7. The intensity of illumination of the three areas FL, LH and HG is obviously different. By narrowing down the cone of rays from the condenser it is possible to cut out the light to LF entirely, in which case the bright area or strip LH will appear relatively at its brightest. If the cone of rays be not stopped down two white lines will appear, one, LF, moving toward M ($=n_1$) and the second, LG, moving toward N ($=n_2$) on raising the microscope tube.

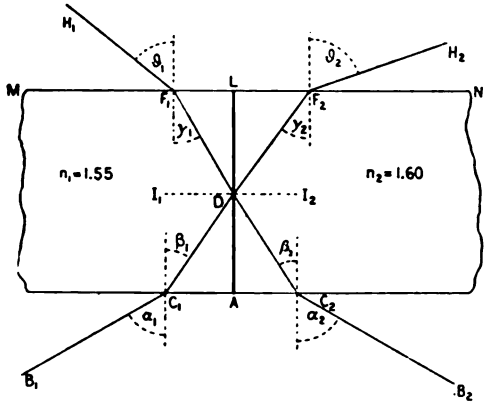


FIG. 56.

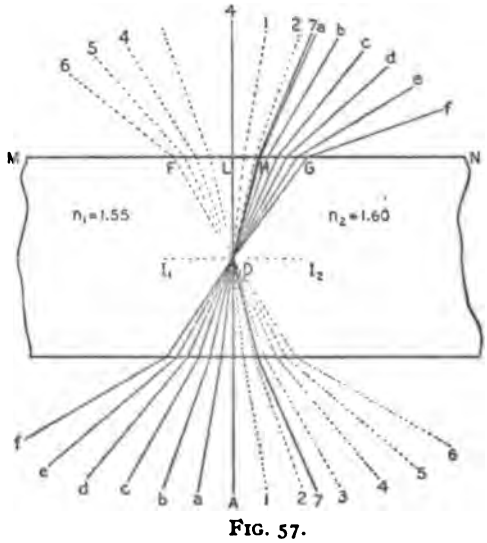


FIG. 57.

The limiting angle for which all the rays from a_2 are reflected varies with the difference in refractive indices n_2 and n_1, as illustrated by the following numerical examples:

(1) Let $n_1 = 1.550$ $n_2 = 1.555$ $n^2_2 - n^2_1 = 0.0155$

Then for $a_2 = 30°$ $\vartheta_1 = 28° 53'$

$a_1 = 30°$ $\vartheta_2 = 31° 05'$

But for $a_2 = 7° 8'$ $\vartheta_1 = 0$ (limiting angle)

$a_1 = 7° 8'$ $\vartheta_2 = 10° 13'$

(2) Let $n_1 = 1.550$ $n_2 = 1.551$ $n^2_2 - n^2_1 = 0.0031$

Then for $a_2 = 30°$ $\vartheta_1 = 29° 47'$

$a_1 = 30°$ $\vartheta_2 = 30° 12'$

But for $a_2 = 3° 12'$ $\vartheta_1 = 0$ (limiting angle)

$a_1 = 3° 12'$ $\vartheta_2 = 4° 31'$

(3) Let $n_1 = 1.700$ $n_2 = 1.701$ $n^2_2 - n^2_1 = 0.0034$

Then for $a_2 = 3° 21'$ $\delta_1 = 0$ (limiting angle)

$a_1 = 2° 34'$ $\delta_2 = 4° 43'$

From these examples, as well as from the formulas, it is evident that the greater the difference between the refractive indices n_1 and n_2, the larger the limiting angle for complete total reflection and therefore the greater the difference in light intensity between both sides. Thus, if a cone of rays of angle $a_2 = 30°$ be used and $n_1 = 1.55$, the refractive index n_2 must equal 1.629 or more for complete total reflection. It is significant that for differences of only 0.001 between n_1 and n_2 the limiting angle necessary to produce complete total reflection is over 3 degrees. By lowering the condenser and stopping down the diafram this angle is easy to obtain and with it the most favorable conditions for the observation of the Becke line. The greater the difference in refractive indices between two adjacent plates, the wider the

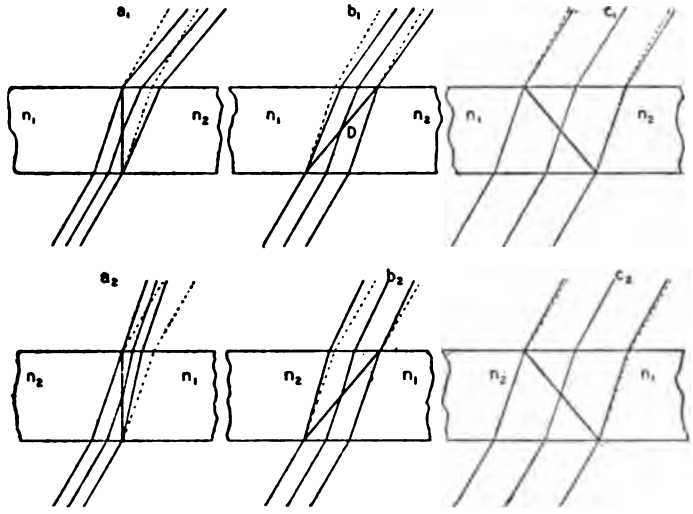


FIG. 58.

cone of rays permissible to show differences in refraction clearly. By closing the iris diafram or by lowering the condenser and observing the phenomena thereby produced, it is possible, therefore, to estimate approximately the relative difference in refringence between two adjacent mineral plates or between Canada balsam and a mineral plate embedded in it.

In case the junction plane between the two mineral plates is not vertical, but inclined, the deflection of the rays is illustrated for the different possible cases in Fig. 58. These are drawn angle true for the refractive indices $n_1 = 1.55$, $n_2 = 1.60$ and for an angle of inclination of 40° of the junction line with the normal to the section. In all cases it is evident that the rays are deflected toward the higher refracting substance. But in the case where n_1 is above n_2 (Fig. 58, b_1, c_2) and a cone of rays is used, as in the Becke line method, either total reflection of the rays may take place and they be deflected toward n_2 or part or all of the rays may be refracted in n_1, as in Fig.

58, b_1; in this case the rays emerge in n_1, although actually bent toward n_2, and if the microscope be focussed on the point D the maximum intensity will appear on the side of n_1; and a false inference might be drawn from the position of the Becke line relative to the junction line between the two minerals. As the objective is raised, the actual line would recede from the junction line on the side of n_1 and one might easily conclude that n_1 had the higher refractive index. Careful attention to these phenomena is sufficient, however, to obviate all such errors. The dotted lines in Fig. 58 indicate the paths of the rays which are transmitted just beyond the boundary line between n_1 and n_2.

Still another factor influences the appearance of the image, and is made use of in practical microscope work. The average thickness of a thin section of rock is about 0.02 mm.; the equivalent optical length of any given plate is its thickness increased in the ratio of its refractive index (Fig. 59).

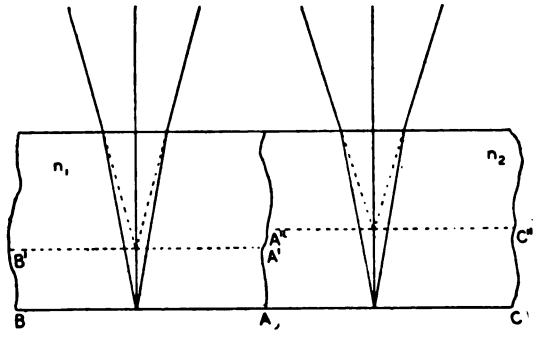


FIG. 59.

If, therefore, the objective be focussed sharply on the lower surface AB of one of the mineral sections of refractive index n_1, the lower surface AC of an adjacent plate of different refractive index will not appear, as seen through the plate itself, to be in sharp focus and the objective must be raised or lowered to obtain a good image, with the result that certain of the mineral grains stand out in relief, while others appear to be below the general level, the rule being that the higher the refractive index of a plate the more pronounced its relief. As the surfaces of thin sections are not highly polished, the phenomena of irregular refraction and total reflection appear on the surfaces of grains whose refractive index is greatly different, either higher or lower, from that of the enveloping Canada balsam of the mount. Such mottled, pock-marked surfaces (shagreened surfaces) are characteristic of a great difference in relative refringence between the mineral plate and the surrounding Canada balsam.* The scattering of light under these conditions is an important factor bearing on the development of the image by the optical system of the microscope.

*The average refractive index of Canada balsam has recently been redetermined by W. Schaller and F. Calkins on several hundreds of thin sections of different sources and ages and found to average about 1.540 (Amer. Journ. Sci. (4), **29**, 324, 1910) and to vary between 1.535 and 1.545. The refractive index of the uncooked balsam is about 1.524; see also E. A. Wülfing, Sitzber d. Heidelberger Acad. d. Wissen. Math.-natur. Kl. 1911, 1–26.

THE IMMERSION METHOD.

This method is based on the principles of oblique illumination outlined above and was apparently first described by O. Maschke in 1871, and again by him in greater detail and with the correct explanation in 1880.* Maschke used and described the effects not only of oblique but also of central illumination. To obtain oblique illumination, he either shifted the substage diafram of the Abbe condenser or shut off half the field by means of a black opaque strip placed between the objective and the grain under observation. In his description he includes a list of refractive liquids suitable for use with natural minerals. He used and observed the white-line effect, now usually known as the Becke line.

In 1880 Thoulet† suggested the Thoulet solution as an appropriate refractive index liquid which can be diluted with water and a series of refractive liquids thus prepared. His method for determining the relative refractive index of the mineral consisted in noting its disappearance in the liquid (absence of relief and of the shagreened surface) and did not furnish results of a high order of accuracy.

In 1884 and 1885, C. Christiansen‡ in considering the cause of white pigments, came to the conclusion that they consist of colorless bodies in a finely divided state, just as powered transparent glass appears white. He found that on immersing a quantity of powdered glass in a glass trough with transparent sides and filled with a suitable liquid, not only did the white color disappear, but the emergent transmitted light was practically monochromatic and of the color for which both glass grains and liquid had the same refractive index, the remaining light having been scattered, by refraction and total reflection, at the surfaces of the innumerable glass particles immersed in the liquid. By changing the composition of the liquid slightly or by heating it, he was able to obtain monochromatic light of different wave-lengths. He found that if such a mixture of glass grains and refractive liquid be placed in a hollow glass prism and then observed in sodium light, a sharp sodium line was visible and from it the refractive index of the mixture could be determined. After the grains of glass had settled to the bottom the clear liquid above had a different refractive index from that of the mixture. If the volume of the powder be V_1, that of the liquid V_2, V_1+V_2 that of the mixture, and n_1, n_2, and N the corresponding refractive indices for sodium light, then the relation obtains

$$(V_1+V_2)N = V_1n_1+V_2n_2 \qquad (1)$$

Theoretically this law of Christiansen states that the time in which light passes through the mixture, is equal to the sum of the times in which it passes through the components. This equation may also be written in the form

$$(V_1+V_2)\ (N-1)\ = V_1\ (n_1-1)+V_2(n_2-1)$$

and agrees accordingly with Gladstone's law of the refractive equivalent. If the compositon of the liquid is changed slightly, then

$$(V_1+V_2).N' = V_1n_1+V_2n'_2 \qquad (2)$$

*Pogg. Ann., 145, 565-568, 1872; Wiedemann's Ann., 11, 722-734, 1880.
†Bull. Soc. Miner, 3, 62-68, 1880.
‡Untersuchung über d. optischen Eigenschaften von fein verteilten Körpern, Wiedemann's Ann. d. Physik, 23, 298, 1884; 24, 439, 1885.

On combining equations (1) and (2) we find

$$\frac{N-n_2}{N'-n'_2}=\frac{N-n_1}{N'-n_1}$$

from which equation the refractive index n_1 of the substance can be determined with a high degree of accuracy. (Probable error in the fourth decimal place.)

Although not strictly germane to the present subject, these observations and conclusions of Christiansen have been outlined above because of their bearing on the apparent refractive index of fine microscopic mineral aggregates embedded in glass, in which case the observed refractive index of the whole as determined by the immersion method is neither that of the mineral nor of the glass. Although such conditions may never arise in the study of natural rocks, in artificial silicate melts they have been observed and have been so interpreted. In case the mineral particles are birefracting and distributed irregularly and in overlapping aggregates, the scattering of the light by refraction and reflection is pronounced because of the differences in the refractive indices of differently oriented particles and the slide appears dusty and not properly transparent.

In 1892 J. L. C. Schroeder van der Kolk* described in detail practically the same method outlined by Maschke except that he obtained oblique illumination by placing a metal strip directly above the condenser and immediately below instead of just above the grain under examination. He added greatly to the number of liquids suitable for such work and published a list† of over 300 minerals arranged according to their refractive index, with notes on special individual features useful for their determination. The immersion method is frequently called the Schroeder van der Kolk method. In 1896 H. Ambronn‡ described an immersion method for determining, under the microscope, the relative dispersion of two adjacent bodies and suggested its use in practical microscopic diagnosis. He observed both the spectral colors produced by dispersion in the rear focal plane of the objective and also the colored fringes formed along the margin of an immersed glass plate. With large grains the spectrum in the rear focal plane of the objective can be measured by means of the Bertrand lens and ocular with scale as used for interference figure measurements, and the refractive indices for the different colors may be thus determined, provided the focal length of the objective be known. His method has not been used, however, by microscopists to any extent.

In 1900 several changes were suggested by the present writer§ to facilitate the application of this immersion method to petrographic microscopical work. In place of the opaque strips immediately above or below the preparation, a sliding shutter with different-sized apertures was attached below the polarizer and could be inserted or withdrawn at will. Still simpler and equally efficient is the finger, which is placed just below the polarizer and thus casts a shadow over any part of the field. Practically the same effect

*Zeitschr. f. wiss. Mikroskopie, 8, 456–458, 1892; Kurze Anleitung zur mikroskopischen Krystallbestimmung. Wiesbaden, 1898.

†Tabellen zur mikroskopischen Bestimmung der Mineralien, Wiesbaden, 1900.

‡Farbenerscheinungen an d. Grenzen farbloser Objekte im Mikroskop. Ber. Sachs. Gesel. d. wissen. math. phys. Kl., 1896, 1–8.

§Tschermak's miner. petrog. Mitteil., 20, 239, 1901; also Amer. Jour. Sci. (4), 17, 385–387, 1904.

can be obtained by tilting the reflector itself. The results of shading part of the field by any of these methods are illustrated in Fig. 60, *a*, *b*, in which the refractive index of the mineral grain n_1 is considered less than that of the liquid n, which, in turn, is lower than that of the second grain, n_2. Fig. 60, *a*, is drawn on the assumption that the substage condenser has been lowered to some extent and the observations are made with a low-power objective. With the field thus half in shadow, the mineral section is observed

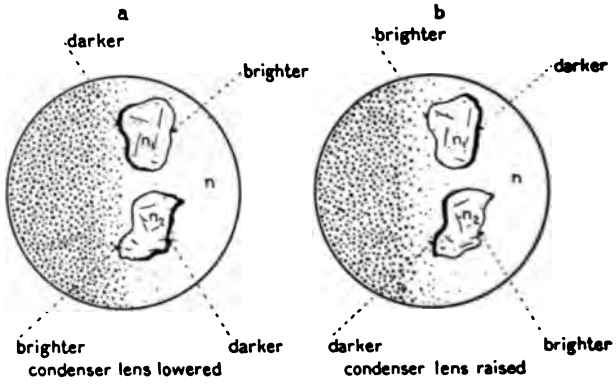


FIG. 60.

at the indistinct edge of the shadow which is purposely not in sharp focus. If the edge of the mineral section adjacent to or partially within the half shadow is brighter than the surrounding field, while the opposite edge is darker, the mineral has a higher refractive index than the liquid; and *vice versa*, if the bright edge be on the opposite side of the grain away from the shadow the mineral has a lower refractive index. In case both liquid and mineral have the same refractive index for yellow light the edge of the grain next the shadow appears orange-red, while its opposite edge is pale blue (Fig. 61).

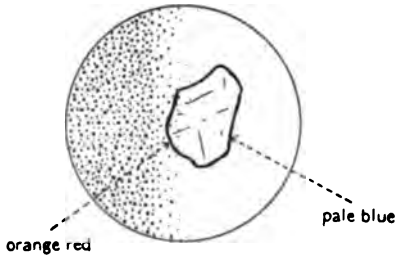


FIG. 61.

It should be noted that, if the condenser be raised so that the cone of rays impinging on the grain is convergent instead of divergent (beyond the point where the edge of the finger or stop appears in sharp focus), the above phenomena are reversed (Fig. 60, *b*). This condition is convenient, since it allows the observer to reverse the phenomena quickly and thus to check his determination in uncertain cases (Figs. 60, *a*, *b*). Unless care be taken, however, to set the condenser correctly, false inferences as to the relative refractive indices can easily be drawn. After some practice the above rule is applied automatically, but, until then, it is well to test it on substances of known refractive indices.

If high-power objectives be used the phenomena are less distinct unless it happens that the numerical aperture of the condenser exceeds that of the

objective noticeably and only marginal rays are used. This principle, which is familiar to biologists as that which underlies dark-ground illumination in which both reflection and refraction are essential factors, is rarely used by petrologists, but is required whenever high powers are employed. Two methods* have been suggested for this purpose: (*a*) by the use of a condenser of wide aperture in which the central rays are stopped out (hollow-cone illumination), or, (*b*) by using a narrow beam of light from the condenser and a small stop in the eye-circle of the ocular, as first suggested by S. Exner† in 1885 in his microrefractometer, which in turn has been applied by F. Becke‡ to microscopic work.

The present writer has also had constructed the small device shown in Plate 1, Fig. 2, which fits above the ocular and carries a rotating plate *c*, into which small disks of cover-glass have been inserted; on these in turn small, thin, opaque brass disks of different sizes (0.5 to 3 mm. diameter) are cemented and serve as central stops when placed in the eye-circle of the ocular. With this device both dark-ground illumination and oblique illumination as well can be satisfactorily obtained. In the examination of very minute particles fairly high powers are frequently necessary and this method of oblique illumination is then the only feasible one not involving complicated apparatus. In applying the method care should be taken to place the small opaque disks precisely in the plane of the Ramsden disk above the ocular. As a general rule, it is unnecessary to use high powers and the above device is rarely required.

THE BECKE LINE METHOD.

In principle this method,|| which is described in the standard text-books, does not differ essentially from that described by Maschke. As noted above, a narrow cone of incident light is required and the concentration of light along the margin of the higher refracting of two adjacent mineral plates is used to determine their relative refringence. On raising or lowering the objective, the intense band of white light (Becke line) is observed toward or away from the margin, the rule being that on raising the objective the Becke line moves toward the mineral of the higher refractive index. High-power objectives are ordinarily used in tests with the Becke line, because of their higher numerical aperture and consequent sharper resolution in depth. If the aperture of the substage condenser be not sufficiently decreased, the light is concentrated on both sides of the boundary plane; there are apparently two Becke lines present, moving in opposite directions, and one is at a loss to determine which is the correct one. The remedy is obviously to reduce the aperture of the cone of illumination either by lowering the condenser or by closing the iris diafram, or by adopting both methods. (See page 53.)

*For detailed descriptions of these methods and of the principles underlying them, see Sir A. Wright, Principles of Microscopy, London, 1906.

†Archiv. f. Mikroskop. Anat., **25**, 97, 1885.

‡S. W. A. **102**, 358–376, 1893; T. M. P. M., **13**, 385–388, 1893.

||F. Becke, S. W. A., I, Abt., **102**, 358, 1892; T. M. P. M., **13**, 385, 1892. Compare also the recent paper by G. W. Grabham (Miner. Mag., **15**, 335–349, 1910), in which a detailed discussion of the Becke line method is given and certain features are emphasized which are not ordinarily taken into consideration, especially the effects produced by obliquely incident rays and by inclined junction planes. See also J. L. C. Schroeder van der Kolk, Zeitschr. Wissen. Mikrosk, **8**, 456–458, 1892.

Under the same conditions, the accuracy of the Becke line method and of the oblique illumination method is about the same when applied to mineral grains immersed in refractive liquids. With monochromatic illumination (strong sodium light) the refractive index of an isotropic clear mineral grain can be determined with an accuracy of ±0.001, even though the diameter of the grains is only 0.01 to 0.02 mm. In case ordinary daylight illumination be used the accuracy for about 550 $\mu\mu$ is less, about ±0.002 to 0.003 on favorable sections.

For birefracting mineral grains or plates the different refractive indices (γ' and α') of the plate are ascertained by using plane polarized light and allowing the vibrations of the transmitted light to take place along the principal ellipsoidal axes of the section. The principal refractive indices of the mineral are determined by using grains normal either to a bisectrix or to an optic axis and determining the value of the different transmitted waves.*

In general it may be stated that the method of oblique illumination and the Becke line method are universally applicable, but that in certain instances the one method is better suited to meet the conditions than the second, and vice versa. As a rule the method of oblique illumination has certain advantages, since it requires only that the field be shaded by placing the finger in front of the polarizer or reflector and is used with low-power objectives so that the relative refractive indices of the whole aggregate of sections in the field can be seen at a glance, while in the Becke line method a higher power objective is advisable, the substage diafram must be closed or the condenser lowered, and practically only one mineral grain can be tested at a time. The oblique illumination method throws at once the elements of the field into relief, and the contrast between the higher and lower refracting grains is sharply brought out. This last feature of the method of oblique illumination is especially useful in the determination of orthoclase in a thin section when it appears only in minute grains and is surrounded by plagioclases. The refractive indices of orthoclase are considerably lower than those of the plagioclases or of Canada balsam and its sections appear, therefore, as small pits in the general background of the slide and are readily recognized; whereas, if the Becke line method were used, the edges chosen for test might not include an orthoclase edge and the orthoclase might be overlooked altogether.

In actual microscopic work, both with thin sections and with powder sections mounted in liquids, the two methods are used indiscriminately and often both are applied, the one serving as a check on the results obtained by the other.

Objection might be made to the application of these methods, all of which involve oblique illumination, to birefracting minerals in which the refractive index varies with the line of propagation of the transmitted wave. For a general section this objection might have force and the results obtained be correspondingly inaccurate, but for the oriented grains, normal either to one of the ellipsoidal axes or to an optic axis, on which proper de-

*For the determination of the relative refractive indices of two adjacent mineral plates in the thin section J. W. Evans (Centralblatt für Miner., 1910, 188) has recently recommended that the section be examined under parallel nicols with the planes of vibration of the adjacent crystal plates equidistant from the principal nicol planes.

terminations can be made, the observer is able to select roughly those obliquely transmitted waves which vibrate normal to a principal section, and these remain the same even for different angles of inclination. In this respect, the Becke method, if applied to a vertical edge as illustrated in Fig. 57, is superior to the method requiring greater oblique illumination.

Other methods of refractive index determination of mineral plates in the thin section, as that of Duc de Chaulnes* and its later modifications, by E. Becquerel and A. Cahours,† by A. Bertin,‡ by C. Viola,|| and by W. O. Hotchkiss,§ are not sensitive nor accurate enough for satisfactory work and, although useful in certain instances, will not be considered further in this paper.

In practical work with refractive liquids the writer has found it convenient to use a set of liquids of refractive indices ranging from 1.450 to 1.840, the refractive index of each successive liquid differing from that of the foregoing by 0.005. The refractive indices of the liquids are determined directly on an Abbe-Pulfrich total refractometer and their constancy checked every three months at least.¶ Experience has shown that the indices of the liquid mixtures do not vary over 0.002 in a year, while the average change in refractive index of a liquid for temperature rise is about 0.001 decrease for every 3° C.

The following is the set of liquids at present used in the Geophysical Laboratory for this purpose. It differs from previous sets** slightly, in that the number of liquids employed is smaller and their dispersion relatively low. The refractive indices were measured on a total refractometer and those above 1.74 by the hollow-prism method in sodium light.†† The glass hemisphere of the Abbe-Pulfrich total refractometer ($n_{Na} = 1.907$) is soft and so readily attacked by chemicals that it should not be exposed to the action even of methylene iodide, as with time this liquid attacks its surface and destroys the polish; the same holds true for the more highly refracting liquids of the list below. The liquids are kept conveniently in small dropping bottles with ground-glass dropper and cap, which interposes two ground joints to prevent evaporation. The set is so prepared that the refractive index of each successive liquid is 0.005 higher than that of the one preceding it. The mixtures are shown in table 4, page 98.

In the preparation of the mineral grains for examination, the following details have been found by experience to be useful. Larger fragments are broken up and reduced to powder 0.05 mm. or less by tapping them (not rubbing) with a pestle in an agate mortar. A short cylinder of brass, into

*Hist. Acad. R. Sci., Paris, 1767; 1770, 162; Mém. Acad. R. Sci., 1767, 423.
†Comptes Rendus, **11**, 867–871, 1840; Pogg. Ann., **51**, 427–432, 1840.
‡Ann. Chim. Phys., **26**, 288–296, 1849; Pogg. Ann., **76**, 611–612, 1849.
||T. M. P. M., **14**, 534–562, 1895; **16**, 150–154, 1897; also Rendiconti, R. Acad. Linc. **5**, 212–216, 1896.
§Am. Geol., **36**, 305–308, 1905.
¶Refractive index indicators have been suggested by Michel-Lévy (Etude sur la détermination des feldspaths, Paris, 62, 1894); and also by Souza-Brandao (Centralblatt f. Min., 14–18, 1904).
Amer. Jour. Sci. (4), **27, 35, 1909; for the measurement of the dispersion of the liquids used in the preparation of the present set, the writer is indebted to Dr. H. E. Merwin, of this Laboratory.
††Anton Pauly (Zeitschr. f. Wiss. Mikrosk., **22**, 344–348, 1905) has recently described an ingenious method for measuring the refractive index of a liquid by means of sections of calcite and siderite cut parallel with the axis. By noting the angle 2ϕ between the positions at which the mineral plate disappears in the liquid, he calculates the refractive index of the liquid from the equation

$$n = \sqrt{\frac{\omega\epsilon}{\epsilon^2 \sin^2\phi + \omega^2 \cos^2\phi}}$$

where ω and ϵ are the principal refractive indices of the mineral. Pauly claims an accuracy of 2 or 3 in the fourth decimal place for this method.

which the pestle fits, serves to confine the fine grains to the mortar during the tapping process. A small platinum spatula (prepared by hammering thin a short piece of platinum wire 1 mm. diameter) serves to convey the mineral powder from the mortar to the object glass, where it is mounted in a drop of the refractive liquid. In actual work care should be taken that everything is perfectly clean in order that no foreign particles are introduced which might lead the observer to false conclusions regarding the composition of the preparation. The drop of refractive liquid should be placed to one side and not directly on the powder to be examined, in order that the dropper may not be contaminated; and the powder can then be immersed by drawing it by means of the tilted cover-slip into the liquid.

TABLE 4.

Refractive indices.	Liquids.
1.450–1.475	Mixtures of petroleum and turpentine.
1.480–1.535	Turpentine and ethylene bromide or clove oil.
1.540–1.635	Clove oil and α-monobromnaphthalene.
1.640–1.655	α-monobromnaphthalene and α-monochlornaphthalene.
1.660–1.740	α-monobromnaphthalene and methylene iodide.
1.740–1.790	Sulfur dissolved in methylene iodide.
1.790–1.960	Methylene iodide, antimony iodide, arsenic sulfide (realgar), antimony sulfide (stibnite) and sulfur.*

THE DIRECT MEASUREMENT OF REFRACTIVE INDICES.

For this purpose special apparatus and specially prepared crystal plates or prisms are required. Many different methods have been suggested for measuring refractive indices and are described in detail in the text-books and need not be repeated here. For the petrologist, the total refractometer method is the simplest and most convenient, as it requires only a single polished surface of any orientation to furnish the three principal refractive indices (α, β, γ) of the crystal. With this method the angles of total reflection are observed for different azimuths of the crystal plate mounted on the glass hemisphere of the total refractometer in a liquid of higher refractive index. In each position of the crystal plate there are, in general, two bands or limiting lines of total reflection; if these lines be observed for all azimuths of the crystal plate, they form two continuous curves from which the refractive indices α, β, γ can be obtained directly, γ being the maximum value of the upper curve, α the minimum of the lower curve, and β either the minimum value of the upper curve or the maximum of the lower. Of these two values for β the correct one is obtained either (1) by means of a second plate, in which case the correct value of β is common to both plates,†

*This last group of mixtures has been prepared by Dr. Merwin, of this Laboratory, whose results will be published in the near future in the American Journal of Science. Measurements by the writer on the highly refracting mixtures cited by F. Zirkel, Lehrbuch d. Petrographie, 2d ed., 1, 40, 1893 (especially mercury sulfide dissolved in aniline and quiniline, with a refractive index about 2.2), gave values considerably lower (n=1.80 or less) than those listed by Zirkel. The original source of these determinations is not given by Zirkel and the writer has not been able to ascertain the exact methods by which these mixtures were prepared so as to give such high refractive indices (see also A. Himmelbauer, Centralblatt Min. 396, 1909).

†Soret, Ch. C. R., **106**, 176, 479, 1888; Arch. Sc. phys. nat. Génève (3), **20**, 277, 1888; Zeitschr. Krist., **15**, 45, 1888; Also B. Hecht, Neues Jahrb. Beil. Bd. (6) 241, 1889; A. Brill, Math. Ann. **34**, 297, 1889; München. Sitzungsber., **13**, 423, 1883. L. Perrot, C. R., **108**, 137, 1889; Arch. sc. Phys. nat. Geneve (3), **21**, 113, 1889; C. Viola, Zeitsch. f. Krist., **31**, 40, 1890; A. Lavenir, Bull. Soc. Min., **14**, 100, 1891.

or (2) by noting* the azimuths of the planes of polarization of the maxima and minima of the boundary curves of total reflection, in which case the plane of polarization of the value β', which is to be discarded, is normal to the plane of incidence, while that of the correct β is inclined to the plane of incidence. Special cases, where the section is cut nearly normal to an optic axis, often give rise to intricate relations which are of theoretical interest but practically never occur in actual work.

Viola and Cornu have shown how it is possible to determine by calculation and also by a graphical method the positions of the principal ellipsoidal axes and of the optic axes relative to the crystal plate under examination; in short, to ascertain practically all of the optical constants of the crystal from the observations on the single polished plate by means of the total refractometer. The following graphical method, which is apparently new, illustrates the principles on which their methods are based.† In the stereographic projection, Fig. 62, let the directions $Z'A$, $Z'B$, $Z'C$, be the observed

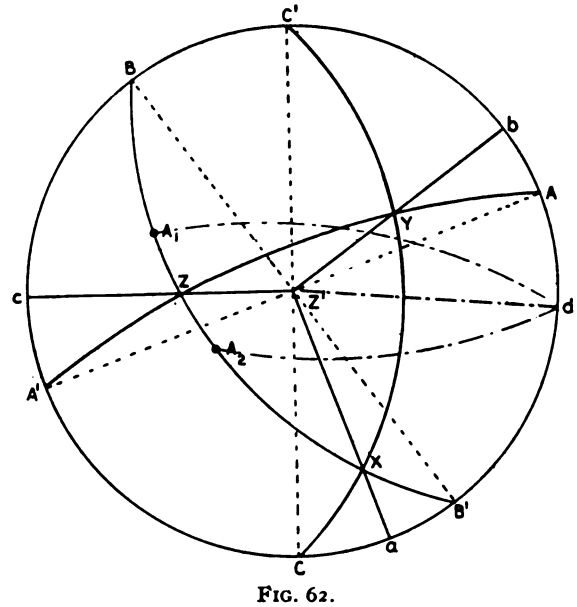


FIG. 62.

azimuths of the maxima and minima of the boundary curves which correspond to the three principal refractive indices. These are then the lines of intersection of the principal planes of the triaxial ellipsoid with the crystal surface on which the measurements were made and which is the plane of projection in the figure. Draw $Z'a$, $Z'b$, $Z'c$ normal to $Z'A$, $Z'B$, $Z'C$; then the three principal ellipsoidal axes must be contained in these planes $Z'a$, $Z'b$, $Z'c$; and their positions are readily found by determining graphically those great circles YZ, YX, and XZ, whose intercepts between the radii $Z'b$ and $Z'c$, $Z'b$ and $Z'a$, $Z'a$, and $Z'c$, respectively, are 90°. From the figure

*C. Viola, Accad. d. Linc. Rend., **1**, 279, 1899; Zeitschr. Krist., **36**, 245, 1902; A. Cornu, C. R., **133**, 125, 463, 1901; Bull. Soc. Min., **25**, 7, 1902.

†Compare F. Pockels, Kristalloptik, 130–131, 1906; also Duparc and Pearce, Traité de Technique Minéralogique et Pétrographique, 390–391, 1907.

it is evident that the principal planes are inclined to the plane of projection; also that if A_1 and A_2 be the optic axes there is a direction d for which the plane of vibration is in the plane of incidence, as required both by theory and observation.

In observations with the total refractometer, it is important that the instrument be in accurate adjustment.* Unfortunately the crystal refractometers now on the market are not designed with reference to adjustment facilities and their adjustment is often an exceedingly difficult and tedious matter. In observing the boundary curves from small mineral plates it is essential that the source of light be imaged in the object plane, and again together with the object in the eye-circle of the ocular of the telescope. In this eye-circle the light from all parts of the field except the mineral plate can be cut off by means of a proper stop or diafram and the boundary curves from plates 1 mm. in diameter readily observed. In the case of a finely crystalline aggregate the writer has been able, in certain instances, to observe the limits on the boundary curves which appear under these conditions as indistinct shadows and which correspond to the greatest and least refractive indices. For practical purposes, it is convenient to prepare a curve plotted on a large scale on which the refractive index corresponding to any observed angle of total reflection can be read off directly to the fourth decimal place. With accurate work, the probable error of refractive index determinations by this method on suitable plates is not over 2 in the fourth decimal place. The interference fringes which appear occasionally when the observations are made in reflected light can usually be decreased by using a higher refractive liquid or by increasing the distance between the plate and the glass hemisphere by means of some small support, as a paper or cork ring.

The total refractometer of Wallérant, which is attached to the microscope and is employed only on thin uncovered and polished sections of rocks, has proved a useful instrument in the hands of Professor Wallérant, who has applied it to the determination of the feldspars and other minerals. The present writer has unfortunately had practically no opportunity to work with this instrument and is not in a position to judge of its fitness for application to fine-grained preparations. Obviously it can not be used on preparations whose granularity is expressed in hundredths of millimeters.

*Compare F. E. Wright: Die Justierung des Abbe-Pulfrichschen Kristallrefraktometers, Zeitsch. f. Instrumentenkunde, 28, 201–206, 1908.

CHAPTER III.

BIREFRINGENCE.

The standard methods for the determination of birefringence involve the use either of the Babinet* compensator, the Chrustschoff compensator,† the Bravais-Biot compensator,‡ the Michel-Lévy comparator, the graduated wedge, or the Michel-Lévy color chart. The methods of application are treated at length in the standard text-books and need not be presented here. Their relative sensitiveness, however, and the degree of accuracy obtainable by the use of such methods in actual work merits consideration, since birefringence is one of the characteristic features of birefracting minerals.

One of the most accurate methods for determining, in monochromatic light, the path-difference between plane polarized light-waves emerging from a crystal plate is by use of the Babinet compensator. After calibration of the compensator in sodium or other monochromatic light, the displacement of the bands produced by a birefracting plate in the same monochromatic light is measured directly on the scale of the Babinet compensator. This displacement is directly reducible to path-differences in millimeters and, if the thickness of the plate itself in millimeters be known, its birefringence can be figured directly from the formula

$$\gamma - \alpha = \frac{l\lambda}{d\delta}$$

in which λ = wave-length of light used expressed in millimeters; δ = distance in scale divisions between two successive bands on the Babinet compensator for the wave-length λ; l = displacement (in scale divisions) caused by inserted plate; d = thickness of plate. If the birefringence for different wave-lengths be desired, it is necessary either to calibrate the compensator for each wave-length used or to calculate the required constants from the known dispersion of quartz.§

As the determination of the thickness of the plate is much less accurate than that of the path-difference, graduated wedges|| have been substituted for the expensive Babinet compensator. These quartz wedges are usually so cut that the path-difference of the emergent waves can be read directly from the scale engraved on their upper surface. In the wedges of Césaro, Amman, Leiss, and Evans an arbitrary scale is used and afterwards calibrated, while in the wedge of Siedentopf and that of the writer the pitch of the wedge is so calculated that, from the scale divisions, the path-difference in $\mu\mu$ is read off directly.

*Jamin. Ann. de Chim. et Phys., **29**, 263, 1850; see also K. E. F. Schmidt, Zeitsch. Inst., **11**, 439, 1891. Wied. Ann. **35**, 360, 1888; J. Macé de Lépinay, Journ. de Phys. (2), **10**, 204, 1891.

†Zeitschr. Kryst., **30**, 389, 1899.

‡Ann. Chim. Phys. (3), **43**, 139, 1855; Biot, Traité de Phys.; see E. Mascart, Traité d'Optique, **2**, 61, 1892.

§See Michel-Lévy, Bull. Soc. Miner. Fr., **7**, 57, 1884; Les Minéraux des Roches, 57, 1888. F. Becke, T. M. P. M., **22**, 378, 1903. V. de Souza-Brandao, Centralblatt f. Miner., 23–29, 1905. C. Hlawatsch, T. M. P. M. **23**, 415–450, 1904; **21**, 107–155, 1902.

||G. Césaro, Bull. Acad. Roy. Belg. Cl. Sciences, 208, 1893. J. Amman, Zeitschr. wiss. Mikrosk., **11**, 440–454, 1894. C. Leiss, Neues Jahrb. B. B., **10**, 426–428, 1896. J. W. Evans, Min. Mag., **14**, 87–92, 1905. H. Siedentopf, Centralblatt f. Min., 745–746, 1906. F. E. Wright, Amer. Jour. Sci. (4), **29**, 417, 1910.

The wedge or birefractometer of Amman consists of a quartz wedge placed in the lower focal plane of the eye lens of a Huygens ocular. It is inserted by means of a micrometer screw until the interference tint of the mineral plate under investigation is exactly compensated and the path-difference thus obtained. In the Césaro wedge compensation is carried only to the sensitive tint either between crossed or parallel nicols. The Leiss modification of the Amman birefractometer covers only half the field and carries, in addition, a micrometer scale directly beneath the wedge, so that accurate settings can be made. On the upper surface of the graduated quartz wedge of Evans a scale is engraved which gives directly the path-difference of the emergent waves. The second wedge of Evans consists of two adjacent quartz wedges in combination, the direction of elongation of the one half being $\mathfrak{c}$, that of the second $\mathfrak{a}$. This wedge is inserted until in the one half the interference color of the mineral plate is exactly compensated; in this position the path-difference of the waves emerging from the second half is twice that of those from the mineral plate alone. The Siedentopf wedge consists of two wedges superimposed and with different ellipsoidal axes parallel with the elongation. Where both wedges have the same thickness a band of exact compensation occurs and the effect of the combination for that particular point is neutral. The wedge is infinitely thin at that point and increases in thickness on either side of the dark achromatic band. The wedge of the writer consists of a quartz plate and a quartz wedge after the manner of the combination wedge* and is so calculated and cut that the divisions on the scale read directly to 10 $\mu\mu$, the zero line of the scale coinciding with the achromatic line. In principle all these wedges are practically identical with the Babinet compensator. They are inserted in the lower focal plane of a Ramsden ocular and require the use of a cap nicol. If the relative dispersion of the substance under examination be not greatly different from that of the quartz, the graduated wedge furnishes good results even in white light. By the use of the graduated wedge, the birefringence formula becomes $\gamma - \alpha = \frac{\delta}{d}$, where δ = direct reading in $\mu\mu$ from the scale on the wedge and d = thickness of the plate (in $\mu\mu$).

If the interference color of the plate under observation is of a higher order than can be compensated by the wedge or Babinet compensator, a birefracting quartz plate of known path-difference can be inserted and the wedge thickened optically by just that much. In the actual determinations of the birefringence parallel beams of polarized light should be used to obviate the disturbing effects of obliquely incident light. The condenser system should be lowered or, if possible, removed altogether. The effect of obliquely transmitted light-waves can be readily observed on any strongly birefracting substance where the interference color changes perceptibly on passing from central to oblique illumination.†

In 1908,‡ the writer described a combination wedge consisting of two adjacent combination quartz wedges so cut that the least ellipsoidal axis $\mathfrak{c}$

*T.M. P. M., **20**, 275, 1901; Jour. Geol., **10**, 23–25, 1902; E. Sommerfeldt has recently suggested (Zeitschr. wissen. Mikrosk., **27**, 448, 1910) that under certain conditions it would be an improvement if this wedge covered only half the field, as in the Michel-Lévy comparator and the Leiss modification of the Amman birefractometer.

†Compare J. L. C. Schroeder van der Kolk, Zeitschr. wissen. Mikros., **8**, 456–458, 1892; G. W. Grabham, Miner. Mag., **15**, 340–341, 1910.

‡Amer. Jour. Sci. (4), **26**, 370–371, October, 1908.

of the one was parallel with the greatest ellipsoidal axis $\mathfrak{a}$ of the second, as indicated in Figs. 44, 45. At the point a of this wedge both plate and wedge have the same thickness and the dark band of exact compensation is observed. The interference color rises slowly on either side of this band, as in the ordinary combination wedge; this part of the wedge especially is an exceedingly sensitive device for detecting anisotropy in very weakly birefracting substances. By combining this wedge with a Bravais-Biot compensator with isochromatically illuminated field it is possible to measure the path-difference of the emergent waves with great accuracy.

Shortly after the appearance of the above paper and without knowledge of the same, J. Koenigsberger* described a modification of the Bravais double plate showing the sensitive-tint, in which very thin mica plates were substituted for the thicker Bravais plates, and a low gray interference color was thus obtained. This combination is useful for the detection of weak anistropy. Following the example of Bravais, Koenigsberger has combined his plate with a Biot-Bravais compensator and is able, with the combination, to determine the path-difference with great accuracy.† With the sensitive-tint double plate Bravais was able to measure differences of slightly less than 1 $\mu\mu$, while Koenigsberger claims an accuracy of 0.0003 λ or about 0.2 $\mu\mu$ for the thinner double plate. The double combination wedge of the writer has the advantage of these plates because of its adjustable sensibility, by virtue of which the interference tint can be changed and the particular color selected which is the most sensitive under the given conditions of observation; although in the double combination wedge the field is not uniform throughout it is practically so for the small mineral grains in the thin section and no error is introduced thereby.

The Fedorow mica comparator‡ consists of a set of 15 superimposed and overlapping thin mica plates, each successive plate being about 2 mm. shorter than the one immediately below it, the whole forming a step-like wedge of known optical intervals. The plates are of such a thickness that the path-difference of the emergent light-waves (of wave-length 510 $\mu\mu$) is 127.5 $\mu\mu$ or one-quarter wave-length. This wedge is useful, but of less general application than the graduated quartz wedges and has not been generally adopted by petrologists.

The Sénarmont-Friedel|| method for measuring the birefringence is based on phenomena obtained by the use of elliptically polarized light and is a less direct method than the above methods. Just below the crystal plate whose birefringence is to be measured and whose ellipsoidal axes are placed in the diagonal position with respect to the nicols, a quarter undulation mica plate is inserted with its ellipsoidal axes parallel with the principal nicol planes. The lower nicol is then rotated until, in monochromatic light, the crystal plate becomes dark or until both field and crystal plate are equally illuminated. In the first case the angle of rotation of the lower

*Centralblatt f. Miner., Dec. 729–730, 1908; 249–250, 746–747, 1909.

†C. Kraft (Bull. International d. L'Acad. d. Sci. de Cracovie. Cl. Sciences, 310–353, 1902) also made use of this type of Biot-Bravais compensator in his work on the Newton color scale and in determining the path-differences (for $\lambda = 550\,\mu\mu$) which produced certain observed interference tints.

‡Zeitschr.-Kryst., **25**, 349–351, 1896; **26**, 251–253, 1896; **29**, 611–613, 1898. In this last paper Fedorow suggests the use of the Bravais-Stöber plate in white light and considers its accuracy twice that of the ordinary wedge.

||H. de Sénarmont, Ann. Chim. Phys. (2) **73**, 337, 1840; Pogg. Ann. Erg. Bd. **1**, 451, 1842; see also, H. Wiedemann, Pogg. Ann., **151**, 1, 1874; Ber. d. Sachs. Gesell. d. Wiss., 1872; the same method was described by G. Friedel, Bull. Soc. Min. Fr., **16**, 19–33, 1893; J. Walker, Phil. Mag. (6) **3**, 541, 1902.

nicol divided by 180° gives directly the phase difference of the waves transmitted by the crystal, this difference being exactly compensated by the mica plate and the lower nicol. The accuracy of this method is dependent on (1) the homogeneity of the monochromatic light; (2) the accuracy with which the quarter wave plate has been cut; (3) the accuracy with which its ellipsoidal axes are placed parallel with the principal planes of the nicols; (4) the accuracy of the setting of the rotating lower nicol. Friedel claims an accuracy of about $\frac{1}{180}$ of the wave-length used under favorable conditions, or about 3 $\mu\mu$ for sodium light. This accuracy is not great and the method has not been generally adopted by petrologists.

W. Nikitin* has recently described a compensator which is intended especially for measuring birefringence on thin plates showing low interference colors. The compensation is effected by a quartz plate (0.07 mm. thick) whose normal includes an angle of 25° with the axis. The path-difference is increased or decreased by rotating the quartz plate, the angle of rotation being determined directly on a graduated circle. Nikitin claims an accuracy of at least 4 $\mu\mu$ for this plate. The birefringence of the plate for the different positions can be calculated by formulas developed by Nikitin.† The accuracy of this device does not exceed that of the Babinet compensator or even that of the graduated quartz wedges, and is, moreover, limited in its application.

In the determination of the birefringence of a given crystal plate in the thin section two distinct measurements are necessary: (1) that of the thickness of the crystal plate, and (2) that of the path-difference between the emergent light-waves (in monochromatic light). The thickness of the plate can be measured either by direct contact by use of the micrometer-screw or of the spherometer, or by means of the fine adjustment screw of the microscope, or indirectly by means of the interference-color or path-difference (in $\mu\mu$) of an adjacent mineral properly cut and of known birefringence.‡ Of these different methods, the second (with fine adjustment screw of microscope) is most convenient, although possibly less accurate. The usual method consists in bringing to sharp focus‖ the upper surface of the plate and then the lower surface as seen through the plate itself, or, if the plate be resting free on the object glass, to focus on the upper surface of the glass at the side of the mineral plate. In the first case the apparent thickness must be multiplied by the refractive index of the mineral to obtain the real thickness. As the average thickness of minerals in the thin section is 0.03 to 0.02 mm. an error of 0.001 mm. in setting the micrometer-screw will produce an error of 3 to 5 per cent in the thickness determination. In ordinary microscopes this error may frequently amount to 0.002 or 0.003 mm. and the resulting error in thickness to 10 per cent. In making this measurement well-defined particles should be selected on the upper and lower surfaces of the plate and the thickness determined in the

*Zeitschr. Kryst., 47, 378–388, 1910.
†Zr. Kryst. 33, 133–146, 1900.
‡E. Mallard, Bull. Soc. Min. Fr., 6, 130–133, 1883.
‖For the accurate determination of the focus on a point, the method of Michel-Lévy (Bull. Soc. Min. Fr. 6, 153–154, 1883) of illuminating the object by a narrow beam of strong monochromatic light is satisfactory. If the point be not in exact focus sharply defined diffraction rings appear. An accuracy of $\frac{1}{1600}$ mm. is claimed for this method.

center or axial portion of the field, where the image is sharpest and the errors from the lens system are least serious.

Suppose the extreme limits of error to be 0.003 mm. or 0.0015 mm. on either side of the correct value; then an error of 5 per cent in the thickness determination may be considered probable. If this probable error be increased to 8 per cent to allow for multiplication by the refractive index and to introduce a safety factor, it can be safely assumed that the thickness of a favorable plate in the thin section, ranging from 0.03 to 0.05 mm. in thickness, can be ascertained readily to within 8 per cent of the correct value. For minerals in powder form, the thickness of the individual grains may be much greater and the thickness determination correspondingly more accurate.

On the graduated wedge the scale divisions correspond to a path-difference of 10 $\mu\mu$ in the emergent light-waves and the error of the determination is not over one division on the scale (or 0.1 mm.), which is less than 2 per cent.

The total probable error of the determination of the birefringence of a mineral plate in the thin section may amount, therefore, to 10 per cent. As the birefringence of ordinary rock-making minerals ranges from about 0.005 to 0.050, an error of 10 per cent is confined to the third decimal place.*

In determining the birefringence $(\gamma-\alpha)$ or $(\gamma-\beta)$ or $(\beta-\alpha)$ of a mineral, the optical position of the mineral plate under examination is ascertained by observations in convergent polarized light. In actual work it is not always easy to find a plate cut precisely perpendicular either to the optic normal or to the bisectrices, and it is of interest to know the percentage error caused by using sections inclined at low angles to the correct directions. For a given plate the birefringence can be calculated approximately from the usual formula,

$$\frac{\gamma'-\alpha'}{\gamma-\alpha}=\sin\theta'\,.\,\sin\theta$$

in which θ and θ' are the angles which the normal to the plate makes with the two optic axes (optic binormals) respectively. A graphical solution of the equation is given in Plate 5, in which the abscissæ and ordinates represent the angles θ and θ' respectively; the curves indicate the percentage ratio $\frac{\gamma'-\alpha'}{\gamma-\alpha}$. From this plate the values of θ and θ' for any given birefringence ratio can be found directly with sufficient accuracy for practical purposes.†

In Figs. 63–68 these relations are shown graphically in stereographic projection. In each figure the angular distance between any two successive concentric circles is 10°. Thus in Fig. 63 the positions of the sections are indicated whose birefringence is 2 per cent less than the maximum birefringence $(\gamma-\alpha)$ exhibited by a plate exactly perpendicular to the optic normal. The position of these lines of equal birefringence is different for different optic axial angles, as indicated by the lines for $2V=0°, 45°$ and 90°;

*An average of the birefringences of the 118 minerals listed under birefringence on pp. 292–295 of Rosenbusch-Wülfing gives 0.040 as the mean value, while the value of the members midway between the two limits is 0.020 to 0.025. This value represents more nearly the mean value of the birefringence of rock-making minerals than the arithmetical mean 0.040.

†Compare also Duparc and Pearce, Traité, etc., 229, 1907.

but it is evident from the figure that, in general, it is safe to assume that an inclination of 10° with the true optic normal section will cause an error of not over 2 per cent less than the correct value $(\gamma - \alpha)$ and often much less.

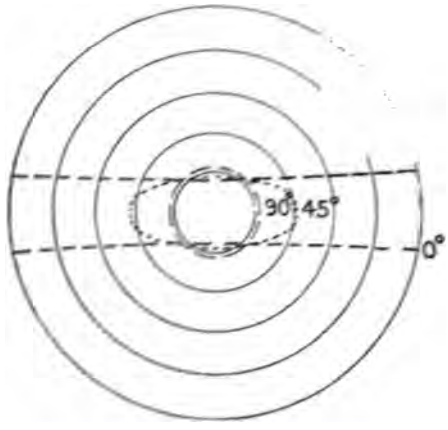


FIG. 63.

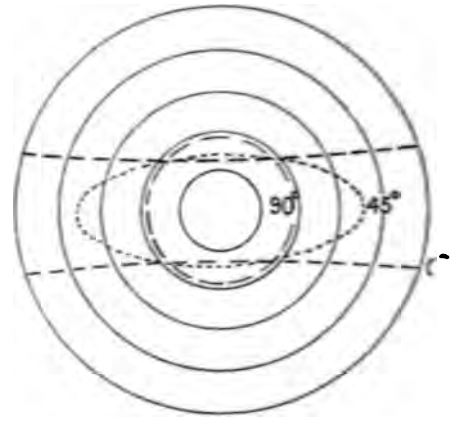


FIG. 64.

FIG. 63.—In this stereographic projection plat (circles 10° apart) the positions of the directions in a biaxial crystal whose birefringence $(\gamma' - \alpha')$ is 2 per cent less than that of the optic normal $(\gamma - \alpha)$ are indicated for the optic axial angles $2V = 0°$, 45°, and 90°. The optic normal coincides with the central point of the figure.

FIG. 64.—Stereographic projection plat showing positions of the directions for which the birefringence $(\gamma' - \alpha')$ is 5 per cent less than that of the optic normal $(\gamma - \alpha)$, which coincides with the center of the concentric 10° circles. These curves are drawn corresponding to the optic axial angles $2V = 0°$, 45°, and 90°.

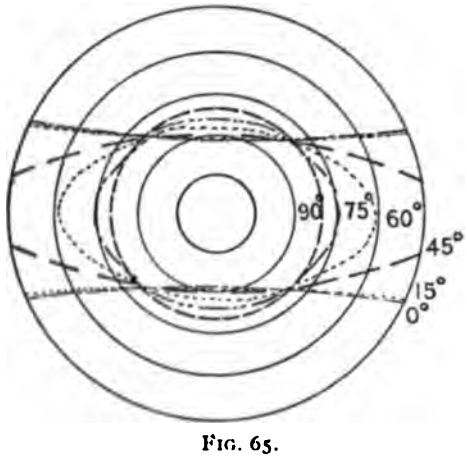


FIG. 65.

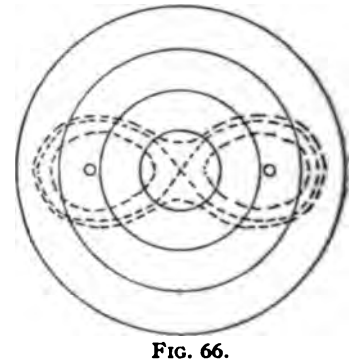


FIG. 66.

FIG. 65.—Like Fig. 64, except that the directions are indicated whose birefringence is 10 per cent less than that of the optic normal located at the center of the projection plat. The positions of the curves corresponding to optic axial angles $2V = 0°$, 15°, 45°, 60°, 75°, 90°, are indicated in this figure.

FIG. 66.—In this figure the directions whose birefringence is 10 per cent less or greater than that of the acute bisectrix (optic axial angle $2V = 45°$) are shown by the dotted curves. The dotted curve which passes through the center point (acute bisectrix) marks the directions whose birefringence is equal to that of the acute bisectrix $(\gamma - \beta)$ or $(\beta - \alpha)$, as the case may be.

In Fig. 64 lines of equal birefringence 5 per cent less than the correct value $(\gamma-\alpha)$, are drawn for different optic axial angles and show that inclinations of 15° produce errors of 5 per cent and less of the true value $(\gamma-\alpha)$, while inclinations of 20° (Fig. 65), produce errors of 10 per cent and less of the total birefringence. Similarly, for sections normal to a bisectrix, Fig. 66 indicates that for an optic axial angle, $2V=45°$, a plate cut at an angle of 7° with the bisectrix may produce a positive or negative error of 10 per cent or less in the birefringence $(\gamma-\beta)$ or $(\beta-\alpha)$. But in this case the birefringence $(\gamma-\beta)$ or $(\beta-\alpha)$ is only about 14 per cent of the total birefringence and an error of 10 per cent therefore usually applies only to the fourth decimal place. In Fig. 67 the directions for which the birefringence is

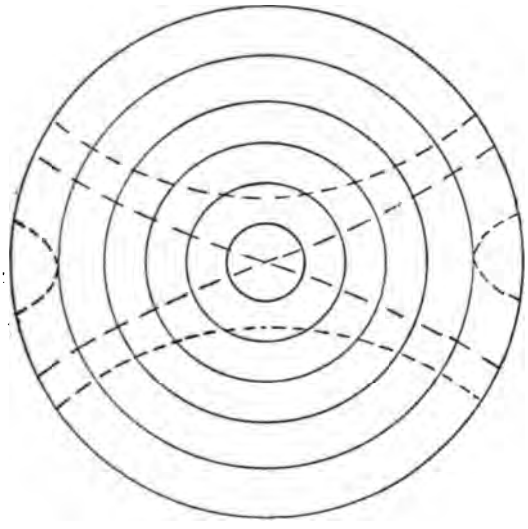


FIG. 67.

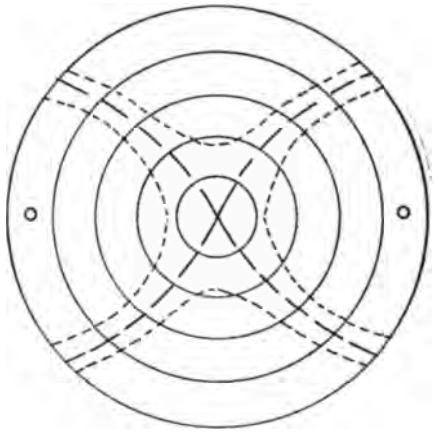


FIG. 68.

FIG. 67.—Similar to Fig. 66, except that the center of the projection plat is the obtuse bisectrix ($2V=45°$). As in Fig. 66, the directions whose birefringence is 10 per cent greater or less than that of the obtuse bisectrix are indicated.

FIG. 68.—Similar to Fig. 66, except that the optic axial angle is $2V=90°$. The dotted curves again represent the directions for which the birefringence is 10 per cent greater or less than that of the bisectrix at the center of the projection plat. In this stereographic plat, as in all preceding, the concentric circles are 10° apart.

10 per cent greater or less than $(\beta-\alpha)$ or $(\gamma-\beta)$—here about 85 per cent of $(\gamma-\alpha)$ for $2V=45°$ (obtuse bisectrix)—approach within 18° of the bisectrix. In this figure the curve indicating an increase of 10 per cent birefringence is 50° and over from the obtuse bisectrix. Plates making an angle of less than 20° with the bisectrix can, therefore, be safely assumed to furnish values of $(\beta-\alpha)$ or $(\gamma-\beta)$ which are not over 10 per cent in error. An inclination of 8° would produce an error of about 2 per cent in $(\beta-\alpha)$ or $(\gamma-\beta)$. In Fig. 68, the rate of change of birefringence for sections at different angles with the bisectrix is indicated on the assumption that $2V=90°$; there an inclination of 12° and over is required to effect a negative error of 10 per cent in the birefringence $(\gamma-\beta)$ or $(\beta-\alpha)$ and 18° or more to effect an equal positive error. Assembling these data, it may be assumed in general that the birefringence of a plate inclined at an angle

of 5° to 10° with the true direction (optic normal or bisectrix) will be in error about 2 per cent of the true value for $(\gamma-\alpha)$, $(\gamma-\beta)$ or $(\beta-\alpha)$; an inclination of 10° to 15°, about 5 per cent, while for 15° to 20° inclination the error may be as much as 10 per cent of the correct value desired. By means of the coordinate scale ocular described in Chapter V, the angular inclination of the section can be ascertained and the probable error due to this cause thus eliminated.

In actual practice, therefore, the method of procedure in the determination of the birefringence of a mineral plate in the thin section or a mineral grain is to measure first the thickness by one of the methods noted above and then to insert the graduated combination wedge and determine, under crossed nicols and in monochromatic light, the path-difference between the interfering light-waves. For less accurate work the direct determination of the interference color and equivalent path-difference, as indicated on the standard Michel-Lévy color chart, is sufficient. The actual error of such a determination should not exceed 10 per cent of the correct value of the birefringence of the section. The probability of finding a section making an angle within 10° of a particular direction is about 1 in 66; and a section within 20° about 1 in 16.

THE NEWTON COLOR SCALE.

The succession of colors produced by the interference of light-waves reflected from the surfaces of two plates of glass or other substance including a thin film of air resembles closely the series of interference colors observed on the insertion of a quartz wedge between crossed nicols and is usually reproduced in text-books on petrology as the Michel-Lévy standard color chart.* In the formation of the Newton color scale the following factors are of importance.

If monochromatic light be used the series of brilliant interference colors is replaced by a set of dark bands or lines with intervening light strips. For any given thickness or path-difference the intensity of the monochromatic light emitted is found from the standard equation

$$I' = I \sin^2 \frac{\pi\Delta}{\lambda}$$

where I = intensity of illumination; Δ = path-difference, and λ = wave-length of light used. For white light the expression for total intensity, T, is

$$T = \int I'_{(\lambda)}\, d\lambda = \int I_{(\lambda)} \sin^2 \frac{\pi\Delta}{\lambda}\, d\lambda$$

In order to solve this expression it is essential that we know the intensity of each color of light in the illuminating source. A standard white light has not yet been adopted. Nutting† has recently suggested as a standard for white light that which is reflected by "a mat non-selective surface of high-reflecting power illuminated at the earth's surface by the midday sun." Even this standard varies considerably on different days, but it

*Bull. Soc. Min. Fr., 6, 145–161, 1883; 7, 43–44, 1884; Minéraux des Roches, 64, 1888.
†P. G. Nutting, Circular, Bureau of Standards, No. 28, 6, 1911.

seems to be the best standard of reference available. Ives has proposed as a standard the light emitted from a black body at 5000°, but this also is not entirely satisfactory for many reasons. In determining the visual intensity of a light source for different parts of the spectrum, it has been found by experience that "the ratio of light to energy (the visual sensibility of the eye) varies with the wave-length and, in the range 1 to 10 meter candles (roughly), with the intensity as well. This ratio depends upon the sensibility of the eye, the so-called visibility of radiation."* Visual sensibility curves for white light have been determined for high intensities and for low intensities and found not to agree precisely. Both curves have a single maximum which is located for low intensities at about 510 $\mu\mu$ and for high intensities at about 545 $\mu\mu$. At low intensities, below 1 meter candle down to the threshold value, the vision is largely achromatic (rod vision) while for intensities above 10 meter candles the vision is chiefly chromatic (cone vision.) Between 1 to 10 meter candles both the rods and cones of the retina of the eye are active and the maximum of the visibility curve lies

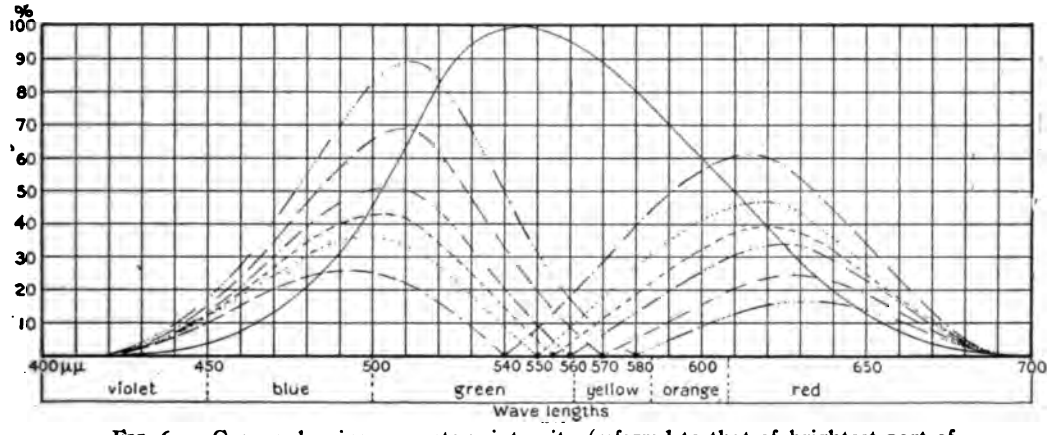


FIG. 69.—Curves showing percentage intensity (referred to that of brightest part of spectrum at 545 $\mu\mu$) of transmitted light for path-differences 540, 550, 560, 570, and 580 $\mu\mu$. The solid-line curve is the luminosity curve. The scale of the dotted intensity curves is ten times that of the solid-line luminosity curve. Thus for a path-difference 550 $\mu\mu$, the visual brightness of the color $\lambda = 500$ $\mu\mu$ is 3.6 per cent of that of the brightest color (545 $\mu\mu$) in the daylight used as source of illumination. For the path-difference 550 $\mu\mu$ the amounts of light transmitted from the red and blue ends of the spectrum are about equal (2.87 per cent in the red and 2.96 per cent in the blue end from the original light, the total amount of light transmitted for this path-difference being 5.83 per cent of the original source).

between 510 and 545 $\mu\mu$. The actual energy distribution of the visible solar spectrum for midday sun has often been measured and found to have a maximum at about 520 $\mu\mu$, the energy curve sloping gradually to about 75 per cent of the maximum at 400 $\mu\mu$ and about 80 per cent at 700 $\mu\mu$.†

From the energy and visual sensibility curves the curve of actual light can be found directly (the visibility being the ratio of the light to the energy $V = \frac{L}{E}$ or $L = E.V$). This is indicated by the full line curve of Fig. 69,

*See P. G. Nutting, circular, Bureau of Standards, No. 28, 7, 1911.
†Nutting, P. G., Circular, U. S. Bureau of Standards, No. 28, 6, 1911; Abbot and Fowle, Annals Astroph. Observatory, 2, 105, 1905.

the data being taken from the curves drawn by Nutting* and compared with the more recent determinations by Ives, the agreement between the two observers being very close. On this curve the maximum intensity at 545 $\mu\mu$ is assumed to be unity and the other intensities proportionately smaller. A graphical determination of the partial areas included by this white light curve indicates that at about 555 $\mu\mu$ the total amount of light on either side is about equal; in other words, 555 $\mu\mu$ is about the center of the visible spectrum when measured in terms of total light.

From this curve of white light visual sensibility, which indicates the approximate distribution of light throughout the visible spectrum of standard white light, it is a simple matter to calculate the relative inten-

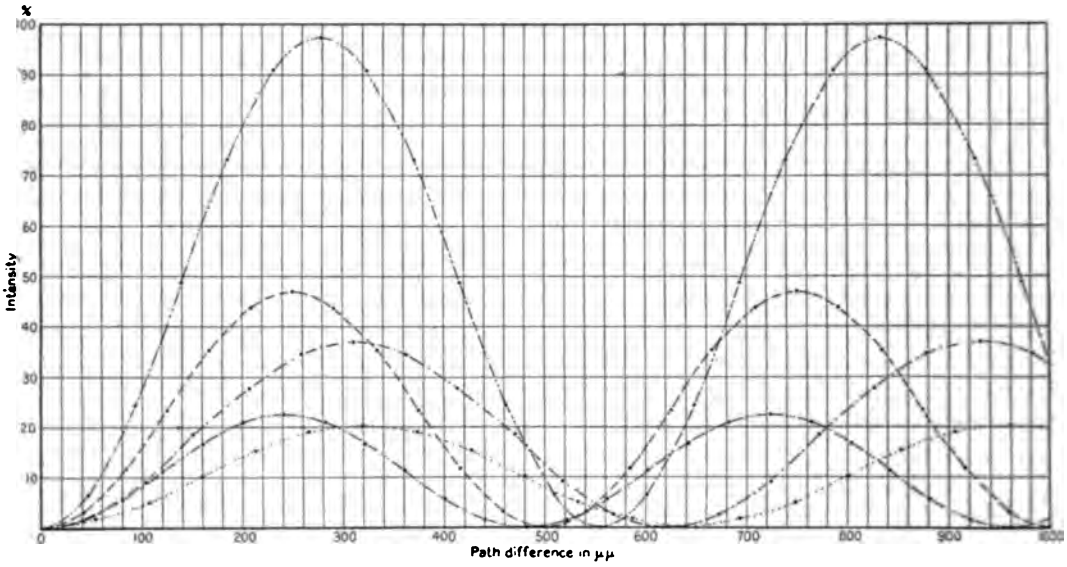


FIG. 70.—In this figure the change in intensity of different colors (λ = 640, 620.3, 555, 500, 481.2 $\mu\mu$) with change in path-difference from 0 to 1000 $\mu\mu$ is illustrated. The relative brightness of the different colors is the same as that observed in white light. The intensity minimum at about 555 $\mu\mu$ is also clearly evident in the figure.

sities of the different spectrum colors for any given path-difference between interfering waves. In plotting the values obtained from the standard intensity equation

$$I' = I \sin^2 \frac{\pi\Delta}{\lambda}$$

the intensity may be chosen as ordinate and either the path-difference Δ or the wave-length as abscissa; in Fig. 70 the path-difference has been chosen for the abscissa and curves for different colors (λ = 481.2, 500, 555, 620.3, 640 $\mu\mu$) are plotted, while in Fig. 69 the wave-lengths are the abscissæ and curves for the different path-differences are shown. In Figs. 69 and 70 the increase in intensity of total light as the path-difference increases from 0 to about 300 $\mu\mu$ is clearly shown (the interference color passing from dark gray to white and pale yellow of the first order); from about

*Circular, Bureau of Standards, No. 28, 1911.

300 to 500$\mu\mu$ the total intensity decreases rapidly to approximately 5.85 per cent of the total possible intensity at 555$\mu\mu$. At this point the total amount of light in the blue end of the spectrum is about equal to that in the red as indicated in Fig. 69. From Fig. 69 and Fig. 70 it is evident that in this region a relatively slight change (5$\mu\mu$) in the path-difference produces a great change in the relative intensities of the red and blue end of the spectrum, while the intensity for the central part of the spectrum changes only slightly and is very low. A slight change in the path-difference at this point will produce, therefore, a decided shift in the interference color from red to blue. Although the color can not be determined directly because of the fact that parts of the spectrum will combine to produce the sensation of white light, and the resulting hue will be simply the residual light combined with this white light, in the present instance the ends of the visible spectrum play the important rôle and the color does shift from dominant red to dominant blue. This path-difference of about 555 $\mu\mu$ produces a tint more sensitive to slight change in path-difference than any other and is the violet of the second order. As the total intensity is low for this tint, both rod and cone vision are active in the eye and the maximun intensity is no longer at 545$\mu\mu$, but lower down the scale; care should be taken, in making the observations, to have a strong source of light, otherwise the actual tint will change color and be no longer the most sensitive tint. The effect of the variation in chromatic sensibility of the eye on the Newton color scale has still to be determined.

In the text-books on petrography the Newton color scale is given with path-differences as determined by observers, especially by Wertheim and Quincke* in the middle of the last century, at a time when practically nothing was known of standard white light. From the above descriptions the dependence of the interference tint obtained on the source of illumination is obvious and must be considered in practical work. The path-difference for the sensitive violet in the Newton color scale is usually stated as 575 $\mu\mu$. This is evidently too high for most conditions of illumination and should be 10 or 20$\mu\mu$ lower. The exact position of the sensitive tint varies with the conditions of the weather, a deep blue sky causing it to shift toward the blue end of the spectrum, while an overcast sky gives more nearly white light. This shift in the sensitive tint can be readily seen in the microscope by observing the interference tints on a graduated quartz wedge, such as that described above. On tilting the mirror so that the light is received from the horizon or a white cloud, and then from the blue sky, a shift of 10$\mu\mu$ in the position of the sensitive tint is often observable. Ordinarily the sensitive tint is at about 550 $\mu\mu$ for quartz. In view of these conditions all determinations of path-difference based on interference colors alone are inaccurate and methods like that of the Michel-Lévy color interference chart or of the Michel-Lévy comparator are less accurate than those requiring monochromatic light.

More accurate determinations of the Newton color scale have been made by A. Rollet† and C. Kraft.‡ The values obtained by Kraft for sunlight reflected from freshly fallen snow agree closely with those recorded by

*G. Wertheim, Ann. de Chim. et. Phys. (3), **40**, 180, 1854; G. Quincke, Pogg. Ann., **129**, 177, 1866.
†Sitzb. Akad. d. Wissen. zu Wien, Abteil. 3, **77**, 177–261, 1878.
‡Bull. intern. acad. sci. Cracovie, Classe des Sci. Math. et Nat., 310–353, 1902.

Rollet and demonstrate the inaccuracy of the data on which the Michel-Lévy color chart is based. Kraft found that not only does the position of the sensitive tint vary with different sources of light (gray cloudy sky, clear sky, sunlight reflected from white surface, Welsbach burner, arc light, incandescent bulb), but its relative width on the color scale also varies, and even the character of the tint changes with the different sources of illumination. In his paper Kraft discusses the different factors which enter the problem and their bearing on the accuracy of the results. Until a definite standard for white light has been set and the Newton color scale expressed in terms of this standard, the values of Rollet and Kraft for sunlight from a mat white surface should be adopted in preference to the old Wertheim-Quincke color scale.

In using quartz or selenite wedges still another factor enters which tends to reduce the accuracy of all white-light determinations. The minerals show dispersion such that the difference between the principal refractive indices for the different wave-lengths differ somewhat.* In table 5 the

TABLE 5.

λ	Wave-length in μμ.	Quartz $\epsilon-\omega$.	Selenite $\gamma-\alpha$.	Calcite $\omega-\epsilon$.	Quartz ρ
G	431	0.00943	.....	.....	.....
F	434	.....	0.00948	0.18122	42.0
Hβ	486	0.00930	0.00932	0.17709	32.7
E	527	0.009183	.....	.....	.....
Cd	533	.....	.....	0.17434	.....
Tl	535	.....	0.00923	.....	26.6
D	589	0.00910	0.00916	0.17194	21.7
C	656	0.00904	0.00905	0.16978	17.3
Li	671	.....	0.00902	.....	16.5
β	686	0.00900	.....	0.16938	.....

birefringence of quartz, selenite, and calcite is listed, also the specific rotation, ρ, of quartz, the values of different wave-lengths having been obtained from the standard books of references, Landolt-Börnstein and Dufet.†

Using white light as a standard and neglecting in the case of selenite the dispersion of the bisectrices, the values for the intensity curves throughout the spectrum for a path-difference of 555 μμ for light of this wave-length are indicated in Fig. 71 (calculated from the standard intensity formula for crossed nicols, see page 108). In the case of quartz, parallel to the axis (Fig. 71) the dispersion shifts the path-difference for the sensitive-tint to about 550 μμ; for a quartz plate normal to *c*, the sensitive-tint is at about 547 μμ; for selenite the path-difference for the sensitive-tint is about 554 μμ, and for calcite it is about 551 μμ.‡

*A. Cornu appears to have been the first to direct attention to this factor. Bull. Soc. Min. Fr., **6**, 135, 1883; compare also C. Hlawatsch, T. M. P. M. **21**, 107, 1902, **23**, 415-450, 1904; F. Becke, Denkschr. d. K. K. Akad. d. Wissen., zu Wien, **75**, 58, 1904. The present writer has unfortunately been unable to obtain a copy of this important paper by Professor Becke and the above reference was taken from the Traité of Duparc-Pearce.

†In a recent paper E. A. Wülfing, Sitz. Ber. d. Heidelberger Akad. d. Wissen, 1-16, 1910, has also listed these values for quartz and selenite and his values agree practically with those of table 5. See also Michel-Lévy, Les Minéraux des Roches, 57, 1888; F. Becke, T. M. P. M. **22**, 378, 1903; V. de Sousa Brandao, Centralblatt f. Miner., 23-29, 1905; H. Dufet, Jour. Phys. 305-306, 1888; D. B. Brace, Phil. Mag. (5), **48**, 345, 1899; E. J. Rendtorff, Phil. Mag. (6), **1**, 539, 1901.

‡These values are only tentative, pending the establishment of the white-light standard at which Dr. P. G. Nutting, of the Bureau of Standards, is at present engaged. As soon as this standard has been definitely fixed, the Newton color scale is to be studied in detail by Dr. Nutting and the writer.

In case a mineral plate appears colored its transmissivity varies throughout the spectrum; the more highly colored it is the greater the departure from uniformity in transmissivity. The interference color chart for such bodies is therefore totally different from the standard Newton chart; abnormal interference colors result, as is evident from the blues and reds in chlorites and zoisite, from the yellows in epidote, etc. Such selective absorption combined with the effects of differences in dispersion changes the series of interference colors from such minerals profoundly.

Among the factors, then, which enter into the consideration of the interference chart are standard white light of known intensity, the visual sensibility of the observer throughout the spectrum with varying intensity of illumination, the dispersion of the mineral plate for different colors, the

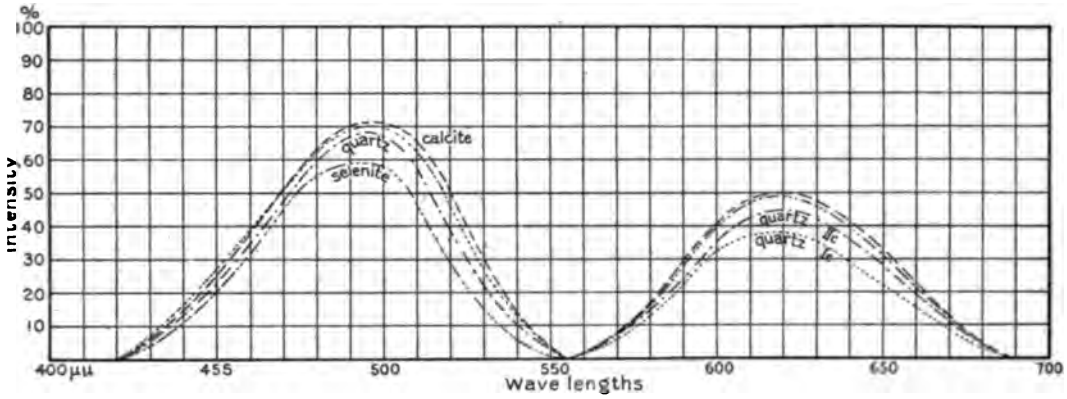


FIG. 71.—In this figure the relative luminosity intensities of light throughout the visible spectrum are shown for plates of quartz (parallel and normal to the principal axis), of calcite and of selenite of such a thickness that the path-difference is 555 $\mu\mu$ between crossed nicols for light of this wave-length. The quartz plate parallel to the axis shows the sensitive tint (red and blue ends of the spectrum about equal) at about 550 $\mu\mu$; the quartz plate normal to the axis, at about 547 $\mu\mu$; the calcite plate at about 551 $\mu\mu$, and the selenite plate at about 554 $\mu\mu$.

selective absorption of the plate for different parts of the spectrum. To control all these factors at one and the same time and to ascertain the influence of each is not possible at present because of lack of precise data. In accurate determinations of birefringence it is accordingly advisable to use strong monochromatic light instead of white light. It is also evident that a revision of the Newton standard interference color scale with reference to a standard white-light scale is necessary. The sensitive-tint plate and its relative accuracy will be considered further in Chapter IV on extinction angles.

G. Césaro* has suggested that compensation be referred to the sensitive tint at 281 $\mu\mu$ between parallel nicols instead of absolute compensation. In view of the errors resulting from the white-light source, and from the dispersion and selective absorption of most mineral plates, this method can furnish only approximate results.

In view of the different relative dispersions between minerals, it is evident that the interference colors from thick plates of a mineral can not be com-

*Bull. Acad. R. Belgique, Cl. Sci. (3), 26, 208, 1893.

pensated for exactly by a wedge of another mineral;* in short, that the use of very thick wedges of quartz or selenite showing interference colors of the higher orders is of little value unless the observations be made in monochromatic light and the wedges calibrated for that light.

H. Tertsch† has recently described a method for measuring the birefringence $(\gamma - \alpha)$ on a section nearly normal to an optic axis. From the two standard equations

$$\left(\frac{1}{\alpha_1^2} - \frac{1}{\gamma_1^2}\right) = \left(\frac{1}{\alpha^2} - \frac{1}{\gamma^2}\right) \sin\theta \sin\theta_1 \qquad \sin^2 V = \left(\frac{1}{\alpha^2} - \frac{1}{\beta^2}\right)\left(\frac{1}{\alpha^2} - \frac{1}{\gamma^2}\right)$$

and the substitution of β for α_1 and γ_1, which is permissible for a section nearly normal to an optic axis, he derives expressions for both α and γ. These expressions involve not only β and the angles θ and θ_1, which the normal to the section includes with the optic axes, but also $\sin^2 V$, and they can not furnish results of a high order of accuracy. Under favorable conditions, however, the method may be of service in ascertaining the birefringence roughly.

SUMMARY.

Briefly stated, the most serious error in the determination of the birefringence of a mineral plate or grain is the measurement of its thickness. If the de Chaulnes method for measuring the thickness be used, another error is caused by multiplying the apparent thickness by the refractive index β instead of the correct index, but this is not serious. The error in the determination of the path-difference of the emergent light-waves is slight, especially if monochromatic light be used together with the Babinet compensator or one of the graduated wedges described above. If white light be used, the character of the white light, the relative dispersion of the mineral plate and of the inserted wedge, the selective absorption of the mineral, are all factors which enter the problem and may seriously affect the accuracy of the measurement of the path-difference. As a general rule experience has shown that the birefringence can be determined on favorable plates or grains in the thin section with a probable error of less than 10 per cent.

*A. Cornu, Bull. Soc. Min. Fr., 6, 135, 1883. †T. M. P. M. 29, 520, 1911.

CHAPTER IV.

EXTINCTION ANGLES.

For a mineral plate in the thin section the term extinction angle signifies the angle between a known crystallographic direction (*e. g.*, cleavage line, or or trace of a crystal face on that plate) and one of its optic ellipsoidal axes $\mathfrak{c}'$ or $\mathfrak{a}'$ or directions along which it extinguishes when these axes are parallel with the principal planes of the crossed nicols. In order to ascertain this angle satisfactorily one must be able not only to measure plane angles accurately, but also to locate correctly the position of the optic ellipsoidal axes of the particular crystal plate. The first condition is easily accomplished and demands no special comment, while the second requirement is extremely difficult to meet satisfactorily without great expenditure of time.

Many methods have been suggested by which the position of the optic ellipsoidal axes of a given crystal section can be located more or less exactly, and all are based on the fact that when the optic ellipsoidal axes are parallel with the principal planes of the crossed nicols the plane polarized light normally incident from the lower nicol passes through the crystal plate without changing its plane of vibration. In case the optic ellipsoidal axes of the plate do not coincide with the planes of the nicols, interference generally takes place and some light passes through the upper nicol. The different methods proposed have for their common object the rendering apparent the extremely small percentage of light which thus emerges from the analyzer when the angle of rotation of the crystal plate from its position of total darkness is very small.

Before considering in detail the different methods for accomplishing this result and their relative merits and defects, it will be well to treat the subject mathematically and to derive the formulas for the intensity of light with special reference to the subject of extinction angles. This treatment is given in some detail in the following paragraphs, since the deductions recorded later are all drawn from these fundamental equations.

MATHEMATICAL FORMULÆ FOR THE INTENSITY OF LIGHT.

The phenomena of light are considered to be produced by periodic changes or disturbances in the ether, transverse to the direction of propagation. Different hypotheses have been proposed which assign different properties to this medium, but no one of the hypotheses yet suggested is satisfactory in all its details. For the purposes of this paper, however, these disturbances may be considered to be vibrations of ether particles about positions of rest and in a plane normal to the line of wave propagation. Adopting this view chiefly as a matter of easy expression, we may assert that in plane polarized light the disturbances or vibrations are confined to a plane, each particle vibrating then with simple periodic motion, to and fro, pendulum-

like, along a straight line. An equation which satisfies such a periodic vibration and which has been found to represent satisfactorily the ether disturbances is the following:

$$y = a \sin \frac{2\pi}{T}(t - t_1) \qquad (1)$$

in which a represents the amplitude, T the periodic time, $\frac{2\pi t_1}{T}$ the initial phase, t the time which has elapsed at any given instant, and y the distance of the ether particle from the position it occupied at the time.

It can be readily proved* that the intensity of light of a given period of vibration (color) varies as the square of its amplitude (a) of vibration. This relation is of importance in determining the relative intensities of the plane polarized waves which emerge from the upper nicol of the microscope after having passed through the lower nicol and an intervening crystal plate in different positions.

Disturbances in the ether which produce light phenomena are ascribed to the action of forces on the ether, and if two or more separate disturbances are simultaneously impressed on the same element the resultant disturbance can be calculated according to the principle of the resolution of forces on the assumption of direct superposition of the forces. If, in the case of plane polarized light, two separate vibrations be imposed simultaneously on an element, the resultant vibration will be in the plane of vibration of the components, and its amplitude (on the principle of superposition) will be the algebraic sum of the amplitudes of the components. The mathematical expression for the resultant vibration of a particle simultaneously impressed by two periodic disturbances of the same period, but differing in phase and amplitude, can be deduced from the equations of the separate vibrations

$$y_1 = a_1 \sin \frac{2\pi}{T}(t - t_1) \text{ and } y_2 = a_2 \sin \frac{2\pi}{T}(t - t_2)$$

The resultant displacement at any time t is

$$y = y_1 + y_2 = a_1 \sin \frac{2\pi}{T}(t - t_1) + a_2 \sin \frac{2\pi}{T}(t - t_2)$$

$$= \sin \frac{2\pi}{T} t \left(a_1 \cos \frac{2\pi}{T} t_1 + a_2 \cos \frac{2\pi}{T} t_2 \right) - \cos \frac{2\pi}{T} t \left(a_1 \sin \frac{2\pi}{T} t_1 + a_2 \sin \frac{2\pi}{T} t_2 \right)$$

$$= A \sin \frac{2\pi}{T}(t - t_3)$$

if we consider

$$a_1 \cos \frac{2\pi}{T} t_1 + a_2 \cos \frac{2\pi}{T} t_2 = A \cos \frac{2\pi}{T} t_3$$

and

$$a_1 \sin \frac{2\pi}{T} t_1 + a_2 \sin \frac{2\pi}{T} t_2 = A \sin \frac{2\pi}{T} t_3$$

By squaring and adding the last two expressions we obtain

$$A^2 = a^2_1 + a^2_2 + 2 a_1 a_2 \cos \frac{2\pi}{T}(t_2 - t_1) \qquad (2)$$

*Preston's Theory of Light, 3d edition, 42–44, 1901.

In this expression $\frac{2\pi}{T}(t_2-t_1)$ denotes the difference in phase of the two component periodic displacements and A the amplitude of the resultant vibration.

In considering the effects which different crystals exert on transmitted light-waves, it has been found, both in practice and theory, that these influences can be predicted accurately and satisfactorily by reference to a triaxial ellipsoid, the optical ellipsoid, the position and relative axial lengths of which vary in general with different minerals and with the wave-length of light employed. Thus the directions of vibration of light-waves emerging normally from a mineral plate are parallel with the major and minor axes of the ellipse which a central diametral plane parallel to the given plate cuts out of the optical ellipsoid for the particular mineral and wave-length used. The determination of the actual position of these directions in the plate is accomplished in polarized light by observing the relative intensity of the transmitted light emerging from the upper nicol (for different positions of the plate) parallel to the principal planes of the nicols.

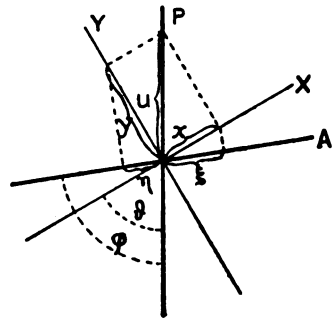


FIG. 72.

Light-waves after emergence from the lower nicol are plane polarized and their vibration is given by the equation

$$u = a \sin \frac{2\pi t}{T}$$

On entering the crystal plate this vibration is resolved into two vibrations in planes normal to each other. If θ (Fig. 72) be the angle included between the major optic ellipsoidal axis of the plate and the plane of the incident vibrations, the equations for the resultant waves are

$$x = u \cos\theta = a \cos\theta \sin \frac{2\pi t}{T} \qquad y = u \sin\theta = a \sin\theta \sin \frac{2\pi t}{T}$$

Each of these vibrations traverses the plate with a different velocity and the time required by the fast wave to traverse the plate of thickness d will be $t_1 = d \cdot \alpha'$, while the time required by the slow wave is $t_2 = d \cdot \gamma'$ where α' and γ' are respectively the refractive indices of the two waves. On emergence, therefore, the equations for the periodic displacements will be

$$x' = a \cos\theta \sin \frac{2\pi}{T}(t - d\alpha') \qquad y' = a \sin\theta \sin \frac{2\pi}{T}(t - d\gamma')$$

On reaching the upper nicol each of these vibrations is resolved further into two component vibrations normal to each other, one of which, however, is annulled by total reflection. If ϕ be the angle between the principal planes of the nicols, then the component vibrations emerging from the upper nicol are

$$\left.\begin{aligned}\xi &= x' \cos(\theta-\phi) = a\cos(\theta-\phi)\cos\theta \sin\frac{2\pi}{T}(t-da') = A_1 \sin\frac{2\pi}{T}(t-da') \\ \eta &= y' \sin(\theta-\phi) = a\sin(\theta-\phi)\sin\theta \sin\frac{2\pi}{T}(t-d\gamma') = A_2 \sin\frac{2\pi}{T}(t-d\gamma')\end{aligned}\right\} \quad (3)$$

and the resultant amplitude

$$A = \xi+\eta = A_1 \sin\frac{2\pi}{T}(t-da') + A_2 \sin\frac{2\pi}{T}(t-d\gamma')$$

The intensity I' of the emergent wave is then proportional to the square of the amplitude A as in equation (2)

$$\frac{I'}{I} = \frac{A^2}{a^2}$$

Equations (3) also show, on developing the sines as in (2), that

$$A^2 = A^2{}_1 + A^2{}_2 + 2A_1A_2 \cos\frac{2\pi}{T} d(\gamma'-a')$$

On substituting the values of A_1 and A_2 in this equation, and noting that

$$\cos\frac{2\pi}{T} d(\gamma'-a') = 1-2\sin^2\frac{\pi}{T} d(\gamma'-a')$$

we obtain

$$A^2 = a^2\left[\cos^2\phi - \sin 2(\theta-\phi)\sin 2\theta \sin^2\frac{\pi}{T} d(\gamma'-a')\right] \quad (4)$$

and finally

$$\frac{I'}{I} = \frac{A^2}{a^2} = \cos^2\phi - \sin 2(\theta-\phi)\sin 2\theta \sin^2\frac{\pi}{T} d(\gamma'-a') \quad (5)$$

But the velocity of light V, period of vibration T, and wave-length λ, are so related that $VT=\lambda$, and if we consider the velocity V the unit of measure, then we may replace T by λ and the equation (5) reads:

$$\frac{I'}{I} = I_1 = \cos^2\phi - \sin 2(\theta-\phi)\sin 2\theta \sin^2\frac{\pi}{\lambda} d(\gamma'-a') \quad (5a)$$

This is the usual expression for the relative intensity of the emergent waves;* it may, however, be brought into more convenient form for practical purposes. To save space, let $\sin^2\frac{\pi}{\lambda} d(\gamma'-a') = K$, where K may have any value from 0 to +1; then

$$I_1 = \frac{1+\cos 2\phi}{2} - K \sin 2\theta \sin 2(\theta-\phi)$$

$$2I_1 = 1+\cos 2\phi - 2K \sin 2\theta \sin 2(\theta-\phi) \quad (a)$$

$$= 1+\cos 2\phi - K(\cos 2\phi(1-\cos 4\theta) - \sin 2\phi \sin 4\theta) \quad (b)$$

$$= 1+(1-K)\cos 2\phi + K\cos 2(\phi-2\theta) \quad (6)$$

*The expression (5a) consists of two terms; the first, $\cos^2\phi$ depending only on the angle between the nicols, while the second contains the expression $\sin^2\frac{\pi}{\lambda} d(\gamma'-a')$, which is a function of the wave-length and varies with the color of light used. It is the "color term" in the equation and to it alone the interference colors are due. (Compare Preston, Theory of Light, 3d ed., 379–380, 1901.)

For a given angle, θ, to find the condition under which the intensity will be zero, the equation (a) of the foregoing can be changed to

$$\begin{aligned} 2I_1 &= 1+(1-2K\sin^2 2\theta)\cos 2\phi+2K\sin 2\theta\cos 2\theta\sin 2\phi \\ &= 1+K_1\cos 2\phi+K_2\sin 2\phi \end{aligned} \tag{7}$$

in which

$$K_1=1-2K\sin^2 2\theta \text{ and } K_2=2K\sin 2\theta\cos 2\theta$$

If $2I_1=0$, then

$$1+K_1\cos 2\phi+K_2\sin 2\phi=0 \text{ or } 1+K_1\cos 2\phi=-K_2\sin 2\phi;$$

on squaring, we obtain

$$1+2K_1\cos 2\phi+K^2_1\cos^2 2\phi=K^2_2-K^2_2\cos^2 2\phi;$$

from which we find

$$\begin{aligned} \cos 2\phi &= \frac{-K_1}{K^2_1+K^2_2}\pm\sqrt{\left(\frac{K_1}{K^2_1+K^2_2}\right)^2+\frac{K^2_2-1}{K^2_1+K^2_2}} \\ &= \frac{-K_1}{K^2_1+K^2_2}\pm\frac{K^2_2}{K^2_1+K^2_2}\sqrt{K^2_1+K^2_2-1} \end{aligned} \tag{8}$$

In order that $\cos 2\phi$ may have a real value, the expression $K^2_1+K^2_2-1$ must be zero or positive. But,

$$K^2_1=1-4K\sin^2 2\theta+4K^2\sin^4 2\theta$$
$$K^2_2=4K^2\sin^2 2\theta-4K^2\sin^4 2\theta$$

Accordingly

$$K^2_1+K^2_2-1=-4K\sin^2 2\theta\,(1-K) \tag{9}$$

The right hand of this equation is a negative quantity, and $\cos 2\phi$ can have a real value only when $K^2_1+K^2_2-1=0$, and this condition is fulfilled only when

(1) $K=0$; or $\sin^2\frac{\pi}{\gamma}d\,(\gamma'-a')=0$, i. e. $\frac{\pi}{\gamma}d\,(\gamma'-a')=n\pi$

(2) $K=1$; or $\sin^2\frac{\pi}{\gamma}d\,(\gamma'-a')=1$, i. e. $\frac{\pi}{\lambda}d\,(\gamma'-a')=(2n+1)\frac{\pi}{2}$

(3) $\sin^2 2\theta=0$; i. e. $2\theta=n\pi$

The value for $\cos 2\phi$ for $K^2_1+K^2_2-1=0$ reduces to

$$\cos 2\phi=\frac{-K_1}{K^2_1+K^2_2}=-K_1=-(1-2\mathrm{K}\sin^2 2\theta)$$

For the three different cases the value of $\cos 2\phi$ becomes

(1) $\cos 2\phi=-1$, i. e. $\phi=(2n+1)\frac{\pi}{2}$

(2) $\cos 2\phi=-(1-2\sin^2 2\theta)=-\cos 4\theta$, i. e. $\phi=\frac{\pi}{2}-2\theta$

(3) $\cos 2\phi=-1$ i. e., $\phi=(2n+1)\frac{\pi}{2}$

In the first and third cases the nicols are crossed. If the nicols are not crossed, it is impossible to obtain total darkness for a given section unless condition (2) is fulfilled

$$\left(\frac{\pi d\,(\gamma'-\alpha')}{\lambda}=\frac{(2n+1)}{2}\pi\right)$$

which states that monochromatic light is used of such a wave-length that the one wave is an odd number of half wave-lengths ahead of the second; in this case

$$\phi=\frac{\pi}{2}-2\theta$$

For total extinction, therefore, θ must be equal to

$$\frac{\pi}{4}-\frac{\phi}{2}$$

If white light is employed, abnormal interference colors will appear because of the abnormal conditions, and at no point will darkness ensue. The condition that all the light be transmitted is

$$I_1=\frac{I'}{I}=1 \text{ or } (1-K)\cos 2\phi+K\cos 2\,(\phi-2\theta)=1$$

which is satisfied if both $\phi=0$ and $\theta=0$.

In case either ϕ or θ be given, the problem of finding the particular disposition of upper nicol or crystal plate, for which the intensity of the transmitted light reduces to a minimum or maximum, involves the first partial differential quotients of the function $2I_1$ (equation 6) after either ϕ or θ. If ϕ be given, the point in question is determined by

$$\frac{\partial(2I_1)}{\partial\theta}=4K.\ \sin 2(\phi-2\theta)=0\ \therefore\ 2\theta=\phi-\frac{n\pi}{2} \text{ or } 2\theta=\phi;\ 2\theta=\phi-\frac{\pi}{2}$$

The second partial differential quotient shows that for $2\theta=\phi$, the intensity is a maximum, while for $2\theta=\phi-\frac{\pi}{2}$ the intensity is a minimum. This relation is of importance in certain of the methods described below.

If θ be given and ϕ is the variable

$$\frac{\partial(2I_1)}{\partial\phi}=-2(1-K)\sin 2\phi-2K\sin 2(\phi-2\theta)=0$$

from this equation we find

$$\tan 2\phi=\frac{K\sin 4\theta}{1-K+K\cos 4\theta}$$

a complicated expression which, for $K=1$, simplifies to

$$\tan 2\phi=\tan 4\theta\ \therefore\ 2\phi=n\pi+4\theta$$

Substituting this expression in the second differential quotient we find that for $\phi=2\theta$ the intensity is a maximum, while for

$$\phi=\frac{\pi}{2}+2\theta$$

the intensity is a minimum.

It is of interest to plot the values derived from equation (6) for different values of ϕ and θ. From the graphical relations thus obtained most of the conclusions in the following pages will be derived.

In Fig. 73, curve V, the rate of increase in intensity of light is given for the special case of $\theta = 0$, where simply the upper nicol is turned and the crystal plate has no effect on the polarization of the waves from the lower nicol. In this case

$$2I_1 = 1 + \cos 2\phi \tag{10}$$

From the curve it is evident that the rate of decrease in intensity with increase of ϕ is very slow at first, but increases rapidly and reaches a flexion point at 45°, after which the intensity decreases with decreasing rapidity to its minimum value 0 at 90°.

In case the nicols are crossed $\left(\phi = \frac{\pi}{2}\right)$, the rates of increase for different values of K are given by the reduced equation

$$2I_1 = K(1 - \cos 4\theta) = 2K \sin^2 2\theta \tag{11}$$

which defines a curve similar in aspect to the foregoing, except that ϕ is replaced by 2θ and the factor K tends to reduce all values proportionately. The curves I to IV of Fig. 73 represent the relative intensities for values of $K = 1$, $\frac{1}{2}$, $\frac{1}{4}$, and 0 respectively. The greatest possible intensity is thus attained when $K = 1$, *i. e.*, when the waves, after emerging from the crystal plate, are an odd number of half wave-lengths apart (in opposite phase); the intensity is zero for all positions of the plate when $K = 0$, *i. e.*, when the distance between the two emergent waves is a whole number of wave-lengths.

In Figs. 74–78 intensity curves are drawn to show the relative intensity of the emergent light for different positions of the crystal plate (θ usually 0°, 15′, 30′, 45′, and 1°) with the principal plane of the lower nicol and for different positions of the upper nicol (ϕ ranging from 88° to 92°). The heavy curve in each figure is the relative intensity curve of the crystal alone (nicols crossed, $\phi = \frac{\pi}{2}$ and θ ranging from $-2°$ to $+2°$). Only this narrow range of intensities is considered, since in general it represents about the order of magnitude of the probable error of a single determination made in the usual manner.

In each of the figures the unit ordinate division represents 0.025 per cent of the total intensity and the unit abscissa division 10′ of arc.

In Fig. 74, K is considered equal to 1 or $\sin^2 2\frac{\pi}{\lambda} d(\gamma' - \alpha') = 1$, which obtains when the one wave is any odd number of half wave-lengths ahead of the second on emergence from the plate; in Figs. 75–78, the relative intensity curves are drawn for $K = \frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, and 0, respectively.

It is not difficult to grasp the meaning of these curves, as the following example will show: let it be required to find the percentage of light which emerges from the nicol in the case of a mineral plate of such thickness and birefrigence that for yellow light the faster waves after emerging from the plate will be precisely half of a wave-length ahead of the slow waves ($K = 1$,

Fig. 74), the direction of extinction of the plate to make an angle of 30′ ($\theta = 30'$) with the principal plane of the lower nicol, and the principal plane of the upper nicol to include an angle of 89° 10′ with the lower nicol ($\phi = 89°\ 10'$). On the 30′ curve of Fig. 74 the ordinate for 89°10′ is 0.104 and the relative intensity is therefore 0.104 per cent of the total intensity.

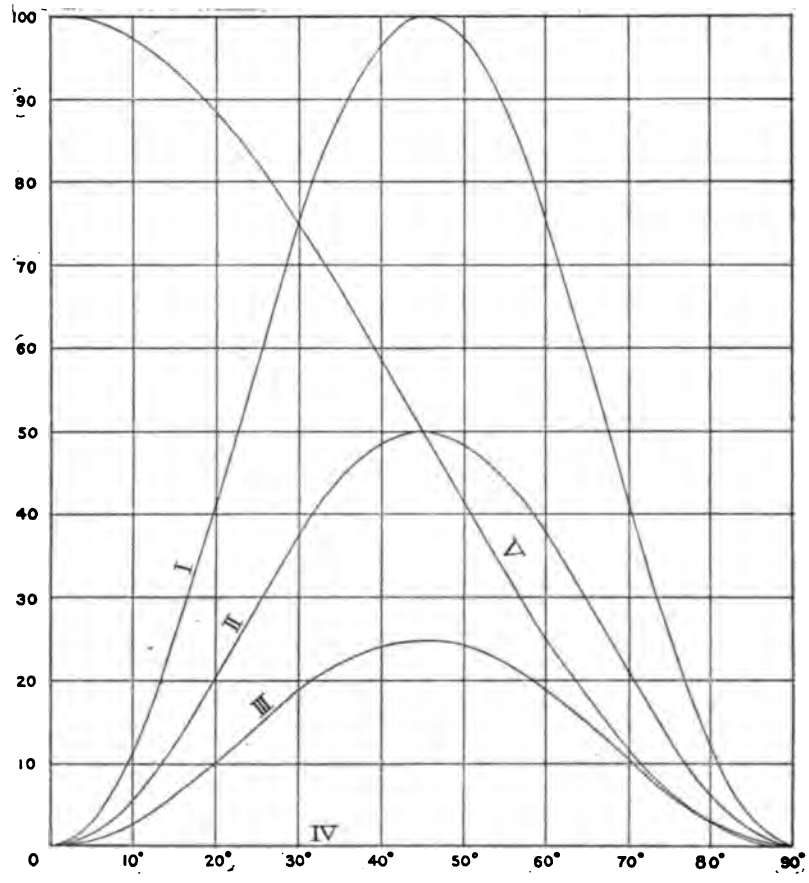


FIG. 73.—Curves showing relative intensity of light emerging from upper nicol after transmission through polarizer, crystal plate, and analyzer, the positions of the crystal plate and also the analyzer ranging from 0° to 90°. The abscissa values refer to angular distances of the major ellipsoidal axis of the crystal plate and also of the plane of the analyzer from the plane of the polarizer. For curves I to IV the nicols are considered crossed $\left(\phi = \frac{\pi}{2}\right)$ and the crystal plate alone rotated from 0° to 90°. In curve I, $\sin^2 \frac{\pi}{2} d(\gamma' - \alpha') = K = 1$; in curve II, $K = \frac{1}{2}$; in curve III, $K = \frac{1}{4}$; in curve IV, $K = 0$. Curve V shows the relative intensity of the emerging light for different positions of the analyzer alone ($\theta = 0, \phi$ ranging from 0° to 90°). Curves calculated from the general equation (6) above.

These figures are well adapted to show graphically certain facts which are evident from a mathematical consideration of the intensity formula. (1) If $K = 0$, which occurs when the one wave is any number of whole wavelengths ahead of the second, the crystal plate is dark and remains dark for

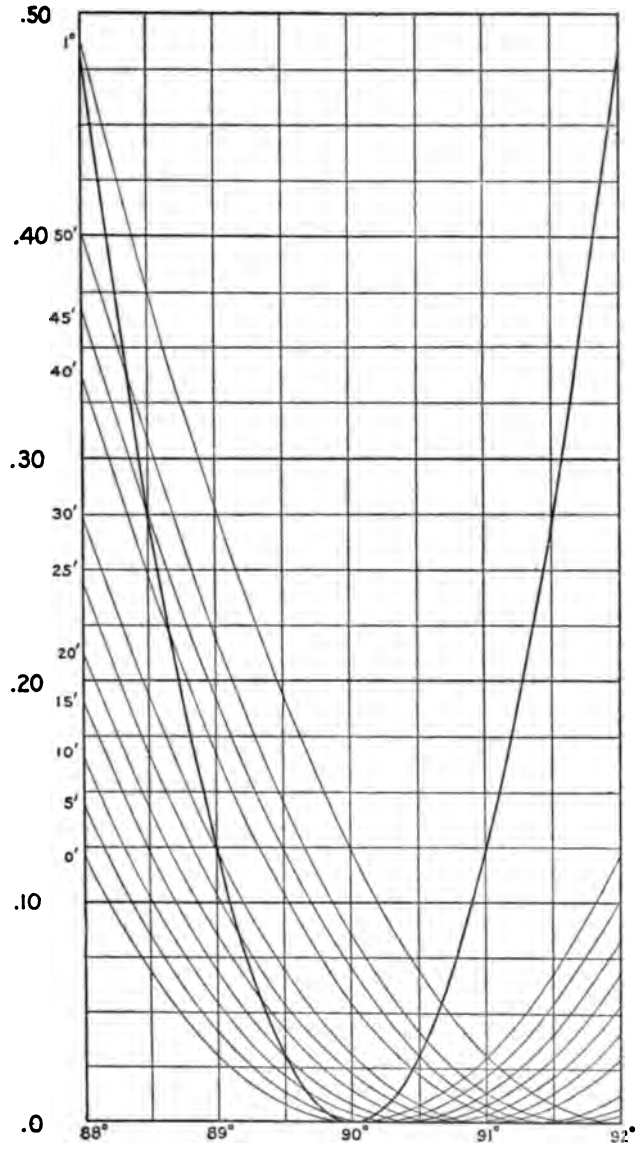


FIG. 74.—Intensity curves for crystal plates making angles 0′, 5′, 10′, 15′, 20′, 25′, 30′, 40′, 45′, 50′, and 1° with plane of polarizer. Analyzer revolved about axis 88° to 92° from lower nicol plane. $\sin^2 \frac{\pi}{T} d_c(\gamma' - a') = K = 1$.

Curves calculated from formula (6) which, for $K = 1$, becomes $I = \frac{1}{2}[1 + \cos 2(\phi - 2\theta)]$.

The heavy curve indicates the relative light intensity under crossed nicols ($\phi = \frac{\pi}{2}$) for different positions of the crystal plate (θ ranging from 88° to 92° or $-2°$ to $+2°$) and $K = 1$. Calculated from the formula $2I_1 = 1 - \cos 4\theta$.

all angles of rotation as indicated by the heavy abscissa line of Fig. 78. (2) In case $K=\frac{1}{4}$, Fig. 77, the intensity curve for the crystal plate, coincides very closely with that for the rotating nicol. The extinction curves, moreover, for the crystal plate at different angles ($\theta=15'$, $30'$, $45'$, and $1°$) with the principal plane of the lower nicol and for different positions of the upper nicol (ϕ ranging from 88° to 92°) are similar and lie close together, so that, in this particular case, methods involving the rotation of the upper nicol

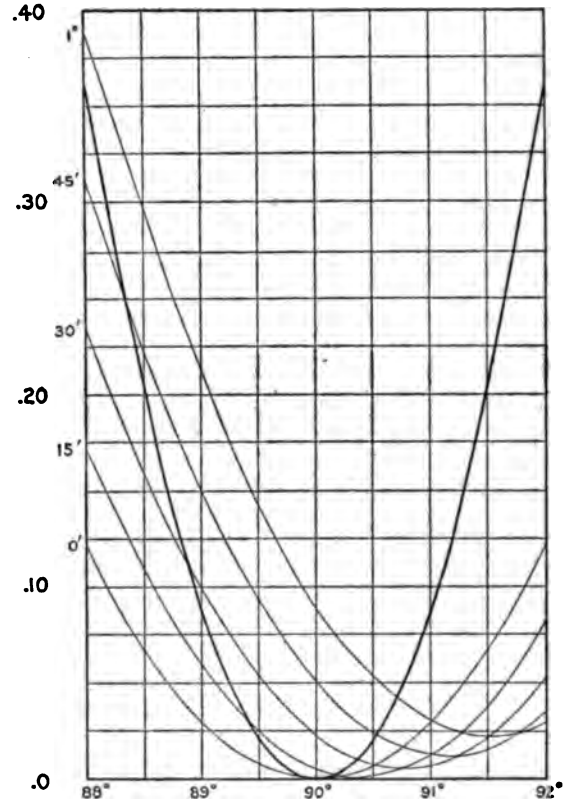


FIG. 75.—Intensity curves for crystal plates at angular distances of 0′, 15′, 30′, 45′, and 1° from plane of polarizer, the analyzer being revolved about axis 88° to 90° from plane of polarizer. $\sin^2 \frac{\pi}{T} d(\gamma'-\alpha') = K = \frac{1}{4}$. Curves calculated from the formula $I_1=\frac{1}{8}[4+\cos 2\phi+3\cos 2(\phi-2\theta)]$.

As in Fig. 74, the heavy curve indicates the relative intensity of emergent light for different positions of the crystal plate (θ ranging from 88° to 92°, or $-2°$ to $+2°$) under crossed nicols ($\phi=\frac{\pi}{2}$) and $K=\frac{1}{4}$. Calculated from the formula $I_1=\frac{3}{8}(1-\cos 4\theta)$.

for the location of zero intensity directions are not greatly different in their degree of accuracy from those in which the nicols remain crossed and the crystal plate is rotated. Nevertheless, even in this instance the former are the more sensitive methods and results attained by their use are correspondingly more accurate. For $K=\frac{1}{2}$, Fig. 76, the extinction curve for the crystal plate alone (nicols crossed and plate only rotated) no longer coin-

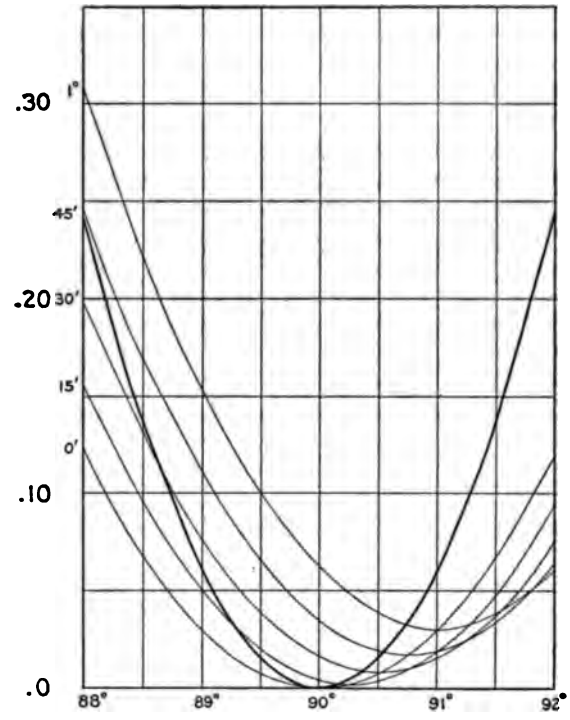


FIG. 76.—This differs from Figs. 74 and 75 only in the value of K, which is $\frac{1}{2}$. Curves calculated from the formula $I_1 = \frac{1}{4}[2 + \cos 2\phi + \cos 2(\phi - 2\theta)]$. The heavy curve from the formula $\phi = \frac{\pi}{2}$, $I_1 = \frac{1}{4}(1 - \cos 4\theta)$.

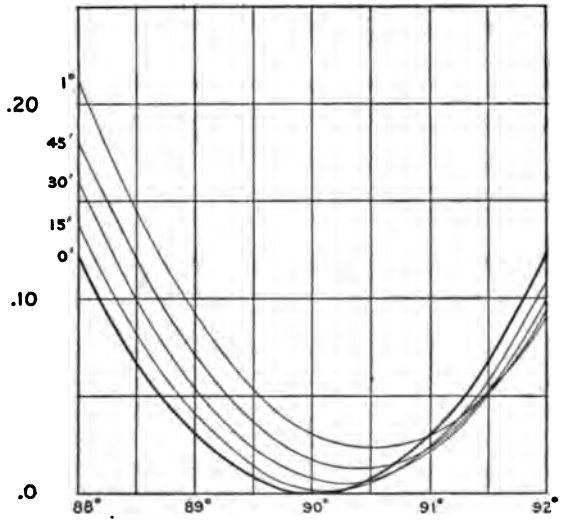


FIG. 77.—Differs from Fig. 76 only in K, which is $\frac{1}{4}$. The curves are expressed by the formula $I_1 = \frac{1}{8}[4 + 3\cos 2\phi + \cos 2(\phi - 2\theta)]$ and the heavy curve by $I_1 = \frac{1}{8}(1 - \cos 4\theta)$.

cides with that for the upper nicol alone, but similar conclusions can be drawn as to the relative sensitiveness of the two methods, the one involving the rotation of the crystal plate (while the nicols remain crossed), and the second the rotation of the upper nicol while the crystal plate remains stationary.

The amount of light required to produce the sensation of light in the human eye is different for different persons. But for a given eye the limit of the actual sensation of monochromatic light (threshold value) is fixed for any particular instant and may be represented by one of the horizontal percentage lines of the figures. Let us assume that for a source of monochromatic light of definite intensity I, the limit for the sensation of light is 0.050 per cent of the total intensity and represented by the first horizontal line above the base line of Fig. 76. Then the curve for the crystal plate alone shows that for all points below that line, *i. e.*, between 89° 04′ and 90° 56′, the crystal will appear absolutely dark and on a single determination an error of nearly ± 1° may be made.

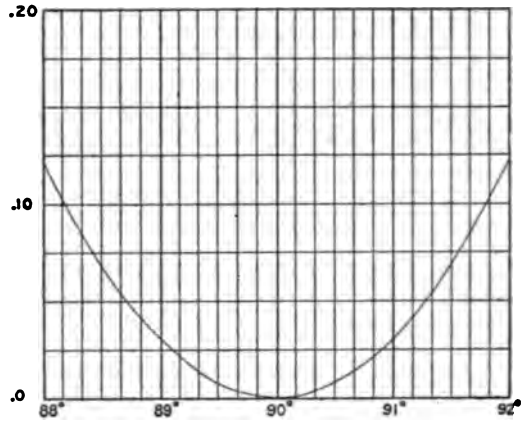


FIG. 78.—In this particular case K is considered = 0 and the general formula reduces to $I = \frac{1}{2}(1 + \cos 2\phi)$, which is independent of θ. In other words, if the thickness of the plate be such that $\sin^2 \frac{\pi}{T} d(\gamma' - \alpha') = 0$, or the emerging waves are any number of whole wavelengths apart, total interference takes place and the plate is dark under crossed nicols for every angle of rotation about its normal axis. The curve indicates the change in intensity of field illumination on rotation of the upper nicol.

If, however, the crystal plate remain stationary, and the upper nicol be rotated through small angles from its normal, crossed position ($\phi = 88°$ to 92°), it is evident from the figure that if, for example, the crystal plate is 30′ distant from its position of total extinction and is still dark under crossed nicols so far as the eye of the observer can detect, the differences in intensity between the field and crystal plate for different angles of rotation of the upper nicol (measured by the ordinate intercepts between the curve 0′ and 30′ of figure) are of such a character that at the point where the illumination of the field can just be observed (88° 43′) the intensity of illumination of the crystal is more than twice as great (0.106 per cent instead of 0.05 per cent), whereas on the other side, where the first indications of illumination

on the central plate can be detected at 91° 41′, the field is lighted up by 0.085 per cent instead of 0.050 per cent of the total intensity. These differences of intensity are of such a character that they can readily be observed, and the sensitiveness of any method involving the rotation of the upper nicol while the crystal remains stationary is, in this case at least, twice as great as that for which the nicols remain crossed and the crystal plate alone is rotated. Similar theoretical conclusions can be drawn from Figs. 74 and 75.

These relations are also directly evident from the intensity formula itself. The plate will appear dark under crossed nicols in monochromatic light if the intensity of the transmitted light is below the threshold value I_0 or

$$I' = I \sin^2 2\theta \sin^2 \frac{\pi d(\gamma' - a')}{\lambda} < I_0$$

For white light this equation for the total intensity becomes

$$I = \int I' d\lambda = \int I_{(\lambda)} \sin^2 2\theta \sin^2 \frac{\pi d(\gamma' - a')}{\lambda} d\lambda$$

For a given light source and thickness of crystal plate the integral quantity

$$\int I_{(\lambda)} \sin^2 \frac{\pi d(\gamma' - a')}{\lambda} d\lambda$$

may be considered constant and

$$I = K \cdot \sin^2 2\theta$$

For small angles θ we may substitute the angle for its sine and obtain

$$I = 4K\theta^2$$

If the plate appears dark, then

$$I = 4K\theta^2 < I_0$$

where I_0 is the threshold value or on an average about 0.001 meter candle.

$$\therefore \quad \theta < \left(\frac{I_0}{4K}\right)^{\frac{1}{2}}$$

Thus if the total intensity $K = 2.5$ meter candles, then the angle θ, which gives the threshold value I_0, is $\left(\frac{0.001}{10}\right)^{\frac{1}{2}} = 0.01$ radians or 35′.

If the upper nicol be rotated and the plate held stationary the intensity equation becomes

$$I_2 = \cos^2 \phi - \sin 2\theta \sin 2(\theta - \phi) \sin^2 \frac{\pi d(\gamma' - a')}{\lambda}$$

The intensity of the field adjoining the crystal plate is then $I_1 = \cos^2 \phi$, where ϕ is the angle between the polarizer and the analyzer and θ the angle between the polarizer and the ellipsoidal axis γ of the crystal plate. The difference in intensity between the plate and the field is then

$$I_1 - I_2 = \sin 2\theta \sin 2(\theta - \phi) \sin^2 \frac{\pi d(\gamma' - a')}{\lambda}$$

If σ be the percentage of the total monochromatic illumination which is necessary to produce a perceptible difference in intensity (0.1 to 5 per cent or more), then in order that the intensity of the mineral plate be appreciably different from that of the field under the conditions of illumination

$$I_1 - I_2 = \sigma I_2 = \sin 2\theta \sin 2(\theta - \phi) \sin^2 \frac{\pi d(\gamma' - \alpha')}{\lambda}$$

but this expression reduces to

$$\frac{\sigma I_2}{K \sin 2\theta} = \sin 2(\theta - \phi)$$

if we consider $\sin^2 \frac{\pi d(\gamma' - \alpha')}{\lambda} = K$, where K denotes the relative proportion of the total monochromatic illumination which is emitted from the crystal plate, the remainder being destroyed by interference. From this equation it is evident that the sensitiveness of the method, so far as it is based on intensity relations alone, is largely dependent on the value of K. If the thickness of the plate be such that the intensity of the emergent light is a large fraction of the total illumination and the photometric sensibility (least perceptible increment of intensity), σ, be small (1 to 2 per cent), then the least value which $\sin 2\theta$ can assume is $\frac{\sigma I_2}{K}$ since the maximum value of the right hand of the equation is unity. Under the most favorable conditions $\sigma = 0.015$ and $K = 1$ or $\sin 2\theta = 0.015$ or $\theta = 26'$. If K were only a small fraction of the total intensity I_2, θ becomes proportionately larger and the method less accurate.

If white light be used, still another factor enters which tends to increase the sensibility of this method. On rotating the upper nicol, the intensity relations of the different colors throughout the spectrum are shifted by the addition of the light from the field, as indicated by the second term of the intensity equation, and this gives rise to a change in the interference color. Abnormal interference colors result, which for a slight rotation of the nicol to the right are noticeably different from those formed on rotating the nicol to the left. The eye is sensitive to this color change and, if the mineral shows brilliant interference colors between crossed nicols, it is possible to determine the position of total extinction of the plate by this method of nicol rotation with great accuracy (10′ or less). If the interference colors of the plate are first-order grays, the change in color is hardly perceptible; the field is relatively flooded with light and the method correspondingly less sensitive.*

In case the half-shade principle be used to increase the accuracy of the determination of the positions of total extinction of a crystal plate, the comparison is between adjacent halves of the crystal plate rather than between the crystal plate and the field of the microscope alone. The eye recognizes differences in intensity between the halves of the analyzer field. Let σ be the photometric sensibility, 2δ the angle between the halves of analyzer field, I_1 and I_2 the observed intensities and $I_1 - I_2 = \sigma I_2$.

*In monoclinic minerals dispersion of the bisectrices often gives rise to abnormal interference colors when viewed in white light under crossed nicols near the positions of total extinction. Abnormal interference colors may also arise from imperfect adjustment of the nicols and care should be taken not to confound the two possible causes—dispersion of the bisectrices or imperfect adjustment of the microscope.

Then

$$I_1=\cos^2\left(\frac{\pi}{2}-\delta\right)-\sin 2\theta \sin^2\left(\theta-\frac{\pi}{2}+\delta\right).\sin^2\frac{\pi d(\gamma'-\alpha')}{\lambda}$$

$$I_2=\cos^2\left(\frac{\pi}{2}+\delta\right)-\sin 2\theta \sin^2\left(\theta-\frac{\pi}{2}-\delta\right).\sin^2\frac{\pi d(\gamma'-\alpha')}{\lambda}$$

$$\therefore I_1-I_2=\sigma I_2=K \sin 2\theta\{\sin 2(\theta+\delta)-\sin 2(\theta-\delta)\}$$

$$=2K \sin 2\theta \cos 2\theta \sin 2\delta$$

$$=K \sin 4\theta \cdot \sin 2\delta$$

This equation indicates that as θ decreases, δ must increase; furthermore, that if the photometric sensibility were to remain constant, the minimum reading for θ would be $\sin 4\theta=\frac{\sigma I_2}{K}$. But the sensibility varies with the absolute intensity of illumination of the field, with the color used and with the condition of the eye of the observer. The effect of these factors is difficult to estimate accurately and to express in mathematical form. It is evident, however, that the limiting value of θ varies with the angle δ and that under given conditions of illumination that angle δ for which θ is a minimum is evidently the best. It is important, therefore, in practical work where the conditions of illumination and the sensibility of the eye are widely variable, that the half-shade apparatus be so constructed that δ can be varied and the angle, for which the sensibility is a maximum, be thus readily obtained. In this arrangement, as in the method based on the rotation of the upper nicol, the abnormal interference colors, which result when white light is used and when the mineral plate is not precisely in the position of total extinction, change rapidly for a slight rotation of the crystal plate and are of great service in determining accurately the position of total extinction.*

With a given color of monochromatic light extinction angles should be determined on plates of such a thickness that K is about 1 (the two emergent waves are a whole number of half wave-lengths apart). Thus, if sodium light be used, the plates should show in white light an interference color of about straw-yellow of the first order but not sensitive violet, since for this particular thickness the two waves are about 589 $\mu\mu$ (555 $\mu\mu$) apart and the yellow waves are totally destroyed, with the result that the plate appears practically dark in all positions. It follows, furthermore, that a plate which is well adapted for determinations in one kind of monochromatic light may be useless for another color. Plates for which the path-difference of the emergent waves is between 200 and 300 $\mu\mu$ are best adapted for the measurement of extinction angles.

It has been found that the insertion between the crossed nicols of specially cut plates and wedges of birefracting substances, as quartz and selenite, often serves to increase the accuracy of the measurement of the extinction angle on a given plate. The principle here involved is that of the superposition of birefracting plates, the action of which is to produce a resultant which differs from that of either component. It is possible to select a

*A half-shade apparatus of variable sensibility was constructed by the writer in 1907 (Amer. Jour. Sci. (4), 26, 375, 1908) but did not prove entirely satisfactory because of the depolarization of the transmitted waves by the total reflecting prisms used.

wedge or plate of such a character that the interference phenomena produced by it alone are extremely sensitive to the slight changes produced by a second crystal plate when it is not precisely in the position of total extinction.

From the mathematical standpoint, the insertion of a second plate involves a new set of conditions for the vibrating ether elements and the equations for the resultant are correspondingly more complex. Their derivation, however, is exactly the counterpart of that for the intensity of a single crystal plate and the final result, only, need be given here. If the nicols be crossed and θ_1 be the angle which γ'_1 of the crystal plate of thickness d_1 includes with the principal plane of the polarizer, and θ_2 the angle between γ'_2 of the inserted plate or wedge of thickness d_2 and the polarizer plane, and $d_1(\gamma'_1-\alpha'_1)=\Delta_1$ and $d_2(\gamma'_2-\alpha'_2)=\Delta_2$, then the relative intensity is given by the standard equation

$$I_1 = -\sin 2(\theta_2-\theta_1)\sin 2\theta_1 \cos 2\theta_2 \sin^2\frac{\pi}{\lambda}\Delta_1$$

$$+\sin 2(\theta_2-\theta_1)\cos 2\theta_1 \sin 2\theta_2 \sin^2\frac{\pi}{\lambda}\Delta_2$$

$$+\cos^2(\theta_2-\theta_1)\sin 2\theta_1 \sin 2\theta_2 \sin^2\frac{\pi}{\lambda}(\Delta_1+\Delta_2)$$

$$-\sin^2(\theta_2-\theta_1)\sin 2\theta_1 \sin 2\theta_2 \sin^2\frac{\pi}{\lambda}(\Delta_1-\Delta_2)$$

From this formula the relative intensity can be calculated for any given values of θ_1, θ_2, Δ_1 and Δ_2.

In case the crystal plate is of such a thickness that $\sin^2\frac{\pi}{\lambda}\Delta_1=1$ and at the same time the inserted plate is also of a thickness that $\sin^2\frac{\pi\Delta_2}{\lambda}=1$, this equation reduces to

$$I_1=\sin^2 2(\theta_2-\theta_1)$$

an expression for a curve similar in every respect to those of Figs. 74–78 but which is zero for $\theta_2=\theta_1$ and also for $\theta_2=\frac{\pi}{2}+\theta_1$, and reaches its maximum of 1 at $\theta_2=\frac{\pi}{4}+\theta_1$. The intensity from the second plate alone is $I_2=\sin^2 2\theta_2$. If this intensity be compared with that of the two superimposed plates, the difference in intensity between the two fields is

$$I_1-I_2=\sin^2 2(\theta_2-\theta_1)-\sin^2 2\theta_2$$

If this difference be just perceptible

$$I_1-I_2=\sigma I_2=\sin^2 2(\theta_2-\theta_1)-\sin^2 2\theta_1$$

From this equation it is evident that if the angle θ_1 is to be small, θ_2 must also be small, because σI_2 is a small quantity. In other words, the sensibility of the method is greatest when the ellipsoidal axes of the inserted test plate include only a small angle with the principal planes of the nicols. If both test plate and crystal plate are not strongly birefracting, abnormal interference colors result under these conditions and aid in determining the position of total extinction accurately.

In the Calderon method described below, the calcite plates are purposely so thick that they show the white interference colors of higher orders in white light, in which case the thickness is so great that for a number of different colors throughout the spectrum the path-difference of the emergent waves is a whole number of wave-lengths; in other words, in the Calderon method it is permissible for practical purposes to consider the plate of such a thickness that for white light the expression $\sin^2 \frac{\pi\Delta}{\lambda}$ is unity; the angle θ_2, therefore, should be small in order to secure the best results, so small in fact that the illumination of the field is just visible.

In several of the other methods cited below for the exact location of the ellipsoidal axes of a given plate, use is made of quartz plates or wedges, which are cut normal to the principal axis, and which rotate the planes of vibration of normally incident, plane polarized light. For the purposes of this paper it is not necessary to enter into the mathematical discussion and theory of the rotatory power of quartz, but simply to apply the known laws of rotatory polarization as they were first proved experimentally by Arago and Biot on this mineral. A quartz plate perpendicular to the principal axis rotates the plane of normally incident, plane polarized waves, through an angle which is proportional to the thickness of the quartz plate and also approximately proportional to the inverse square of the wave-length used.* The rotation effected by two superimposed plates is, moreover, the algebraic sum of the rotations produced by each separately.

By using, therefore, a properly constructed quartz wedge, it is possible to counteract exactly the effect, in plane-polarized monochromatic light, of any crystal plate in any given position with respect to the nicols, by rotating the new planes of vibration, determined by the crystal plate back to the original planes of the nicols.

In the intensity formula (5)

$$I_1 = \cos^2 \phi - \sin\ 2\theta\ \sin\ 2(\theta-\phi)\ \sin^2 \frac{\pi}{\lambda}\, d(\gamma'-a')$$

this rotation affects the angle θ only and, if the nicols be crossed, then

$$I_1 = \sin^2 2\theta \sin^2 \frac{\pi}{\lambda} d\ (\gamma'-a')\ \text{(Equation (11), page 121)}$$

In all measurements of extinction angles, however, θ is a small quantity and in place of the sine we may use the angle itself without sensible error; accordingly,

$$I_1 = 4K\theta^2$$

This formula, which for small angles, θ, states that the light intensity is proportional to the square of the angle θ, will be employed later in the description of a new combination quartz wedge for use in determining extinction angles.

In certain other methods convergent polarized light is employed and the disturbing effects of an intervening crystal plate observed whose optic ellipsoidal axes are not precisely parallel with the planes of the nicols. The intensity formulæ applying to such conditions are similar to those for plane

*For more accurate expressions of the relation of the specific rotation of quartz to the wave-length, see P. Drude, Lehrbuch d. Optik, 1st ed., 381, 1900; P. G. Nutting, Phys. Rev., 18, 24, 1903.

polarized light and the general deductions from the latter may be considered to apply to the phenomena in convergent polarized light. The methods involving convergent polarized light, however, have several important defects which render their general application cumbersome and unpractical.

In the foregoing pages, the intensity formulæ for light transmitted by crystalline plates under different conditions have been developed and the attempt has been made to treat the subject in such a way that the results attained shall be directly applicable to the practical methods for determining extinction angles under the microscope.

METHODS.

Extinction angles can be measured either in plane-polarized light or in convergent polarized light; in plane polarized light the positions of total extinction are determined either by observing the relative intensities of monochromatic light under special conditions or by means of the interference colors resulting from the use of white light; in convergent polarized light they are ascertained by noting the symmetry relations of an interference figure obtained from a specially prepared plate. In all measurements of extinction angles it is imperative that careful attention be given to the source of light, especially if monochromatic light be used. The source should be as intense and steady and uniform as possible in order that the variation in the source of light itself be not mistaken for actual differences in the microscopic field. The rays of light incident on the preparation should, moreover, be as nearly parallel as it is possible to obtain them. To meet these requirements satisfactorily requires both time and patience, but in order to attain the best results they can not be overlooked.

Assuming the microscope to be in perfect adjustment and the source of light satisfactory, any one of the following direct methods may be used for measuring the extinction angle of a particular crystal plate.

PARALLEL POLARIZED LIGHT.

(1) The ordinary method, which consists in turning the crystal plate under crossed nicols until the position of maximum darkness is attained, is in general use and is equally well adapted for white light and for monochromatic light. With it any degree of accuracy can be attained provided a sufficient number of measurements be taken to reduce the probable error. In applying this method it is customary to note, not only the positions of maximum darkness attained by the crystal when rotated clockwise from a position of bright illumination, but also when rotated counter-clockwise from such a position. This was the method used by Max Schuster in his measurements of the extinction angles of plagioclase feldspars. He determined for each cleavage flake the position of total extinction eighty times for clockwise rotations of the plate and eighty times for counter-clockwise rotations, and averaged the two readings. His work in this line remains unsurpassed, even to the present time.

To increase the accuracy of each determination on a crystal plate under crossed nicols, different schemes have been devised, all of which depend on the disturbing influence of the plate on inserted plates or wedges of birefracting substances. Each of the inserted plates or wedges is constructed

in such a way that the interference phenomena which it presents are markedly influenced by the slight disturbing effects from the crystal plate when it is not precisely in its position of complete extinction.

SENSITIVE-TINT PLATE.

In the discussion of the Newton color scale (page 108) attention was directed to the fact that when observations are made in white light, the interference colors which result on the insertion of a quartz or selenite plate in the diagonal position between crossed nicols resemble closely the Newton color scale. Under these conditions the intensity equation for waves emerging from the quartz wedge reduces to $I_1 = \sin^2 \frac{\pi d(\gamma' - \alpha')}{\lambda}$, which is identical in form with that for the interference of white light reflected from two surfaces of glass or other material separated by a thin film of air. It will be recalled that the interference colors obtained from the wedge or crystal plate are dependent in a measure on the source of illumination used; also that the position and hue of the sensitive tint are not precisely the same for quartz, selenite, and air. A plate which shows the sensitive tint allows only about 6 per cent of the total light to be transmitted, with the result that such plates are low in visual intensity.

The methods with the sensitive-tint plate are based on the fact that a slight change in path-difference in the transmitted light-waves produces a decided change in the interference color; also that for certain colors the eye is very sensitive to a slight change in hue. The sensitive-tint plate is most effective on, and practically limited to, colorless plates showing low interference colors of the first order. Its efficiency is seriously impaired in the case of deeply colored minerals which veil the true interference color and also with birefracting minerals thick enough to show interference colors higher than red of the first order. It can, moreover, only be used with white light and accordingly can not take cognizance of the dispersion of the bisectrices in the monoclinic and triclinic systems. Its color is, moreover, dependent to a certain degree on the source of light used. Methods with this plate are, therefore, not of general application and can be employed to advantage only under specially favorable conditions. In the case of exceedingly minute and weakly birefracting grains, it is often of advantage both in parallel and in convergent polarized light to rotate the sensitive-tint plate until its ellipsoidal axes practically coincide with the principal nicol planes. With this arrangement the field intensity is very low and the faint differences in color from the small mineral grain are more readily detected in the dull background than in the field flooded with light when the plate is inserted in the diagonal position. This applies with equal force to the double plates described in the paragraphs below.

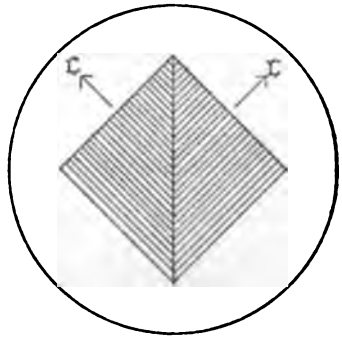


FIG. 79.

BRAVAIS*-STÖBER† PLATE.

This device is cut to show the sensitive violet interference color and consists of two plates in combination instead of a single one. A single sensitive-tint plate of mica or quartz is cut along a line at 45° with the directions of extinction; one half is then turned through a vertical angle of 180° and cemented to the other as indicated in the figure. By this combination plate, which is placed in the focal plane of the ocular, the interference color is made to fall in the one half and to rise an equal amount in the second, thus doubling the sensitiveness of a single plate (Fig. 79). This plate is intended for use only in white light, but under certain conditions it may serve to good advantage in monochromatic light.

THE KOENIGSBERGER PLATE.‡

The Koenigsberger plate is a modification of the Bravais plate and consists of two very thin mica plates arranged as in the Bravais plate and placed in the focal plane of the ocular; it shows, however, a very low interference color instead of the sensitive tint. This plate was originally constructed for the detection of weak anistropy, but under certain favorable conditions it can be used for the determination of the position of total extinction. As the shift in path-difference on insertion of the plate is very slight, this method can only be applied satisfactorily to crystal plates whose interference color is changed perceptibly by a slight change in path-difference between the emergent waves.

THE DOUBLE COMBINATION WEDGE.§

On the principle of the Bravais-Stöber plate, the writer has had a combination wedge prepared in which the interference colors range from total darkness to green of the second order. This wedge (Fig. 80) was made by taking an ordinary combination wedge|| showing the zero interference band exactly in the center and green of the second order on each end, and cutting the wedge in half longitudinally parallel to the ellipsoidal axes; the edges were then polished and the halves again recemented—the one half, however, having been rotated first through 180°, so that in the resultant combination wedge the phase difference of the adjacent half at any point of insertion is always equal and opposite when no crystal plate is in the field. By this method the principle of the Bravais-Stöber plate is extended to cover interference colors from total darkness to blue-green of the second order, and to allow the observer to select an interference color which, in combination with that of the mineral plate examined, is most sensitive. The low gray tints

*Comptes Rendus, **33**, 113, 1851; Ann. de Chim. et de Phys. (3) **43**, 129, 1855; Pogg. Ann., **96**, 397, 414 1855.

†Zeitschr. Kryst., **29**, 22–24, 1898.

‡Centralblatt für Miner, 729–730, 1908; 249–250, 746–747, 1909.

§This wedge was prepared with great care by Voigt & Hochgesang, Göttingen, Germany, and the writer desires to express his appreciation of the interest taken by the firm in the same. The compensation on different ends of the wedge, however, proved to be of slightly different value, with the result that, although the dark zero interference bands were precisely adjacent, the interference colors near the ends of the wedge did not coincide exactly. This slight defect can be eliminated by combining two quartz plates (45 mm. long by 5 mm. wide and of such a thickness as to show interference color green-yellow second order, the ellipsoidal axis c of the one to be parallel to the long direction, while in the second a is parallel) with two wedges of the same pitch (45 mm. long by 5 mm. wide and ranging in interference colors from yellow of the first order to violet-gray of the third order, and likewise the ellipsoidal axis c parallel to the direction of elongation in one and a to the same direction in the second); the wedge of long direction c to be combined with the plate of long direction a. In this manner the plate and wedge compensate in the center of the wedge and the interference colors rise to about blue-green of the second order at both ends. See Amer. Jour. Sci. (4), **26**, 371, 1908.

||Compare F. E. Wright, Tschermak's min. petr. Mitth., **20**, 233–306, 1901; also Jour. Geol., 33–35, 1902.

of this wedge (particularly the dark band region on both sides of which the interference colors rise and thus divide the field into four quadrants and produce an effect similar to that of the Bertrand ocular and of the Koenigsberger plate) have been found specially useful with minerals showing interference colors from red first order to blue second order. This wedge is held in a brass carriage, which in turn slides in the wedge holder shown in Plate 6, Fig. 3, and is viewed by the Ramsden ocular.

CALDERON* CALCITE PLATE.

This device also is placed in the focal plane of the ocular and consists of two adjoining calcite plates so cut that the direction of extinction in each plate makes an angle of about 3.5° on opposite sides of the common line of junction. The plate is so thick that the interference color is white of the higher orders and, when used alone, without an intervening crystal plate, lights the entire field under crossed nicols with a dull gray tone. If a crystal plate, whose lines of extinction do not coincide with the principal nicol planes, is then observed, the field appears divided into two unequally illuminated halves; and only when the extinction directions coincide with the nicol planes is the intensity of illumination in both halves equal. Calderon claims an accuracy of $\pm 2'$ with this ocular, but for a single determination and for general preparations the probable error is considerably larger (10′ to 15′). The principle on which this method is based is evident from the intensity formula, for in case the ellipsoidal axes of the plate do not coincide precisely with the principal nicol planes, they make unequal angles with the optic ellipsoidal axes of the calcite (in the one half, this angle is $3.5°+\theta$ and for the second $3.5°-\theta$) and this produces at once a marked difference in the intensity of illumination.

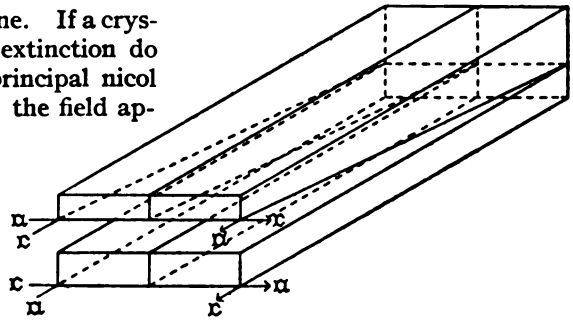


FIG. 80.

QUARTER-UNDULATION PLATE OF H. TRAUBE.†

This plate consists of two adjacent quarter-undulation mica plates so cut that the optic axial plane of each includes an angle of 3.5° (Fig. 81) with the common line of junction, and for slight deviations of a crystal plate from its correct extinction direction the two halves appear unequally lighted, and only when the crystal is precisely in its position of total extinction do the halves show the same intensity of illumination.

*Zeitschr. Kryst., 2, 69–73, 1878. The calcite twin plates of a Calderon ocular were tested by the writer and found to be inaccurately ground. The plate was 3.18 mm. thick and cut at an angle of about 45° with optic axis. The extinction angle in each half of the plate was measured in convergent polarized light by means of the dark bar in the center of the field and found to be +4.4° on the one half and −3.2° on the other. Extinction angles measured with this ocular, using the junction line of the plate as the line of reference, would therefore be out 0.6° from this source alone. The field of the ocular is, moreover, small and unfavorably lighted because of the thickness of the plate and of the wide dark junction line across the center of the field, which in turn disturbs the exact matching of the halves of the field.

†Neues Jahrbuch, 1898, I, 251.

TWINNED SELENITE PLATE.

The use of a twinned selenite plate has been recommended recently by E. Sommerfeldt* for the accurate adjustment of the ocular cross-hairs to the planes of the nicols. But the same twins can be made to serve admirably in the measurements of extinction angles. The extinction angle which the ellipsoidal axis makes in each plate with the twinning plane is 37.5°, and if the twinning line on such a plate be turned to the diagonal position with the crossed nicols, the extinction angle on each side of the nicol measures $45° - 37.5° = 7.5°$, but in the opposite halves different ellipsoidal axes are adjacent to the principal nicol plane. The result of this arrangement is a change in intensity dependent not only on the angle, but also on the different compensations of the path-differences in the two plates. If white light be used, this results in a rapid change in the interference color of the two halves of the selenite plate if the crystal be only a small angular distance from its position of total extinction.

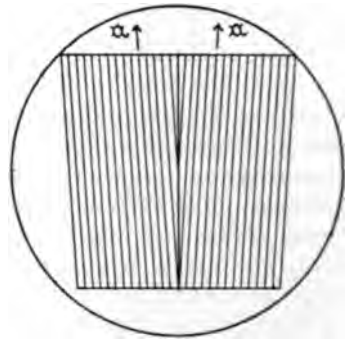


FIG. 81.

The writer has had such a plate cut, showing the sensitive tint and also a wedge, so that on insertion different interference colors, or intensities in monochromatic light, can be used for which the eye under given conditions of observation is most sensitive. These plates, as well as the preceding, are viewed by a low-power objective when used for the adjustment of the cross-hairs of the ocular, the junction line serving for the vertical cross-hair. For the determination of the positions of total extinction the Ramsden positive ocular has been found by experience to be best suited and a specially constructed holder to be convenient (Plate 6, Fig. 3).

ARTIFICIALLY TWINNED QUARTZ PLATE.†

Still another advantageous arrangement can be had by cutting, on a polished quartz plate parallel to the principal axis, a vertical edge making an angle of from 3° to 6° with the principal axis. The quartz plate is then divided transversely to the polished edge and the polished edges cemented together, thus producing an artificial twin whose two halves extinguish at equal and opposite angles from the common line of junction. Such bi-quartz plates may then be ground to a thin plate showing either the sensitive-tint or dull gray of the first order or to wedge form, thus increasing the range and usefulness of the device. Still lower interference figures can be obtained by superposing two polished plates of quartz (the one with the ellipsoidal axis c parallel with the elongation and the other with the ellipsoidal axis a parallel with this direction) after the manner of the combination wedge and then preparing from this a combination wedge divided

*Zeitschr. f. wissensch. Mikroskopie, 24, 24–25, 1907.
†F. E. Wright, Amer. Jour. Sci. (4), 26, 374, 1908.

into longitudinal halves, the ellipsoidal axes of the two halves including an angle of 5° to 10°. With such a combination wedge all interference colors from complete darkness to second-order colors can be obtained and the most sensitive conditions obtained for accurate measurements under the given conditions of illumination.

All of the preceding plates, the Bravais-Stöber, the Koenigsberger, and the Traube, the selenite twin plate, the quartz double combination wedge, and the artificially twinned quartz plate and wedges of the last paragraph, can be made possibly somewhat more sensitive by dividing the field into quadrants instead of halves, after the example of Bertrand in his rotatory polarizing quartz plates described below, but the attendant increase in difficulty of construction and in cost would probably offset any advantage gained thereby.

BI-NICOL OCULAR.

In the practical application of these different types of plates the angle θ has been small (2° to 4°) and found to furnish good results, but in each case there is a particular angle θ which is best adapted for the observations; the limit of sensitiveness of different eyes introduces, moreover, a variable element of such wide range that the angle θ can not be calculated and fixed once for all. In order, therefore, to have control over all angles θ and thus in each instance to be in a position to procure the best possible conditions, the bi-nicol ocular* was constructed, but after completion was found to suffer from a defect which it was difficult to overcome satisfactorily, namely, the depolarizing effect of the total reflecting prism pairs on the transmitted light-waves when the planes of the rotating nicols are not parallel with the planes of the polarizer and analyzer. As a result, a certain amount of false light is introduced into the field and tends to veil the sharp contrast of the two halves and thus to decrease the sensitiveness of the instrument.

BERTRAND PLATE.

In place of birefracting plates, which introduce an entirely new set of conditions in the path of light-waves and which complicate the expression for the relative intensity correspondingly, Klein† and Bertrand‡ have used the rotatory power of quartz plates, cut normal to the principal axis, on the plane of polarization of normally incident, plane polarized waves. As shown above, the total effect of such a quartz plate in monochromatic light is merely to increase the angular distance θ in the intensity formula (6). This power of rotation of quartz varies with different wave-lengths and with the thickness of the plate. If white light be used, interference colors result, as indicated in Figs. 69, 70, and 71. In Fig. 70 the change in intensity of different colors with change in thickness of the plate is shown, while in the dotted line of Fig. 71 the distribution of intensity for different wave-lengths throughout the spectrum is indicated for a quartz plate normal to the optic axis and 3.6585 mm. thick, for which the angle of rotation for the color 555 $\mu\mu$ is 90°; this color accordingly suffers total reflection in the

*Amer. Jour. Sci (4), 26, 374, 1908.
†Neues Jahrbuch, 9, 1874.
‡Zeitschr. Kryst., 1, 69, 1877; Bull. Soc. Min., 1, 27, 1878.

upper nicol. A comparison of this curve with the other curves of Fig. 71 shows clearly that the distribution of colors for a quartz plate of this thickness and cut normal to the axis is analogous to that resulting from a sensitive-tint plate of quartz or of selenite. The sensitive-tint is not precisely the same in both cases, nor is the resultant total visual intensity, but for practical purposes the quartz plate normal to the axis and about 3.557 mm. (rotation = 90° for light of wave-length 547 $\mu\mu$) thick serves well in place of the sensitive-tint plate. Its color depends on the kind of illumination used, and it is not satisfactory when used with deeply colored plates. To increase its sensitiveness, Bertrand combined two plates (2 mm. in thickness) of right-handed quartz with two plates of left-handed quartz, so that each right-handed plate is adjacent to a left-handed plate. This plate is inserted in the focal plane of the ocular; the sharp junction lines serve as cross-hairs. The Bertrand plate can be used in monochromatic light, provided that for the particular wave-length used its angle of rotation is not a multiple of π, in which case darkness ensues and the observed effect is nil. By rotating the upper nicol it is possible in white light to bring out the sensitive interference tint over the entire field covered by the Bertrand plate, and in such a position that a very slight turn of an intervening crystal from its position of total extinction is sufficient to disturb this equality of interference color and to divide the field into four quadrants, the opposite sections of which are similarly colored, while adjacent sections are differently colored. Even with colored mineral sections it is possible with the Klein or Bertrand plates to obtain, by rotating the upper nicol, a tint which is sensitive, under the conditions of observation, to slight changes in path-difference and thus to the extinction direction of the mineral.

The Bertrand plate is best adapted for use in white light; but it may also be used in monochromatic light, provided its thickness be correct for the particular wave-length of light employed.

In the case of quartz plates cut normal to the axis the conditions for maximum sensibility under given conditions of observation are readily derived from the standard equations above. With crossed nicols the effect of the inserted quartz plate in monochromatic light is to rotate the plane of vibration of the waves from the lower nicol through an angle δ. For two adjacent quartz plates, the one right-handed and the second left-handed, the intensity of the field for the first plate becomes $I_1 = I \sin^2 \delta_1$; and for the second plate $I_2 = I \sin^2 \delta_2$. Both fields will appear equally lighted if $\delta_1 = \delta_2$. For small values of the angle δ, the intensity varies with δ^2.

If now a crystal plate be inserted below the quartz plate, it can readily be shown that the intensity of the field for monochromatic light will be

$$I_1 = \sin^2 \delta + \sin 2\theta \sin 2(\theta + \delta) \sin^2 \frac{\pi d(\gamma' - a')}{\lambda}$$

in which θ is the angle included between the polarizer and the ellipsoidal axis γ' of the section. For the second half of the quartz plate (left-handed circular polarization) the intensity equation is

$$I_2 = \sin^2 \delta + \sin 2\theta \sin 2(\theta - \delta) \sin^2 \frac{\pi d(\gamma' - a')}{\lambda}$$

The difference in intensity between the two fields is accordingly

$$I_1 - I_2 = \sin 4\theta \sin 2\delta \sin^2 \frac{\pi d(\gamma' - \alpha')}{\lambda}$$

If $\theta = 0$, both sides of the field appear equally illuminated with the relative intensity

$$I_0 = \sin^2 \delta$$

The ratio of the least perceptible intensity difference between the two halves when a crystal plate intervenes to the intensity of the field of the quartz plate alone determines the setting of the plate at total extinction.

$$\frac{I_1 - I_2}{I_0} = \frac{K \cdot \sin 4\theta \cdot \sin 2\delta}{\sin^2 \delta} = 2K \cdot \sin 4\theta \cdot \cot \delta$$

The more sensitive the conditions, the smaller the angle θ; if, therefore, the reciprocal of θ be taken as a measure of the sensibility, then for small angles of θ we have

$$\frac{1}{\theta} = \frac{8I_0 \cdot K}{I_1 - I_2} \cdot \cot \delta$$

This relation indicates that the sensibility increases with the value of K and also with that of cot δ. The most sensitive conditions are obtained accordingly when K is large and δ small.

THE QUARTZ HALF-SHADE PLATE OF S. NAKAMURA.

In a recent paper,* S. Nakamura has discussed the problem of the sensitiveness of the half-shade system and arrived at practically the same conclusions as those noted above. He suggests the use of a double quartz plate of 0.04 mm. thickness instead of 3.5 mm. or 7 mm. thick as in the Bertrand ocular, and by actual tests finds the theoretical deductions valid and the plate useful. The thickness of 0.04 mm. is equivalent to an angle $(90 - \phi)$ of about 0.87° on each side of the junction line. Under certain conditions of strong illumination this angle is undoubtedly the best, and with the plate the accuracy of the measurements thereby attained equal to that of any of the other measuring devices.

BI-QUARTZ WEDGE PLATE.†

It is possible, however, to construct a combination wedge of quartz plates of such a character that any angle of rotation from 0° to any other value, positive or negative, can be had on insertion of the wedge, thus adapting to wedge form the advantage of the rotating bi-nicol ocular. This has been accomplished by combining two plates of quartz cut normal to an axis and of specified thickness, the one of right-handed, the other of left-handed quartz, each with a wedge of quartz of the opposite sign of rotary polarization, as indicated in fig. 82.

The effect of this combination is to produce zero rotation in each half wedge where plate and wedge have the same thickness; as the wedge is

*Centralblatt f. Mineralogie, 267–279, 1905. Compare also J. Macé de Lépinay, Jour. de Phys. (2), 4, 367, 1885; (3), 9, 585, 1900; Zeischrift Inst. Kunde, 21, 90, 1901. P. G. Nutting, Bull. U. S. Bureau of Standards, 2, 249–260, 1906.

†F. E. Wright, Amer. Jour. Sci. (4) 26, 377–378, 1908.

inserted or drawn out from this point of zero rotation the angle of rotation increases proportionately in a positive sense on one side of the junction line of the combination and in a negative sense on the opposite half. This combination wedge, which is introduced at the focal plane of the ocular (Plate 6, Fig. 3), divides the field under crossed nicols into two halves, whose intensities of illumination at any instant are equal, provided no intervening crystal plate is present or is rendered inactive by the parallelism of its ellipsoidal axis with the principal planes of the nicols. As soon as the crystal is turned even a very small angle out of this position, the intensity of illumination of the two fields is no longer equal. By inserting or withdrawing the combination wedge, the most advantageous angle of rotation in the two fields can be procured, so that the difference in intensity between the two halves is most apparent. In effect this wedge is identical with that of the bi-nicol ocular noted above, but is much simpler in construction, and requires no adjustment; the one condition which must be fulfilled for satisfactory results is that the wedge be not tilted on insertion; the optic axis must remain parallel with the optic axis of the microscope, otherwise disturbing bire-

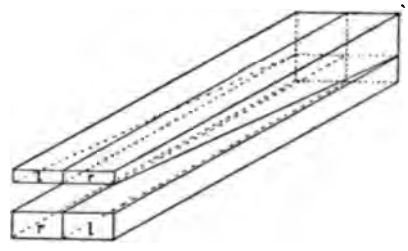


FIG. 82.—Bi-quartz wedge plate. Consists of two quartz plates with superimposed quartz wedges, all cut normal to the axis, the right-handed and left-handed elements arranged as indicated. Cementing material is Canada balsam whose refractive index is 1.54 while ω for quartz is 1.544, a difference so slight as to render inappreciable the exceedingly slight deviation of the waves caused by the slight wedge surface of the wedge. This inclined surface is mounted next the Canada balsam and care is taken (by inserting a thin glass strip at the thin end) to make the upper and under surfaces of the completed wedge parallel. In preparing the wedge it is necessary that the edges be ground and polished in order that the central division line be as sharp as possible. The two halves are eventually cemented side by side with Canada balsam and any disturbing influence thus eliminated which might arise from total reflection on the sides. At the point where wedge and plate have the same thickness the rotation is zero and a dark band traverses the wedge under crossed nicols. On the wedges which have been constructed, the rotation for sodium light at the one end has usually been about $\pm 1°$, while at the other end it has been either $\pm 3.2°$ or 10° or 15°. Specifications for a wedge of any angle of rotation at the thick end can readily be given.

fringence phenomena appear. The wedge carriage should, therefore, slide in an accurately fitting holder, such as that shown in Plate 6, Fig. 3. If the wedge be inserted horizontally to the point where wedge and plate have the same thickness, the effect of the combination on the plane of polarization of transmitted light is nil and a straight black vertical band appears in each half of the field similar to the zero band of the Babinet compensator, Fig. 83. By means of this band the position of total extinction of an inserted plate can be found with great accuracy, for the setting is thus made to depend upon the exact alinement of two black bands and the photometric principle

of comparing two dimly lighted fields is for the most part eliminated. The same line can be used to advantage for the adjustment of the nicols.*

METHODS INVOLVING ROTATION OF THE UPPER NICOL.

In all of the preceding methods the nicols have been considered crossed and the crystal plate has been turned. The intensity formula shows, however, that the relative intensity is dependent not only on the angle θ of the crystal plate but also on ϕ, the angle between the principal planes of the nicols. It was shown in the mathematical treatment that this method is in general more sensitive than the method based on the rotation of the crystal plate under crossed nicols. The mode of application of this method to any particular crystal plate is obvious and consists simply in placing the crystal under crossed nicols in its position of apparent total extinction and then observing, either in white or monochromatic light, the changes which occur on rotating the upper or lower nicol through small angles from its normal position. In case the crystal is actually in its position of total extinction, the crystal and field attain their position of maximum darkness simultaneously and show the same increase in its intensity of illumination; if, however, the crystal be not in position of total extinction, but a small + angle, as 30′ distant, then for a position of the nicol $+2°$ from its normal position the crystal plate will appear lighter than the field; and, *vice versa*, for the nicol $-2°$ from its normal position the crystal plate will appear darker than the field. This method† is extremely simple in manipulation

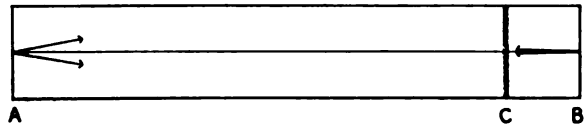


FIG. 83.

and does not require special apparatus. Weinschenk,‡ in describing the adjustment of the nicols in the microscope, uses the interference phenomena which occur under these conditions for the accurate adjustment of the nicols, but does not appear to have applied, conversely, the principle to the practical determination of the optic ellipsoidal axis in a given crystal plate.

CONVERGENT POLARIZED LIGHT.

Several different methods have been proposed which require convergent polarized light and are based on the change in aspect of symmetrical interference figures caused by the intervening crystal plate when it is not precisely in the position of zero extinction. The idea underlying these methods is that the eye can detect more readily slight changes in the shape of a

*In a review of a paper by the writer on the "Application of the bi-quartz wedge plate to polarimeters and saccharimeters" O. Schönrock (Zeitschr. Instrumentenkunde, **30**, 1910) has criticized the writer's description on the basis that this wedge had been described in 1900 by Macé de Lépinay (Jour. Phys. (3), **9**, 585, 1900). A comparison of the two descriptions is sufficient, however, to disprove this statement of the reviewer, as Macé de Lépinay's wedge consists simply of a thin bi-quartz wedge without the quartz plates and does not therefore, show zero rotation, nor the black line noted above. Schönrock suggested in a review of Macé de Lépinay's paper (Zeitschr. Instrumentenkunde, **21**, 90, 1901) the use of two bi-quartz wedges, but even with that arrangement failed to obtain the sensitive black line of zero rotation, which is one of the features of the bi-quartz wedge plate. If the construction suggested by Schönrock were carried out the fields would be uniformly lighted, if strictly parallel polarized light were used; to obtain different angles of rotation with this arrangement it would be necessary, moreover, to move the upper half over the lower half and this, mechanically, is not advantageous.

†F. E. Wright, Amer. Jour. Sci. (4), **26**, 379, 1910.

‡Zeitschr. Krystall., **24**, 581–583, 1895.

symmetrical interference figure than proportionate changes in intensity or color. Theoretically these methods suffer from serious defects which it is difficult to remedy. Convergent polarized light postulates a cone of obliquely incident light; the plane of polarization of each wave impulse of this cone suffers at the boundary surfaces of the crystal plate more or less rotation, depending on the angle of incidence, the optical constants of the crystal, the azimuth of the crystal section, and the enveloping medium. The lenses of the microscope, moreover, rotate the plane of polarization of all oblique waves whose line of propagation is not contained in the principal nicol planes. These factors together tend to modify the phenomena which appear in the interference figure so that the observed positions of total extinction may not be precisely correct for the section under investigation. Plane parallel polarized light should be used as far as possible in all such determinations.

The practical application of such methods, moreover, is not entirely satisfactory. The first method of this type was proposed by Kobell* in 1851, who used a plate of calcite normal to the optic axis as his test plate. The microscope was arranged for convergent polarized light and the crystal plate with the calcite test plate above it placed on the microscope stage and turned until the interference figure appeared perfectly normal and undistorted. Practically, the following objections apply to this method. The optical system of the microscope requires changing each time to meet the new conditions; during the observations the crystal itself is lost sight of, and in the case of minute crystals or crystals with undulatory extinction this is a serious drawback. Moreover, it is tacitly assumed that in the crystal plate itself for directions other than normal to its surface the planes of polarization remain parallel, which in general is only approximately true even for small fields which include only a small angle with the normal.

In the Brezina† method a more complicated interference figure is produced by two calcite plates cut at a small angle with the optic axis and cemented together one above the other in such a way that the optic axes of the two are in the same plane and at equal angles with the normal. The interference figure from such a combination is noteworthy because of a dark vertical bar through the center of the field. A slight rotation of an intervening crystal plate displaces this bar noticeably, but the same objections noted in the Kobell method apply with equal force to this method, with the result that neither method is made use of at the present time by working petrologists. Both these methods, in fact, were suggested before the petrographic microscope had come into general use.

THE RELATIVE SENSITIVENESS OF THE DIFFERENT METHODS.

The term position of extinction or extinction direction means practically that position of a birefracting plate for which light-waves are transmitted without changing their plane of vibration and for which no light passes the upper nicol, *i.e.*, the field is actually just as dark as though no birefracting crystal plate were there. A rotation of the plate through a very small angle from its position of total extinction allows a small percentage of the total amount of incident light through the upper nicol and the field is very dimly illuminated. For a given angle of rotation, the actual amount of

*Pogg. Ann., 95, 320, 1855. †Described in Schrauf's Lehrb. d. Phys. Min., 2, 219–220, 1868.

transmitted light can be increased only by increasing the original source of light, the sensitiveness increasing with the square root of the intensity. Since, however, it is not possible to increase the intensity of such a source indefinitely, and the human eye is sensitive only to a certain minimum limit, the threshold value, the position of actual extinction can only be determined within a definite degree of exactness. By means of the above devices, however, certain phenomena are introduced which increase the accuracy of such a determination, even though the field of original illumination remains the same. That method or device is obviously the best for which the probable error of a single determination under the same conditions is the least.

In comparing the relative accuracy of the methods described above, it will facilitate the presentation to assume definite conditions and then by means of the theoretical intensity curves (Figs. 73–78) to test the results attainable by the different methods under the most favorable conditions.

Let it be assumed that under the conditions of experiment the eye of the observer is of such sensitiveness that he is able to detect 0.05 of one per cent of the total light intensity; in other words, he can just detect the difference between the dark field of the microscope under crossed nicols and a crystal section turned at such an angle as to allow 0.05 per cent of the total intensity through the upper nicol. For all positions of the crystal, then, for which the intensity of the emergent light is less than 0.05 per cent, the crystal will appear absolutely dark. The heavy curves in Figs. 74 to 78 indicate the relative intensity of illumination of a crystal under crossed nicols for all positions of its major ellipsoidal axis from 88° to 92° or −2° to +2° with the plane of the polarizer; in Fig. 74 there is an interval of 38′ at least on each side of the total extinction position, for which the eye is unable to detect any interference illumination. The maximum error on a single determination under the most favorable conditions is, in this case at least, ±38′, while for Fig. 75 it is ±44′; for Fig. 76 ($K=\frac{1}{2}$) ±55′; for Fig. 80 ±1°17′; for Fig. 78 ($K=0$) the crystal is dark for all positions.

In any crystal, therefore, the conditions are most favorable when the plate is of such thickness that $K=1$ or the emergent waves are half a wave-length apart (in opposite phase). Conversely, having given a crystal plate, not all wave-lengths are best adapted for extinction-angle measurements. If yellow sodium light be used, a plate showing the sensitive violet interference tint is worthless, since for that tint the path-difference is about 555 $\mu\mu$, nearly a whole wave-length of Na light (589 $\mu\mu$), and for this difference $K=0$. If sodium light be used, then plates should be chosen for which the phase difference of the two emerging waves is $\frac{(n+1)\lambda}{2}$, bright yellow of the first order or pure yellow of the second order or green-yellow of the third order, etc. This is an important consideration and applies to all methods involving the intensity equations.

The visible spectrum extends roughly from about 400 $\mu\mu$ to 700 $\mu\mu$ and for this range of wave-lengths the maximum intensity in the Newton interference color scale (Fig. 69) is obtained for a path-difference between 200 and 350 $\mu\mu$ or at about 280 $\mu\mu$. For a path-difference of 555 $\mu\mu$, the total light emerging is only about 6 per cent of the total and for the major part of the spectrum the phase-difference (for path-difference, 555 $\mu\mu$) is such that

K is a small fraction not greatly different from zero, as shown in Figs. 69 to 71. Plates showing interference colors from first-order red to second-order blue are the least favorable, therefore, for the measurement of extinction angles by methods based on intensity differences. Plates, on the other hand showing interference colors gray to yellow of the first order are best suited for such measurements. If the methods involving interference tints (chromatic intensity) be used, however, these objections do not hold with equal force. Experience has shown that in case the mineral plate does show red or blue interference tints of the first and second orders the best determinations can be made either by the method of rotating the upper nicol or by the bi-quartz wedge plate, and the extinction direction is fixed by noting the absence of abnormal interference colors on rotating the nicol very slightly or on inserting the wedge.

After this digression on the most suitable sections for the measurement of extinction angles, Fig. 74 may again be considered and the relative accuracy of the different methods under the same conditions of experiment deduced.

The heavy curve (Fig. 74) indicates that for the assumed threshold value sensitiveness, 0.05 per cent of the total intensity, an error of at least $\pm 38'$ on a single determination is possible if the crystal plate alone be rotated under crossed nicols. On the other hand, if the crystal plate remains stationary and the upper nicol alone is rotated, the other intensity curves of Fig. 74 are valid, each curve indicating the intensity of illumination of the crystal plate for a specified angular distance from its position of total extinction during the rotation of the upper nicol from 88° to 92°. These curves indicate that the probable error with this method is less than half as great as in the preceding method, for if the crystal be only $\pm 15'$ distant from its position of total extinction, differences in intensity can even then be detected on rotating the upper nicol.

The changes in intensity of illumination of the microscopic field on rotation of the analyzer are indicated by the $0'$ curve, while for the crystal plate the $15'$ curve is applicable for illustration. At $88°43'$ (Fig. 74) the field is just beginning to show detectable illumination (0.05 per cent of total intensity), while for the same angle the crystal is illuminated with 0.097 per cent of the total intensity, nearly twice as great and easily noticeable. In this position the crystal plate appears, therefore, decidedly lighter than the field. On the other side of 90° the crystal plate passes the threshold limit of vision under the assumed conditions at $91°42'$, while for the same angle the microscopic field is illuminated by 0.097 per cent of the total intensity; in this case the field is appreciably brighter than the crystal and the difference can be readily detected by the eye.

If white light be used, these differences are accentuated by the abnormal interference colors which appear in the crystal plate when it is not precisely in the position of total extinction. This method of rotating the upper nicol has the advantage, furthermore, of not being dependent on the accuracy with which the nicols are crossed, since all data are referred at once to the plane of the polarizer. It is not, however, so advantageous in very weakly birefracting or deeply colored mineral plates; and for such plates may become less sensitive than the ordinary method.

More sensitive methods can be obtained with devices which allow the phenomena on both sides of the 90° position to be observed simultaneously. This is the purpose of the Bravais-Stöber plate, the double combination wedge, the Koenigsberger plate, the Calderon plate, the Traube plate, the artificially twinned quartz plates and wedges described above, and most effectively by the new circularly polarizing bi-quartz wedge plate; also by the bi-nicol ocular, though less satisfactorily. In each of these last two devices the plane of polarization of the incident waves is turned through equal angles on both sides of the junction line between the two halves, so that the field appears equally lighted throughout, while if the crystal plate be not in the position of total extinction it will appear lighter than the field in the one half and darker in the second. Since, however, there is an angle best suited under the given conditions to show these differences most clearly, it follows that the best results can be had with a plate or apparatus in which the angle ϕ can be varied at will. This condition of variable sensibility is met by both the circularly polarizing wedge plate and the bi-nicol ocular; by use of the bi-quartz wedge plate the probable error of the determination of the extinction position of any crystal plate is at least one-fourth that of a determination after the usual method by rotating the crystal plate under crossed nicols. Experience has shown that with favorable sections extinction angles can be determined by the use of the bi-quartz wedge with a probable error of less than $\pm 10'$ on a single trial.

Still another method for obtaining the most favorable conditions of experiment with a given plate is that suggested on page 136 with the artificially twinned quartz wedge. The two halves of this wedge extinguish at a small angle (*e. g.*, 3°) on opposite sides of the line of junction, and by inserting the wedge that particular interference color, or phase difference if monochromatic light be employed, can be produced for which the given angle of revolution (3°) is the best. This wedge, however, is less favorable than the circularly polarizing bi-quartz wedge, since its twinning line must be inserted precisely parallel with the plane of the polarizer, while with the circularly polarizing bi-quartz wedge the rotation of the planes of polarization of transmitted waves is entirely independent of the line of junction of the adjacent halves.

In the preceding pages special emphasis has been placed on those methods for measuring extinction angles which are of general application and which are based on intensity differences. The other methods, which are of limited application and can be used only in white light on favorable sections, depend on differences in interference colors produced by slight deviations of the crystal plate from its position of total extinction. Although these methods are serviceable in many instances, their application and the results obtained thereby are so dependent on the conditions of experiment that they are difficult to treat satisfactorily in a general way. Experience has shown that they are not more sensitive than the other methods and usually much less so. This is true both of the selenite sensitive-tint plate and of all combinations of the same.

EXPERIMENTAL TESTS.

To test the different methods under different conditions, different mineral plates were chosen and the position of complete extinction on each determined by the different methods under precisely the same conditions of

illumination with white light. On an anhydrite plate showing white interference tints of the higher orders the maximum error of a single determination by rotating the crystal plate under crossed nicols was found to be about 1.1°; by revolving the upper nicol alone, 0.4°; by inserting the bi-quartz wedge plate, about 0.1°; by using the Calderon ocular, about 0.5°; by means of the Bertrand ocular, about 0.1°; with the anhydrite section the sensitive-tint plate is of no value, since the interference color of the anhydrite plate itself is so high that the violet of the inserted plate has no effect and any occurring differences in intensity are in a strongly lighted field and not easily discernible.

Similar measurements were made on an apatite plate parallel to the principal axis and showing the interference tint, red of the first order. The maximum error of a single determination of the position of total extinction on turning the crystal plate alone under crossed nicols was found to be 0.9°; on rotating upper nicol about 0.2°, accurate because of abnormal interference colors which appear when the plate is turned only a slight distance from its correct extinction position; on inserting the bi-quartz wedge 0.2° to 0.3°; with the Calderon ocular, about 0.3°; with the Bertrand ocular, about 0.3°; the sensitive-tint plate is again of no value, since the interference color changes comparatively slowly as the crystal is rotated.

A section of nephelite parallel to the principal axis and showing the interference color, yellow first order, gave the following results: On turning the crystal plate alone, possible error 0.4°; on rotating upper nicol, less than 0.1°; with bi-quartz wedge plate less than 0.1°; Calderon ocular about 0.2°; Bertrand ocular less than 0.1°; sensitive violet plate still of very little value as a method; only slight changes in color for large angles of rotation of plate.

On a plate of colorless gehlenite of very low interference color, dull gray of the first order, the sensitive-tint plate proved as satisfactory as any other, and more so than the method of turning the crystal plate under crossed nicols or of rotating the upper nicol or the Calderon ocular. The Bertrand ocular and the bi-quartz wedge plate proved about as favorable, the probable error being slightly less than 0.5°.

A plate of strongly pleochroic tourmaline was also used and the following results obtained: Probable error of determination on turning crystal plate alone, about 1.6°; the method of rotating the upper nicol is of little value because of deep natural color of mineral and consequent inability to match fields; with the bi-quartz wedge plate 0.3°; Calderon ocular, about 0.4°; Bertrand ocular, about 0.5°. The sensitive-tint plate is useless because of the strong natural color of mineral which veils the true interference colors.

The results of these tests show that the theoretical deductions from the general equations are in general valid, but that in certain instances other factors (as natural color and very low birefringence) become dominant and tend to render some of the methods less sensitive and to favor the use of other (for general purposes, less suitable) methods. The bi-quartz wedge-plate, however, seems to apply in all cases with equally favorable results and to equal in sensitiveness any of the methods, whether of limited or of general application.

CHAPTER V.

OPTIC AXIAL ANGLE.

In petrographic microscopical analysis the optic axial angle is an important characteristic which separates birefracting minerals into two great classes—uniaxial and biaxial. Optic axial interference figures have long attracted the attention of observers and are still considered among the most interesting and beautiful phenomena in the whole realm of optics. Until recently they have been studied solely in convergent polarized light and the methods applicable thereto are in consequence better known and more fully developed than those requiring the use of parallel polarized light and the universal stage methods first successfully applied by Fedorow. We shall accordingly begin with the methods for measuring the optic axial angle of minerals in convergent polarized light.

CONVERGENT POLARIZED LIGHT.

There are several different lens combinations which can be used to advantage for obtaining and observing interference figures under the microscope in convergent polarized light;* and of these, the one suggested by Amici† in 1830 has been found to be the best suited for optical measurements. With this method the primary interference image, which is formed in the upper focal plane of the high-power objective, is magnified and reproduced as a secondary image in the upper part of the microscope tube, where it can be observed either with a magnifying glass or an ocular with cross-hairs and micrometer scale. The small Amici-Bertrand§ lens, by means of which this change of microscope to conoscope is effected, must be inserted at such a point between the ocular and objective that the secondary interference image observed through the ocular is as sharp and clear as possible.

Both theory and observation show, however, that all parts of the interference figure thus formed are not in perfect focus at the same time. Fig. 32, page 39, illustrates the path of a light beam from the condenser lens to the eye of the observer. From this figure it is evident that the upper focal surface of the objective for light-waves entering in all possible directions is not a plane, but consists of two coaxial convex, warped, eggshell-shaped surfaces, the mean locus of which roughly approximates a spherical surface. In the introduction it was shown that in view of the corrections of the objective for a fixed object and image distance, it is not possible to correct the objective so that the image in the upper rear focal plane shall also be a plane image free from the errors of central and oblique spherical and chromatic aberrations. These defects are common to all interference figures and can

*The conditions best adapted for observations in convergent light have been discussed in detail by S. Czapski, Neues Jahrb. B.B. 7, 506–515, 1891; and by E. A. Wülfing, Neues Jahrb. B. B. 12, 405–446, 1898.

†Ann. Chim. Phys. (3), 12, 114, 1844.

§E. Bertrand, Bull. Soc. Min. Fr., 1, 27, 97, 1878, 3, 98, 1880; 8, 29, 1885; for other lens combinations see A. v. Lasaulx, Neues Jahrb. 1878, 377; C. Klein, Nachr. d. K. Ges. d. Wiss. zu Göttingen, 1878, 461; Sitz. Br. K. Preuss. Akad. d. Wiss. 1893, No. 18; H. Laspeyres, Zeitschr. Kryst. 4, 460, 1880; 25, 380, 1896; A. Lacroix, Minér. d. l. France, 15–16, 1893; F. Becke, T. M. P. M., 14, 375, 1895; H. Lenk, Zeitschr. Kryst. 25, 379, 1896; Neues Jahrb. B. B. 10, 429, 1896.

not be eliminated. Still other factors enter into the formation of the interference figure, which are difficult to control and which tend to modify the figure to some extent, with the result that all measurements on interference figures are encumbered with a relatively large probable error exceedingly difficult to reduce under the ordinary conditions of observation.

As the surface, on which the interference figure is formed (upper focal plane of the objective), is not a flat but a curved surface, it is necessary to focus the secondary microscope, consisting of Bertrand lens and ocular, on points midway between the center and margin of the field and also to use two small aperture stops, one in the eye-circle of the ocular and the second either directly in front of the Bertrand lens or in its upper focal plane, in order to reduce the parallax and to increase the depth of focus of the secondary microscope. With these precautions the interference figure appears fairly sharp, even though magnified up to 15 diameters.

Different methods have been suggested for determining the exact position and angular equivalent of any point in a given interference figure. These methods consist in introducing scales of definite construction and value, either in the average plane of the interference figure itself or in one of its conjugate planes, and then ascertaining the angular equivalent of the divisions of the scale by means of minerals whose optic axial angle is known or by use of the apertometer. Having once determined this relation between the scale and its angular equivalents, the angular position of any point in the field is ascertained by direct observation with the scale. The conversion of the scale divisions into their angular equivalents is accomplished ordinarily by use of the Mallard constant of the microscope or by direct calibration of the scale by use of the apertometer, as first suggested by Dr. J. S. Flett.

THE MALLARD CONSTANT.

In 1882 Mallard* proved that the interference figure observed in the microscope is approximately an orthographic projection of the optical phenomena in space. In the measurement of optic axial angles only a low-power auxiliary microscope (magnification 3 to 15 diameters) is used; the rays from the interference figure which pass through the microscope subtend only a very small angle and may be considered practically parallel. The distance, therefore, of the different points in the interference figure from the center of the field is approximately the sine of the angle which the rays forming these points include with the axis of the microscope. It should be noted that these optic phenomena are observed as they appear in the objective itself, *i.e.*, modified by their refraction in the glass.

The actual distance d of a point in the interference figure is accordingly proportional to the sine of the angle which the ray it represents includes with the axis of the microscope; this angle can be reduced to its equivalent angle E in air by means of the refractive index n of the glass, while the distance D, as observed in the ocular, is the magnified image of "d." If M be the magnifying power of the auxiliary microscope, then $D = M \cdot d = \frac{M}{n} \sin E$, or $D = K \sin E$, wherein K is the Mallard constant of the microscope for the fixed position of the optical system used in the observations.

*E. Mallard, Bull. Soc. Min. Fr., 5, 77–87, 1882.

Conversely if the angle E be known and the distance D be measured, the constant K can be calculated from the above equation and this in turn applied to the measurement of optic axial angles whose values are not known.

This method, however, does not furnish a check on the value of K thus obtained, and its validity can not be considered verified for other angles E, unless many similar sections of different biaxial minerals be taken and the K of the microscope for each angle E be ascertained. The Mallard equation above assumes that the loci of the focal points of waves entering in all directions lie on a perfect spherical surface, an assumption which actual microscopic objective lens systems do not fulfill in the strict sense of the word.

The validity of this formula has been tested by the writer by several different methods* and also by E. A. Wülfing† on four mineral plates of known optic axial angle. The observations show that the accuracy of the Mallard equation is not the same for all objectives, certain objectives furnishing values which agree closely with the values of calculation, while marked differences between observation and calculation occur in others. As a rule, the agreement is close and not unsatisfactory.

TABLE 6.

E Degrees.	D Horizontal scale	D Vertical scale.	Sin $E=\frac{D}{K}$ $K=30.4$.
5°	3.5	.....	D
10	7.0	7.0	7.0
15	10.4	.....	.....
20	13.9	13.8	13.8
25	17.2	.....	.....
30	20.4	20.2	20.2
35	23.4	.....	.....
40	26.2	26.0	26.0
45	28.8	.....	.....
50	31.2	31.0	31.0
55	33.3	.....	.....
60	35.2	35.0	35.0
65	36.6	.....	.....
70	37.9	37.6	38.0
75	38.8	.....	.....
80	39.5	39.0	39.8

The simplest and most convenient method for testing the Mallard equation and for calibrating a microscope for all possible angles is that of Flett, who uses an Abbe-Zeiss apertometer for the purpose. The micrometer scale is introduced as usual in the focal plane of the ocular and the angle corresponding to any number of divisions of the scale is read directly on the apertometer. In this manner an objective can be calibrated rapidly for all possible angles within the field of vision and an empirical, correct table prepared which is independent of the Mallard constant, thus obviating all errors due to the fundamental assumption on which the Mallard formula is based.

*Amer. Jour. Sci. (4), 14, 327, 1907. †Rosenbusch-Wülfing, Mikroskop. Physiogr. 1, 1, 330, 1904.

The results of the calibration of a 4 mm. Zeiss apochromatic objective by this method (with Bertrand lens and cross-line micrometer disk* in the focal plane of the Ramsden ocular and with a small aperture stop in its eye-circle) are given in table 6, from which it is evident that the Mallard formula obtains throughout practically the entire field, the appreciable differences being confined to the margin of the field, where the measurements at best are less satisfactory because of (*a*) increasing indistinctness of the dark axial brushes, (*b*) the crowding together and decreasing distance between successive equal angular intervals, (*c*) the distortion caused by the analyzer.

TABLE 7.

	Observed.		Calculated.			Observed.		Calculated.	
E	*Dv*	*Dh*	*Dv*	*Dh*	*E*	*Dv*	*Dh*	*Dv*	*Dh*
0°	0	0	0	0	26°	181.	186.	.....	.....
1	8.5	8.	.....	.....	27	187.5	192.5	.....	.....
2	16.5	16.	.....	.....	28	194.5	199.	.....	.....
3	23.	22.5	.....	.....	29	199.	206.5	.....	.....
4	29.	30.	.....	.....	30	205.5	212.5	206.1	211.2
5	36.5	37.5	35.9	36.8	31	213.5	218.5	.....	.....
6	44.	44.	.....	.....	32	219.	225.	.....	.....
7	52.	51.	.....	.....	33	225.	230.5	.....	.....
8	58.5	57.5	.....	.....	34	230.5	237.5	.....	.....
9	65.	65.5	.....	.....	35	236.5	243.5	236.5	242.3
10	71.5	73.5	71.6	73.4	36	243.	250.5	.....	.....
11	78.5	80.	.....	.....	37	249.	256.5	.....	.....
12	85.	87.5	.....	.....	38	254.	261.5	.....	.....
13	92.5	94.5	.....	.....	39	259.	266.5	.....	.....
14	100.5	102.	.....	.....	40	264.	272.	265.	271.5
15	107.5	109.5	106.7	109.3	41	269.	277.	.....	.....
16	113.5	117.	.....	.....	42	273.	283.	.....	.....
17	119.5	124.5	.....	.....	43	279.5	288.	.....	.....
18	125.5	131.5	.....	.....	44	286.5	293.5	.....	.....
19	132.	138.	.....	.....	45	290.	299.	291.5	298.7
20	139.5	144.5	141.0	144.5	46	295.	303.5	.....	.....
21	146.5	152.5	.....	.....	47	300.	308.	.....	.....
22	154.	158.5	.....	.....	48	.....	.....	.....	.....
23	160.	164.5	.....	.....	49	.....	.....	.....	.....
24	166.	171.	.....	.....	50	.....	.....	316.6	323.6
25	173.	179.	174.2	178.5					

The validity of the formula was furthermore checked for another microscope with another lens system (No. 9, Fuess, with Amici-Bertrand lens, system 7 to 9, and double-screw micrometer ocular with Ramsden ocular) and by another method. Two polished plates of calcite were used, the one cut after 0001 and the second at an angle of 23°17′ with 0001 as measured on a two-circled goniometer, the adjustment on the goniometer having been accomplished by means of fresh cleavage faces along the edge of the plate. These plates were placed successively on a carefully adjusted Fedorow-Fuess universal stage (large model) fitted with a special universal condenser lens† and the positions of the optic axis were measured on the micrometer ocular

*Amer. Jour. Sci. (4), 24, 341, 1907. †Amer. Jour. Sci. (4), 24, 332, 1907.

screws as the stage was turned about its horizontal axis from degree to degree on both sides from the horizontal position. By means of the two plates a continuous set of readings was obtained for D for angles E ranging from 0° to 47°. These values were ascertained for both scales (horizontal and vertical) of the double-screw micrometer ocular described on page 155 and are listed in table 7.

The values for D listed under the heading "calculated" in this table were calculated from Mallard's formula by taking the average value of K for both the horizontal and vertical scales of the double-screw micrometer ocular; for the vertical micrometer scale, $K_v = 412.3$; for the horizontal, $K_h = 422.4$. The agreement between theory and practice for this objective (Fuess No. 9) as indicated in table 5 is remarkable.* The screw of the horizontal micrometer registered 0.005 mm. for each division on the head, while the vertical scale screw, which was constructed later and on a different lathe, was a trifle coarser and registered a slightly greater movement for one division on its head. For this reason the values K_v and K_h are slightly different. On an average a movement of 6 divisions or 0.03 mm. corresponded to 1°, so that with the bi-micrometer ocular the probable error is nearly 10′ from the setting alone and in ordinary work with interference figures differences of 1° may be expected.

If only a single-screw micrometer ocular be used, the section should be cut very nearly normal to the acute bisectrix; otherwise the values become much less certain, but with a double-screw micrometer ocular, or micrometer ocular with coordinate scale, this error can be eliminated directly and equally good values can be obtained on sections only approximately normal to the acute bisectrix, as will be shown on page 155. In all measurements of optic axial angles, it is highly essential that the lens system of the microscope be perfectly centered, otherwise errors of considerable magnitude are possible.

Instead of solving the Mallard equations ($D = K \cdot \sin E$ and $\sin E = \beta \sin V$) by logarithms, graphical methods may be used which are sufficiently accurate for the purpose. The method of Fedorow,† which is a graphical solution of the proportion

$$\frac{\sin E}{1} = \frac{\sin V}{\frac{1}{\beta}} \quad \text{or} \quad \frac{A}{1} = \frac{B}{C}$$

is of general application and is satisfactory in practice. An accurate drawing (Plate 7) is made, which serves for all possible angles and all refractive mineral indices to be encountered. To solve the equation $D = K \sin E$

	H	D	K	H_1
Aragonite...	11°33′	0.325	1.623	10°59′
Muscovite...	24°43	0.700	1.674	24°14′
Topaz.....	39°05	1.075	1.705	(39°05′)
Calcite.....	60°51	1.590	1.821	68°50′

*In the Mikroskopische Physiographie I, 1, 330, by Rosenbusch and Wülfing, the latter gives a series of measurements with an α-monobromnaphthalene immersion objective of R. Winkel and finds differences as high as 8° between observed and calculated values as indicated in the table herewith. The angles under H are half the axial angles for these minerals obtained from measurements on an optic axial angle apparatus, while the angles under H_1 were calculated by Mallard's formula on the assumption that the K obtained for topaz (1.075) is valid for all angles. The differences between observation and calculation are large and indicate, in accord with the observations by the writer, that the determination of the positions of optic axes near the periphery of the field is less accurate than that for more centrally located points.

†Zeitschr. f. Kryst., 26, 225–261, 1896. Compare also Souza-Brandao, Comm. d. Serv. Geologico de Portugal, 5, 118–250, 1903. F. E. Wright, Amer. Jour. Sci. (4), 24, 330, 1907.

or sin $E=\frac{D}{K}$, draw a circle with radius K (Plate 7, preferably in colored ink); the intersection of the ordinate D with this circle determines then the angle E in degrees. To solve the equation sin $E=\beta$ sin V, or

$$\frac{\sin E}{1} \quad \frac{\sin V}{\frac{1}{\beta}}$$

find the intersection of radius $E°$ with the circle for the given refractive index and pass horizontally from this point to the intersection with the outer circle of Plate 7, which point indicates the angle V in degrees. The following are examples:

(1) $K=54.0$ $D=21.1$

Intersection of ordinate D with K-circle is at radius 23°.

(2) $E=42°$ $\beta=1.65$

Pass along the radius 42° to the intersection with the circle $16.5=\beta$ and then horizontally to the outer circle and read $V=24°$.

Having once determined, however, the angular equivalents of the micrometer scale divisions for the particular lens system, it is simpler to plot these values once for all, as a curve from which the angle corresponding to any scale reading can be read off directly. Curves can also be drawn, showing directly the equivalent angle V for any angle E and refractive index β, and *vice versa*. A convenient form for such curves is illustrated in Plate 8, in which the angles E are the abscissæ, the angles V the ordinates, and the curves the refractive indices.* But having once determined K or the equivalent angle curve, it is possible to prepare, once for all, a set of curves (with the angles V as abscissæ, the readings D as ordinates, and the refractive indices as curves derived from the equation sin $E=\frac{D}{K}=\beta$. sin V or $D=K.\beta.\sin V$), from which the angle V can be read off directly, thus obviating one set of operations. Such a set of curves is then valid only for the particular microscope and lens system with which the observations were made.

Mallard's method for measuring the optic axial angle is one of the most satisfactory of the microscopic methods and if sections showing the required phenomena are available Mallard's method should be adopted without question, especially if the measurements can be made with a double-screw micrometer ocular, or with a micrometer ocular with coordinate micrometer scale. The limits of error of measurements of $2V$ by the Mallard method should not exceed 1° to 2° on clear interference figures.

THE BECKE DRAWING-TABLE METHOD.

In place of the single-screw micrometer ocular, which in itself is of very limited application, F. Becke† has substituted a graphical method in which the observed optical phenomena are projected by a camera lucida on a rotat-

*Another form of plot has been used by Dr. Merwin of this laboratory in connection with work on the alkali feldspars; in his plot the refractive indices were chosen as ordinates, and the angles E as abscissæ and the angles V as curves. For general work, however, the form of the curves in the arrangement of Plate 8 seems more favorable.

†F. Becke, Tschermak's Min. petr. Mittheil., 14, 563, 1894; 16, 180, 1896.

ing drawing-table fixed in position relative to the microscope. Accurate drawings of the interference phenomena are then prepared and serve in place of the actual interference figure. This method has been fruitful in its results, and with practice the necessary manipulative skill can be acquired to obtain trustworthy axial-angle values. The accuracy of the method is dependent on several factors—the accuracy with which the drawing is prepared, the exactness with which the drawing-table is centered, and the care with which measurements are made on the finished drawing.

The actual field of the projection does not measure over 25 mm. in diameter, and a difference of 1° of E corresponds to a difference in D of about 0.25 mm., a distance which is easily measurable. With unusually sharp axial bars and nice adjustment of the optical system, it is theoretically possible to obtain an accuracy of about $\frac{1}{3}$ to $\frac{1}{2}$°; in practice, however, a greater accuracy than 1 to 2° can not be claimed for the method.

The writer has not seen the revolving drawing-table described by Professor Becke, and has used in his work a small rotating disk graduated in degrees and supported by an arm which, in turn, is attached to the microscope stand by means of a collar.* This device was constructed in the mechanical workshop of the Geophysical Laboratory. The results obtained with it have proved satisfactory and the manipulation with the same convenient.†

Having once fixed the position of this table so that its axis of rotation coincides (after reflection in the camera lucida) with the optical axis of the microscope and is also at the proper distance from the eye for distinct vision, its constant K, corresponding to the K of the microscope in the formula $D = K \sin E$, can be determined by one of the methods described above.

With the drawing of an interference figure thus properly prepared, it is possible to determine the angular distance—polar angle ρ and longitudinal angle ϕ, of any point in the projection—and to plot the same in stereographic or orthographic or angle projection, and thus to measure the angular distance between any two points, as those between optic axes occurring in the field of vision.

In a recent article,‡ Professor Becke has described an ingenious method by which any section, in which only one optic axis appears in the field, can be used for the measurement of the optic axial angle, although the values obtained are only close approximations to the correct value of $2V$. He utilizes the fact that sections of biaxial minerals, cut approximately normal to an optic axis, exhibit, in convergent polarized light, dark axial bars which resemble hyperbolas in the diagonal position and whose degree of curvature is dependent on the optic axial angle $2V$. For any given position of the stage, the points along the dark bar of the interference figure correspond to those directions of light-wave propagation in space whose lines of vibration are contained in the principal plane of the lower nicol (polarizer) and for which the extinction angle is zero.

To measure graphically the optic axial angle of a given mineral from the degree of curvature of its dark axial bar (zero isogyre) on a section approxi-

*Amer. Jour. Sci. (4), 24, 332, 1907.

†H. Tertsch (T. M. P. M. 29, 171–172, 1910) has described recently an ocular which is so arranged that the image of the interference figure is projected by the Bertrand lens on a glass surface on which it is traced directly.

‡F. Becke, Tschermak's Min. petr. Mitth., 24, 35–44, 1905.

mately normal to an optic axis by this method, the axial bar is first drawn when in a position parallel to the horizontal cross-hair (Fig. 84), the straight line $A_1 C$ in this position being the trace of the plane of the optic axes; the microscope stage and drawing-table are then rotated in the same direction through some convenient angle 30° or 45° and the axial bar drawn in the new position ($A_1 H$ of Fig. 84). These drawings are repeated after rotation of the microscope stage or drawing-table alone through 180° (A'_1C' and A'_1H'); the center O of the projection then bisects the distance $A_1A'_1$.

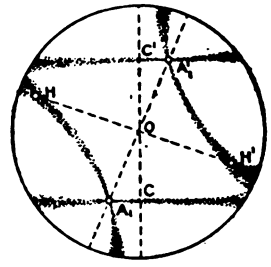


FIG. 84.

The point H in the projection is any convenient point on the achromatic brush or zero isogyre, and is therefore a direction in the crystal along which light-waves are propagated whose plane of vibration on emergence coincides with the extinguishing plane of the analyzer. The plane of vibration for the point H is thus known, and the law of Biot-Fresnel can be applied directly to find by construction the second optic axis A_2.

By means of the Mallard formula the polar angular values ρ equivalent to the distances OA_1 and OH in the interference figure are first deduced, and these in turn are reduced to true angles within the crystal by means of the refractive index formula

$$\sin r = \frac{\sin i}{\beta}$$

where i is the angle observed in air, r is the angle within crystal desired, and β is the mean refractive index of the crystal. The error committed in using β for the reduction of the angle equivalent to OH instead of the actual value is not great and can be neglected, since the latter does not differ appreciably from β in minerals of ordinary birefringence.

The form of graphical construction used by Becke in applying this rule is shown in Fig. 85. The observed points H and A_1 are first plotted (by means of their observed longitudinal angles ϕ and reduced polar angles ρ) on tracing paper above the stereographic projection plat of Wulf (Plate 4); the tracing paper is then rotated about the center, O (tracing paper held in place by a needle-point through O) until H coincides with the vertical diameter of the underlying plat and the great circle PK, whose intersection with the vertical diameter is 90° from H (polar circle to H), is then sketched. Similarly, the great circle DE, containing A_1 and the extremities of the horizontal diameter, is located and drawn. The great circle HT, which indicates the plane of vibration of H, is then determined by Becke as the one tangent at H to the straight line LQ parallel to the trace FOI (in the projection) of the principal section of the lower nicol.* The great circle HA_1, passing through H and A_1 is then sketched and its intersection A'_1 with the polar circle PK is accurately determined. The projection of the second optic axis A'_1 is found by making $A'_2T = A'_1T$ (Biot-Fresnel's law). The intersection of the great circle HA'_2 with the plane of the optic axes A_1A_2 determines then the position A_2, and the angle A_1A_2 in projection is $2V$, the angle between the optic axes.

*Recently Professor Becke (T. M. P. M., 28, 293, 1909) has suggested a simpler method of construction which leads to the same result (see footnote, page 160).

The actual time consumed in this operation is not great, and the values obtained are approximately correct. The objection to its use lies chiefly in the subjective factor involved, namely, the skill required in drawing accurately the phenomena observed, and also in the nice adjustment of all parts of the instrument.

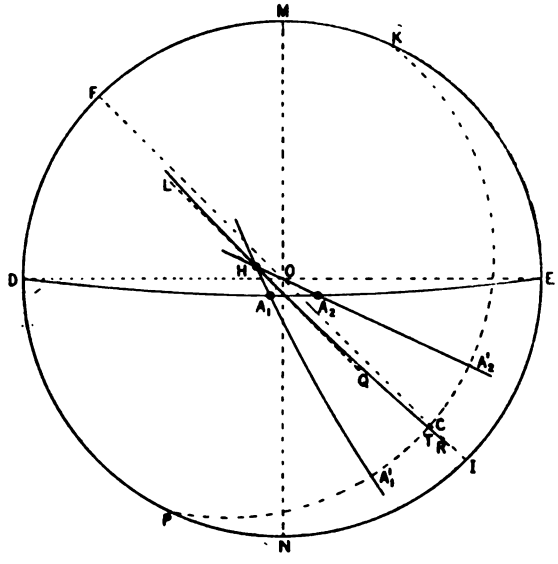


FIG. 85.

The location of the optic axis A_1 is at all times more accurate and trustworthy than that of H, owing to the indistinctness and width of the black axial bars near the margin of the field—in consequence, partly, of the rotation of the plane of polarization of the obliquely incident light-waves.

METHODS WITH THE DOUBLE-SCREW MICROMETER OCULAR; ALSO WITH THE COORDINATE MICROMETER OCULAR.*

In seeking for more accurate and at the same time simpler methods than those of Professor Becke described above, the writer has used (in place of the usual single-screw micrometer ocular, with a movement in one direction only) a double-screw micrometer ocular with movements in two directions normal to each other. This was first constructed in the workshop of the Geophysical Laboratory (Plate 2, Fig. 2).† By its use it is possible to determine the position of any point in the interference figure accurately by means of two micrometer-screw readings, which correspond to rectilinear coordinates in the orthographic projection and vertical small circle coordinates in the stereographic projection. By means of the constant K of the microscope for each of these micrometer movements, V and H, which must have been determined previously by means of known angle values (table 7,

*Amer. Jour. Sci. (4) **24**, 336, 1907; (4), **31**, 97, 1911; Jour. Wash. Acad. Sci., **1**, 60–61, 1911.

†The two movements of the double-screw micrometer ocular, H horizontal and V vertical, are effected by fine micrometer screws, reading accurately to 0.005 mm. The construction of this ocular is similar to that of the single-screw micrometer oculars, except that here two screws with corresponding movements are used in place of the single screw. O = Ramsden ocular; S, small stop aperture to reduce errors of parallax. This ocular is now made by R. Fuess & Co., Steglitz, Germany.

page 150), each of these readings is reduced, as usual, to its corresponding angle value for the crystal by means of K and the average refractive index of the crystal,

$$\sin V = \frac{D}{K \cdot \beta}$$

Having given the interference figure from a section of a biaxial mineral, cut so that one axial bar is visible, the course of procedure in measuring the optic axial by means of the double-screw micrometer ocular consists in: (*a*) rotating the microscope stage until the dark axial bar is parallel to the horizontal cross-hair of the ocular; (*b*) moving the horizontal cross-hair by means of the vertical micrometer screw V until it coincides precisely with the center of the dark axial line (Fig. 86, A_1C_1); (*c*) rotating the nicols (not the stage, as is the case with the Becke method) about a suitable known angle (usually 30° or 45°), the exact position of the optic axis A_1 then being the intersection of the axial bar with the horizontal cross-hair (Fig. 86, A_1C_1 with A_1H_1); (*d*) moving the vertical cross-hair by means of the horizontal micrometer screw until it coincides with this intersection and recording both vertical and horizontal micrometer-screw readings; (*e*) the stage is then rotated about an angle of 180°, and similar readings for A_1 taken in its new position, A'_1. This last step is necessary in order to locate the center of the field O (half the distances $C_1C'_1$ and $D_1D'_1$). The position of A_1 is thus fixed accurately and can be plotted directly after proper reduction to true angles within the crystal ($\sin r = \frac{\sin i}{n}$ for each coordinate angle)* in ste-

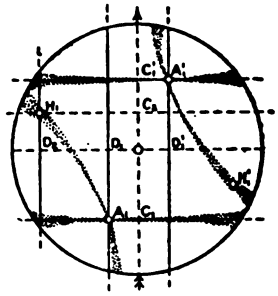


FIG. 86.

*Two methods are available or locating a point H in the interference figure: (1) by means of its longitudinal angle ϕ and its polar angle ρ, as in the Becke method; (2) by means of small circle coordinates, as in the method just described. Before plotting these angles of observation, they must first be reduced (by use of the refractive index formula) to corresponding angles within the crystal plate. The question may then be asked, "Do the reduced angles of the two methods lead to the same point H' in the projection?" That they do is evident from the orthographic projection (Fig. 87, projection of the interference figure as observed) in which O is the zenith, OX the first meridian, H the point observed, LOH its longitudinal angle, $OH(=\sin\rho)$ its polar angle, OL $(=\sin\alpha)$ and OM $(=\sin\beta)$ its small circle coordinates. The reduction of the observed angles α', β', ρ', to corresponding angles α, β, ρ in the crystal plate is accomplished by the equations

$$\sin\alpha' = \frac{\sin\alpha}{n}, \qquad \sin\beta' = \frac{\sin\beta}{n}, \qquad \sin\rho' = \frac{\sin\rho}{n}$$

But in right-angled triangle LOH, $OH^2 = OL^2 + LH^2$ and the length $LH = OM$;

accordingly $\sin^2\rho = \sin^2\alpha + \sin^2\beta$

Therefore

$$\frac{\sin^2\rho}{n^2} = \frac{\sin^2\alpha}{n^2} + \frac{\sin^2\beta}{n^2} \text{ or } \sin^2\rho' = \sin^2\alpha' + \sin^2\beta'.$$

In orthographic projection, accordingly, the points H' obtained by the two methods are identical in position and the reduction of the coordinate angles as observed is therefore permissible. Since these coordinate angles define certain planes in projection, the stereographic or any other projection may be used in place of the orthographic projection. In making the reduction it is assumed that the refractive index n is the same for the different directions H, L, M: this is not strictly correct, but the error introduced thereby is practically negligible for minerals of ordinary birefringence.

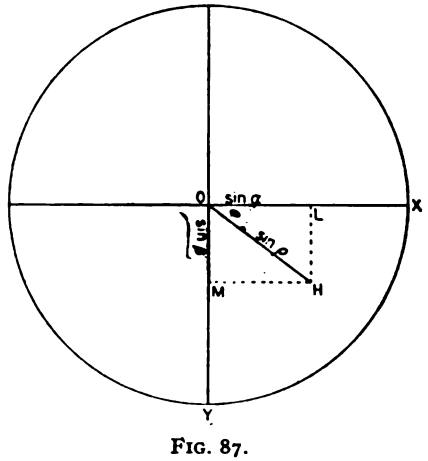


FIG. 87.

reographic (small circles) or orthographic (coordinates) or angle projection. Any point H of the dark curved axial bar can then be determined by two micrometer readings (coordinates from the center), and, after proper reduction to angles within the crystal, may be plotted in the projection. From the projection plot thus obtained, the optic axial angle can be ascertained, either by the Becke method described above or by the following simpler method,* which differs from the Becke method in the determination of the direction of vibration for a dark point H on the zero isogyre in the interference figure.

In the new method two courses of procedure are available for finding the great circle which indicates the plane of vibration for a point H of the dark brush of an interference figure: (a) it is the great circle passing through H and the intersection C of the polar great circle PK (Fig. 88) with the trace

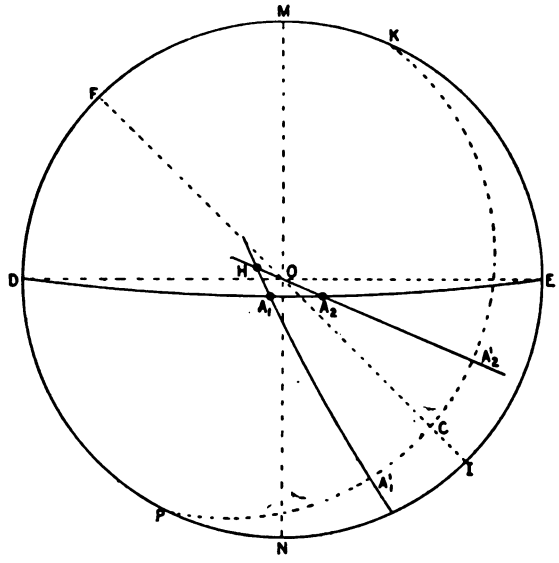


FIG. 88.—In this figure the operations of actual construction are given which are required to measure $A_1 A_2$ from the data in the drawing. The points A_1 and H are first located accurately in the drawing, reduced to angular values and plotted directly on tracing paper in stereographic or angle projection; the great circle PK polar to H, and the horizontal great circle DA_1E through A_1 are then sketched; the point A'_1 is the intersection of the great circle HA_1, containing H and A_1 with the polar circle PK, while the line OC is the trace of the plane of vibration of the lower nicol (polarizer) as it appears on the drawing after the rotation of 30° or 45°; the angle MOF indicating directly the angle of rotation. By construction A'_1C is made equal to A'_1C, and the intersection A_2 of the great circle HA'_2 with the great circle DA_1E determines A_2, the second optic binormal; the angle $A_1 A_2$ is the desired optic axial angle $2V$.

of the principal plane FOI of the lower nicol; (b) it is the great circle containing H and tangent at H to the small circle which parallels in projection the trace FOI of the principal plane of the lower nicol. In actual work, however, it is not necessary to draw this great circle HC, as the point C is the point sought and determines at once the direction of extinction for the given section. The simplified construction is illustrated in Fig. 88, where C is the

*Am. Jour. Sci. (4), 24, 336, 1907.

intersection of the great circle PK with the diameter OC, the trace of the plane of vibration of the lower nicol.

In plotting the angles corresponding to the coordinate readings of the double-screw micrometer ocular, it should be noted that these angles apply to small circles, the angle for each micrometer-screw reading indicating the position, from the center of the projection, of a certain horizontal or vertical small circle. The intersection of the horizontal and vertical small circles thus obtained from the two micrometer-screw readings for a particular point of the interference figure determines the location of that point in the projection.

The reason for adopting this graphical method in preference to the Becke method is apparent when the factors underlying the formation of an interference figure are considered. An interference figure is obtained by passing a cone of convergent polarized light-waves through a crystal plate and

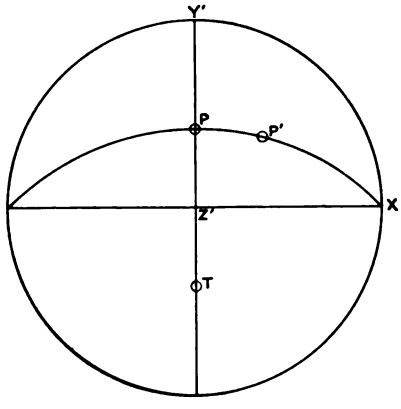


FIG. 89.

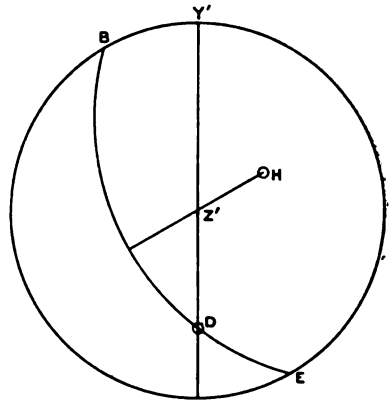


FIG. 90.

observing the interference phenomena as they appear in the rear focal plane of an objective of short focus when examined through an analyzer. In the course of their passage through the microscope, the light-waves emerging from the lower nicol (polarizer) may be considered practically parallel, plane polarized waves. In transmission through the condenser lens system, their directions of propagation are changed and they emerge from it in a sharply divergent cone; but their lines of vibration have remained in the same plane except for the slight rotatory effects of the surfaces of the condenser lenses, which for the moment may be disregarded. That this is the case is tacitly assumed in all microscopic work, since the rotatory effects produced by the condenser and objective lens systems alone, on the plane of polarization of transmitted light-waves, are practically negligible and the field appears approximately dark under crossed nicols. Thus in Fig. 89, if the direction Z' be the axis of the optical system of the microscope, $Y'Z'$ the plane of vibration of the entering waves, and P the direction of propagation of one of these waves after refraction, its direction of vibration will then be along T, at right angles to P and in the original plane of vibration. This same direction of vibration, OT (O being the center of the sphere of projection), obtains for any other point P' in the polar plane to T. A wave

propagated along P', but still vibrating along OT in the original plane of vibration, will be destroyed by total reflection in the analyzer, just as is the wave OP. Since the entire field may be covered with waves similar to OP', whose directions of vibration are contained in the plane of vibration $Y'Z'$, all the waves of the converging cone from the condenser and objective systems are extinguished by the analyzer, and the field appears dark between crossed nicols provided no birefracting crystal plate intervenes. The effect of the lens system of the microscope is, therefore, to change the directions of propagation of transmitted light-waves, but not seriously to affect the plane in which their vibrations take place.* Conversely, if $Z'Y'$ (Fig. 90) be the extinguishing plane of the analyzer, and H any dark point in the field of the interference figure, the direction of vibration for this light-wave H

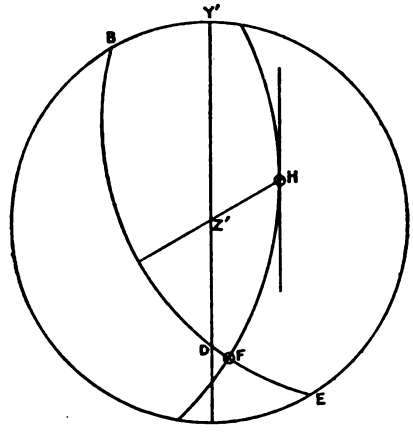


FIG. 91.

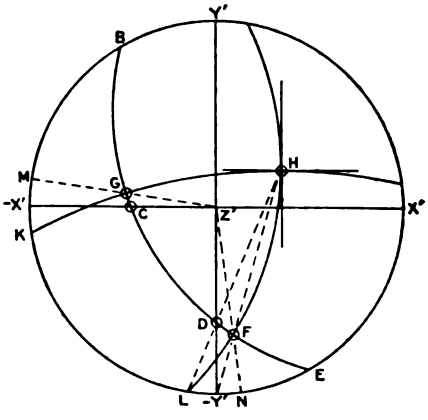


FIG. 92.

must be contained in the plane $Z'Y'$ and also in the polar plane to H'; it is accordingly the direction D. If its direction of vibration be not in the plane $Y'Z'$, it will not be totally extinguished in the upper nicol and the point H will not appear completely dark. Briefly stated, for any dark point H of the interference figure, the direction of vibration is the line of intersection, OD, of the extinguishing plane $X'Z'$ of the upper nicol with the plane BE polar to H. This is the rule of construction given by the writer for finding the plane of vibration of any dark point in the interference figure. As noted above, the rotatory effects of the surfaces, both of the crystal plate itself and of the glass mounts and lenses, are disregarded in this connection. These effects are small, but still noticeable, and the method, in consequence, is only an approximate method.

In Professor Becke's method, outlined above, the direction of vibration for a dark point P on the zero isogyre of the interference figure is found by drawing in stereographic projection the great circle which is tangent to a

*That there is some effect on the planes of polarization of transmitted light-waves is at once evident, even without accurate measurements, from the lack of uniformity in illumination of the field when viewed under crossed nicols in convergent polarized light. A dark cross divides the field into quadrants which are perceptibly lighter than the bars of the cross. This cross is visible in every microscope and is not always due to faulty construction of the objectives nor to strains in the glass. The effect of inclined surfaces is also clearly shown on the margins of air-bubbles included in Canada balsam or glycerin or other liquid mounted between object-glass and cover-slip. Compare F. Rinne, Centralblatt für Miner., 1900, 88–89; also G. Césaro, Bull. Acad. roy. Belgique, Classe Sciences, 459, 1906.

line through H parallel with the plane of vibration $Y'Z'$ (Fig. 91). The intersection F of this great circle with the polar circle of H is then the desired direction. The point can also be found, as Professor Becke has shown recently,* by noting that it is at the intersection of the straight line HFY' (Fig. 92) and the circle BDE polar to H.† This direction of vibration F is not, however, contained in the plane $Y'Z'$ (Fig. 91), the extinguishing plane of the upper nicol; in this case the point H can not be perfectly dark, if the above reasoning be correct. If the extinguishing plane of the nicol were $Z'X'$ instead of $Y'Z'$, the point C would be the direction of vibration for a dark point H, while G would be the equivalent point determined by the method of Professor Becke.

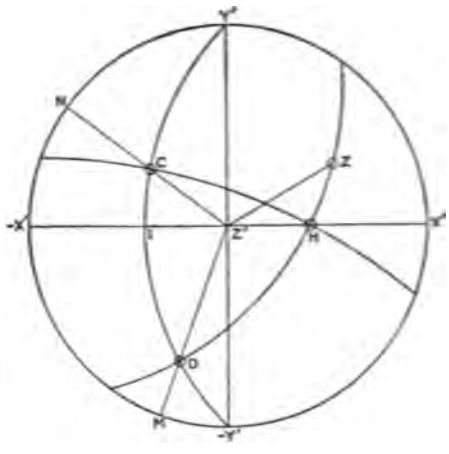


FIG. 93.

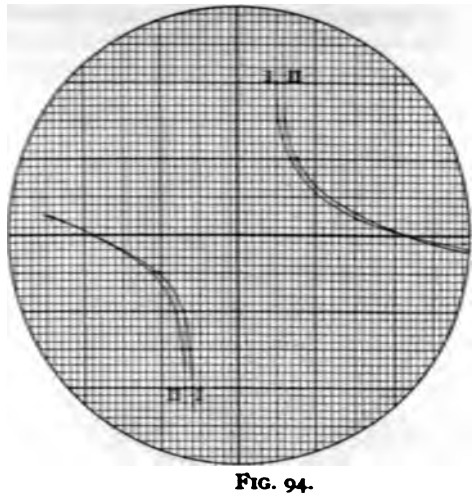


FIG. 94.

According to the writer's method of construction the directions of vibration of any dark point of the interference figure, as viewed through the upper nicol, must lie in the extinguishing plane of the upper nicol. The directions found by Professor Becke's method are not in general contained in this plane and appear, therefore, to be incorrectly located. Objection has been made by Professor Becke‡ to the writer's method because the lines D and C are not 90° apart, while the points F and G are precisely so. In answer to this it may be stated that in any direction within a crystal plate (as H in the uniaxial crystal plate of Fig. 93, Z being the optic axis) two waves are possible whose directions of vibration D and C (Fig. 93) are strictly normal to each other and to the line of propagation H. In the interference figure, however, these directions are not observed along the line of propagation H, but as they appear in projection; and in the plane of this projection the lines of vibration are not 90° apart. To assume, therefore, that the planes of polarization of the two possible waves as observed in the interference figure are tangent to the two lines parallel with YZ through H in stereographic

*T. M. P. M., 28, 293, 1909.

†In this figure the line $Y'H$ cuts the great circle BE at F, as Professor Becke has shown; the line HD intersects the horizontal circle at L; the angles LM, $X'Y'$, KN are right angles; the angle CD is equal to KL, the angle between the lines of projection of the lines OF and OG (O being the center of the sphere of projection) on the horizontal plane. In Fig. 93, the angle $-X'M$ is equal to the angle DI and also to the angle ZHX' or $\omega-\delta$ of the spherical triangle ZHZ'.

‡T. M. P. M., 28, 293, 1909.

projection, obviously introduces an error. If the point appears dark in the interference figure, its direction of vibration must be contained in the extinguishing plane of the analyzer and it is with such points alone that the present problem has to do. Along the line of propagation OH* (Fig. 93) a second direction of vibration is possible at right angles to OD and normal to OH; this direction OC is in general not contained in the plane $X'Z'$ at right angles to $Y'Z'$; but with this direction the present problem is not concerned, its object being solely to find the direction of vibration of an observed dark point in the interference figure, the extinguishing plane of the upper nicol being given.

In the last paragraph one factor, which has profound influence on the phenomena actually observed, has been purposely held in abeyance and must now be considered in detail. Let P be a direction on the axial bar in an interference figure along which plane-polarized light-waves enter at uniradial azimuth. At the upper and lower surfaces of the crystal plate

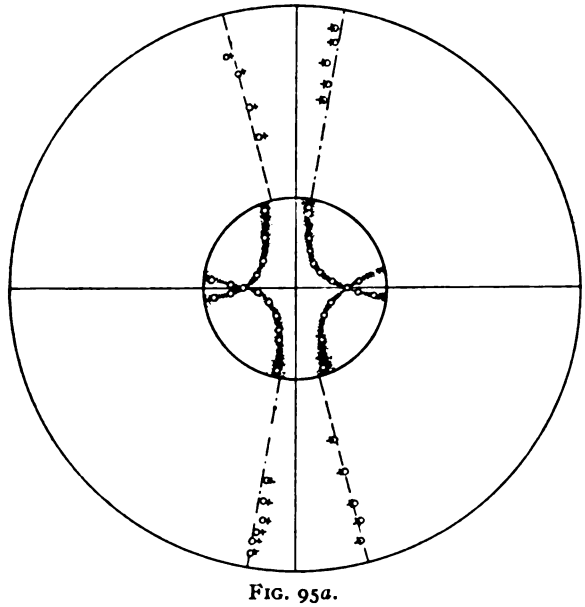


FIG. 95a.

the plane of polarization of these waves suffers a slight rotation and as a result the emergent waves no longer vibrate in their original plane and are consequently not totally extinguished by the upper nicol. The point P is not completely dark. Similarly, let H be a point, adjacent to P on the axial bar of the interference figure, for which one of the emergent waves vibrates in the extinguishing plane of the upper nicol. If this wave alone were considered, the point H would appear completely dark, but along H a second wave is possible whose plane of vibration after emergence neither coincides with, nor is at right angles to the first; it is not completely extinguished by the upper nicol and accordingly illuminates H slightly. The two adjacent points P and H appear, therefore, only approximately dark,

*O being the center of the projection sphere.

and the narrow fringe between them is of about the same degree of darkness. There is, in short, no point of absolute extinction on the bar. The width of the bar increases as the margin of the field is approached (Plate 2, Fig. 3).

As the positions of extinction of the two possible waves emerging in a given direction in air do not coincide precisely and are not exactly 90° apart, there is evidently a range of weak illumination between the positions of uniradial total extinction. For each of the two possible waves, however, the positions of extinction are precisely 90° apart, and if the crossed nicols be rotated through 90° there will be only a slight change, due to surface film effects, in the position of the axial bars in the interference figure from an unmounted plate. On mounted crystal plates the rotary effects of the surfaces of the glass mount enter the problem and there a rotation of the crossed nicols through 90° often produces a small though perceptible shift in the

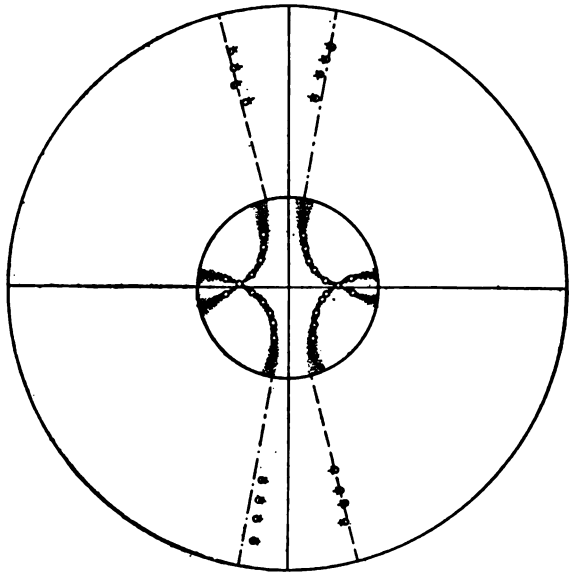


FIG. 95*b*.

position of the dark brushes in the interference figure, as is evident from the series of measurements on the interference figures represented by Figs. 94, 95, *a*, *b*, 96, *a*, *b*. The distortion due to the analyzer may be responsible in part for the observed shift in the positions of the dark brushes in the interference figure on rotation of the nicols.

OBSERVATIONS IN CONVERGENT POLARIZED LIGHT.*

To test the above conclusions, a series of measurements was made in strong sodium light on clear mounted and unmounted cleavage flakes of muscovite and anhydrite. The petrographic microscope (Plate 1, Fig. 3) was first accurately adjusted, a cap nicol being used whose vernier divisions read directly to 3′, which was also the interval of the vernier of the lower nicol; with this arrangement both nicols were situated outside the optical system,

*Amer. Jour. Sci. (4), 31, 202, 1911.

and during each set of observations neither the optical system nor the crystal plate was touched. Experience showed that if the upper nicol remained in the tube, as is ordinarily the case, and was then rotated, a shift of the optical center resulted, which, although almost negligible, was still distinctly noticeable. The cross-line micrometer ocular described below served to locate accurately the different points in the field. The axial bars of the interference figure were plotted directly on cross-section paper as they appeared for different angles of rotation of the crossed nicols. The interference figures were sharp and the dark axial bars clearly defined, although not perfectly dark, and readings could be made to one-half of one division (about 1° in angular coordinates) of the coordinate scale of the ocular.

MUSCOVITE.

Several fresh cleavage flakes of this mineral were observed in convergent polarized light and the positions of the axial bars determined for different angles of rotation of the crossed nicols. The results are plotted in the stereographic projections (Figs. 95*a* and 95*b*), in which the axial bars are drawn for the two positions of the extinguishing plane of the upper nicol (−10° and +15°) as indicated by the dotted lines. The observations were made by using the cross-grating ocular as shown in the photomicrograph (Plate 2, Fig. 3). The observed coordinate values were reduced to their angular equivalents by use of the apertometer, and these in turn were reduced to the corresponding crystal angles by means of the sine formula and the refractive index β. The use of the refractive index β for all directions introduces an error, but experience has shown that this error is not great and, in general, may be disregarded.

Points were located as accurately as possible along each axial bar and then plotted in projection (indicated by small circles, Figs. 95*a* and 95*b*). Although the axial bars were not perfectly sharp, they were well defined and the points were taken along the central line of the bar, the position of each point being determinable to within about 1°, or less for certain positions. In Fig. 95*a* the results obtained from an unmounted cleavage plate are represented; in Fig. 95*b* the interference figure is that from the same plate mounted in Canada balsam between cover-glass and object-glass. In each of these figures the positions of the line of vibration were determined graphically, both by the method of Professor Becke (indicated by small crosses) and by that of the writer (indicated by small circles). A comparison of the relative positions of these small circles and crosses relative to the dotted line which represents the position of extinguishing plane of the upper nicol shows that in a few instances the points as determined by Professor Becke's method are slightly more accurate than the equivalent points of the writer's method; in the majority of instances, however, the small circles are more nearly correct than the small crosses. As a general rule, it may be stated that the order of accuracy of the two graphical methods is about the same, the writer's method having the single advantage of greater simplicity.

A critical comparison of the results of observation on mounted flakes with those on unmounted flakes shows clearly the effects of rotation by the glass surface, causing the axial bars and axes to shift slightly, so that the direct reading of the optic axial angle is not quite the same in the two cases.

The difference is not great, but it is noticeable, and is sufficient to make it advisable to use unmounted plates wherever possible, in optic axial measurements, if results of the highest accuracy are desired; but ordinarily this precaution is unnecessary, since such accuracy is not required.

A rotation of the crossed nicols through 90° also generally produces a very slight shift of the axial bars from mounted plates, as indicated by Fig. 94, which is a direct record to scale of the observed phenomena. In each case the points along the central line of the axial bar were plotted. The position of this central line for an angle of rotation of 15° of the crossed nicols is indicated by the curve *I*, Fig. 94; its position for an angle of rotation of 105° is shown by curve II. These two curves do not coincide and, although such measurements can not be made very accurately, they show that a rotation of the crossed nicols through 90° may, under certain conditions,

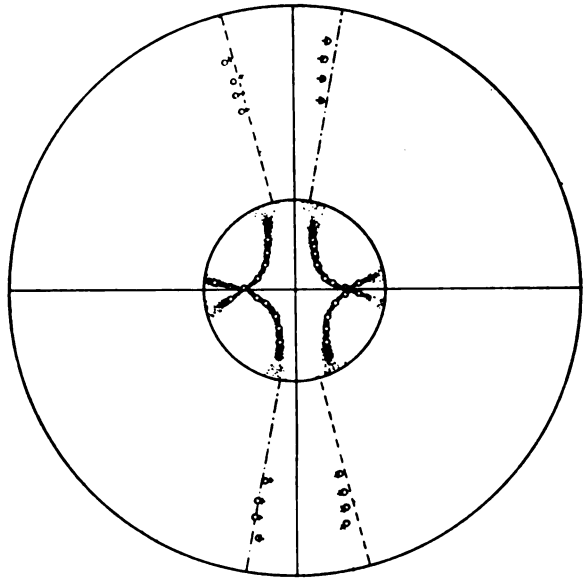


FIG. 96*a*.

cause a slight though perceptible shift of the axial bars of the interference figure of a mounted crystal plate. The amount of shifting rarely exceeds several degrees and is usually less, but it is often sufficient to be perceptible and shows the importance of referring the data, when plotting, to the correct position of the extinguishing plane of the upper nicol. It is, therefore, not immaterial which one of the principal nicol sections be chosen. If the observations themselves were of a higher order of accuracy, this fact would be a serious objection to Professor Becke's method.

ANHYDRITE.

A series of observations on a cleavage plate of anhydrite (unmounted, Fig. 96*a*; and mounted, Fig. 96*b*) corroborates the conclusions stated in the last paragraph. The degree of accuracy of the two methods in question is about the same here as in muscovite. A rotation of the crossed nicols

through 90° also produced a slight shift of the axial bars on mounted plates, as in muscovite.

Both theory and observations show, therefore, that, as a general rule, a uniradial, plane-polarized light-wave, after transmission through a bare crystal plate (preferably a fresh cleavage plate so that the disturbing effects of surface films caused by polishing are not serious), is still plane-polarized, but its plane of polarization has suffered a slight rotation, depending on the direction of transmission, with the result that the plate, when examined under crossed nicols, does not appear perfectly dark. In the thin crystal plates the two refracted waves W_1 and W_2 overlap to a large extent and there exists no position of total extinction for the tilted crystal plate or for obliquely incident waves, even should the upper nicol be rotated alone. In general it may be stated that from an incident plane polarized wave entering a birefracting plate two refracted waves are formed which, on emergence from the plate, are still plane-polarized, but their planes of polarization are not precisely 90° apart. The resultant light, as observed through the analyzer, is consequently elliptically polarized and there is no possible position of total extinction of the plate, but rather a region of minimum illumination which may extend over several degrees.

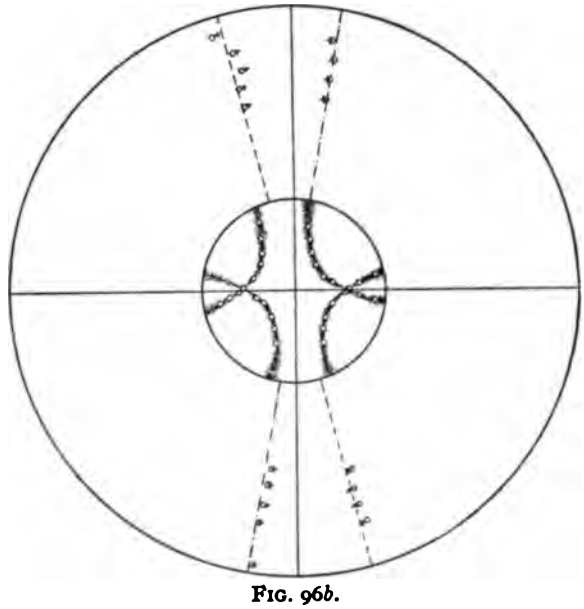


FIG. 96*b*.

These relations have an important bearing on methods based on the determination of the positions of extinction of obliquely transmitted waves and preclude at once a high order of accuracy in the measurements. If the observed crystal plates are mounted in Canada balsam, as in ordinary thin sections, the rotatory influence of the glass and Canada-balsam mount enter the problem and tend to complicate the phenomena still further. The rotatory effect of the glass surfaces of the lens system is still another factor which modifies the phenomena to a certain extent.

If settings be made at the apparently darkest positions of a tilted crystal plate during the rotation of the microscope stage these positions are often several degrees from 90° apart, and, if the observed azimuths of the plane of polarization be taken as the azimuths of the refracted waves within the crystal, errors of several degrees are easily possible. This reasoning and similar observations apply to the zero isogyres in the interference figure. A

point *P* of the zero isogyre, which appears darker than any adjacent lateral point for a given position of the nicols, may then be shifted slightly after a rotation of the nicols through 90°; it should be noted that part of the observed shifting may be due to the distortion caused by the analyzer.

Briefly stated, an obliquely transmitted wave will be extinguished provided its direction of vibration after emergence is contained in the extinguishing plane of the analyzer. The direction of vibration of an observed dark point on the axial bar of an interference figure is therefore the line of intersection of the extinguishing plane of the upper nicol with the polar plane of the given point. As noted above, this construction, suggested by the writer, does not take into consideration the rotatory effects of the surfaces of the crystal plate and the glass mount and is accordingly only an approximate method. The method of Professor Becke is different and consists in finding the intersection of the polar plane with the great circle in stereographic projection which is tangent to a line parallel with the principal section of one of the nicols. The points obtained by these two methods are slightly different, but not sufficiently different to affect the degree of approximation obtainable by such methods. In principle, however, the two methods are fundamentally different and the above detailed discussion* of the factors entering the problem indicates the general validity of the principle on which the method proposed by the writer is based. Neither the Becke method nor that proposed by the writer for finding the plane of vibration of a point *P* on the zero isogyre is strictly correct; but a correct method is not feasible in view of the complex conditions of observation which obtain.

Measurements with the double-screw micrometer ocular or the coordinate micrometer ocular are relatively free from the personal factor which plays an important part in the Becke drawing-table, and the values for 2 *V* obtained by its use are correspondingly more accurate. This method is of general application to all sections cut in such a way that either one or both optic axes appear in the microscopic field. By using projection plats, either stereographic or orthographic or angle, results of a fair degree of accuracy can be obtained in a few minutes.†

*See also Amer. Jour. Sci. (4), 31, 156–211, 1911.

†*A device to aid in the graphical solution of the above problems.*—The measurement of the optic axial angle by the curvature of the dark bar of the interference figure involves two steps: (1) the measurement and location of the optic axis and some point on the dark bar, and (2) certain manipulations in the projection plat. In the second part of the procedure a stereographic or angle projection plat serves as base, and the angular values are plotted directly on thin transparent paper placed directly above the plat and held in center by means of a needle; but this needle is not entirely satisfactory, since it does not hold its place rigidly enough and tends thereby to injure the projection plat below. To overcome this difficulty the writer has constructed the following little device (Fig. 97, one-eighth actual size): A heavy brass bar fits into two brass end-blocks, *C* and *D*; at its center a small brass rod *E*, containing a needle backed by a spring, is introduced. By this device the needle is supported in a vertical position and rigidly; as the distance between the end-blocks *C* and *D* is 44 cm., more than sufficient space is available for the projection plat and overlying drawing. The writer has used this device for several years and has found it satisfactory and a time-saver.

FIG. 97.

Recently, Professor Nikitin (C. Leiss, Zeitschr. Kryst. 47, 379, 1909) has had constructed a graduated porcelain hemisphere which the writer has found satisfactory in optic axial angle projections and slightly more accurate than the projection plats, chiefly because of its lack of distortion toward the margin and consequent acute-angled intersections of great circles. This hemisphere has the advantage of serving as a model in the study of optical phenomena and is a useful piece of apparatus for the petrological laboratory.

On sections in which both optic axes of the interference figure are visible, the exact position of A_1 and A_2 can be measured directly and, after plotting, the value $2V$ can be obtained from the projection by direct reading. Such values should be accurate to 1°.

To form an idea of the relative degree of curvature of the axial bar in the diagonal position for sections of biaxial minerals cut at different angles with an optic axis (binormal) and for the optic axial angles $2V=0°$, 15°, 30°, 45°, 60°, 75°, and 90°, the writer has constructed by graphical methods Figures 98–105.

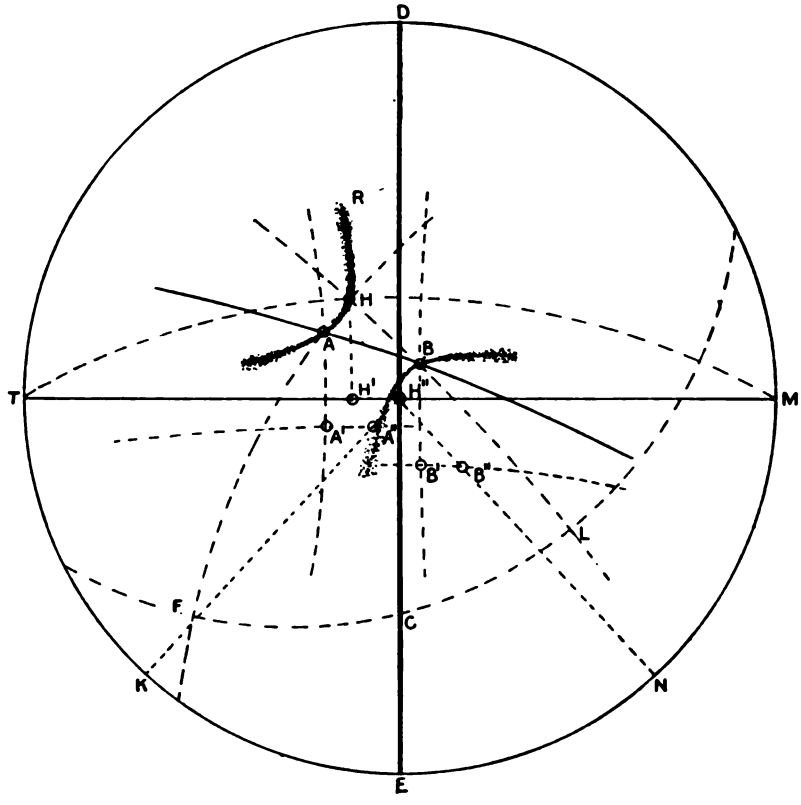


FIG. 98.

The graphical methods of construction adopted for ascertaining these curves are based on the rule of Biot-Fresnel and require the use of the stereographic projection plat. For the sake of completeness, and especially since the figures themselves have been called in question by Professor Becke,* they may be accorded brief description. In Fig. 98 the order of procedure for finding the course of the zero isogyre for a definite optic axial angle is presented. In this figure, which is a stereographic projection plat, the graphical solution of the general problem is indicated, which, briefly, is to find that point H on the great circle THM which is located on the zero

*T. M. P. M., 28, 293, 1910.

isogyre AHR, the positions of the two optic axes A and B (optic axial angle $2V$ in this particular case measuring 30°) being definitely fixed, as well as the plane of vibration DE of light-waves from the lower nicol. To find this point H of the great circle THM, the stereographic projection plane is first rotated about TM until the great circle THM coincides with the straight line $TH'M$. By this process A is transposed to A' and the position of B is changed to B'. The projection is then rotated about DE as an axis until H' coincides with H'' and radii $H''A''K$ and $H''B''N$ include equal angles with the nicol plane DE. The angle of revolution required to bring A' to A'' or B' to B'' is noted and the projection then rotated back to its original position, the center H'' passing thereby first to H' and then to H, which is the desired point. That H is actually the correct point can be proved, furthermore, by drawing the great circles through HAF and HBL and noting that the angles FC and CL on the great circle FCL polar to H are equal.

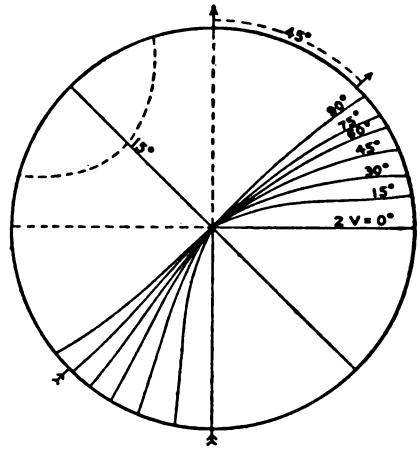


FIG. 99.—This figure illustrates the positions in the interference figure of the dark curves of total extinction (axial bars) as they would appear in the field were observations made in air on a series of biaxial minerals having a mean refractive index of 1.60 and the optic angles indicated, and cut normal to one of the optic axes (binormals). From the figure it is evident that the radius of curvature of the axial bar increases with the optic axial angle, so that for $2V = 90°$, the axial bar is practically a straight line.

This method of construction is perfectly general. After the stereographic plats had been finished they were transferred to equivalent orthographic plats. (In these figures the position of the plane of vibration is indicated by arrows pointing NE.) In Figs. 99 to 101 the directions in the orthographic plat were corrected for the passage of the light-waves from the crystal plate to air by means of the usual sine relation $\sin i = n \sin r$, where i = the angle of incidence, r = angle of refraction, and n = average refractive index; for minerals of medium birefringence the error arising from substituting β for the correct refractive index is negligible.*

In figures 99 to 101 the dark lines represent the curves of total extinction (zero isogyres or axial bars) of the interference figure in orthographic projection, the plane of the optic axes making an angle of about 45° with the plane of vibration of light-waves from the lower nicol. In the construction of the figures neither the rotation of the boundary surfaces on the plane of polarization of the transmitted light-waves nor the elliptic polarization phenomena, due to repeated total reflection within the crystal plate, was taken into account. For this reason the figures do not represent interference figures as actually observed under the microscope, but rather as they would appear were the Biot-Fresnel rule strictly valid. At a distance from the

*See also Amer. Jour. Sci. (4), 24, 338-340, 1907.

optic axis the zero isogyres become wider and less distinct and also somewhat more symmetrical because of the factors noted above, whereby the effects of the two refracted waves W_1 and W_2 are superimposed in different intensities. Although the figures are not strictly true to nature, they show approximately the course of the zero isogyres; and experience has shown that the general conclusions deduced from them are valid.

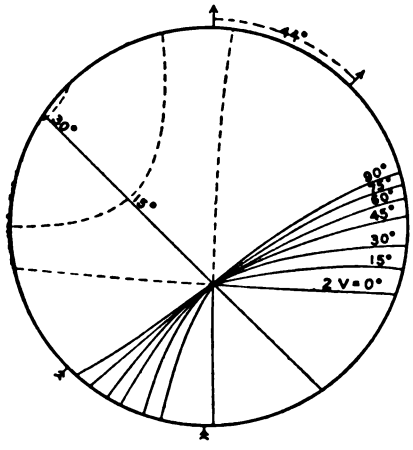


FIG. 100.

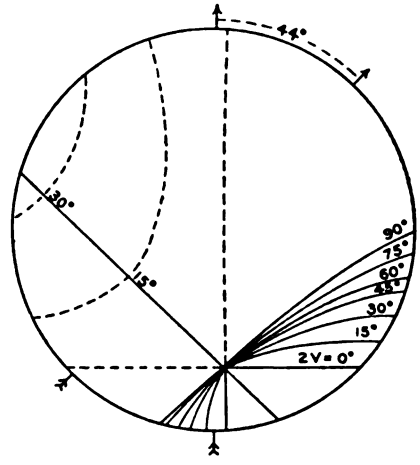


FIG. 101.

FIG. 100.—The conditions of construction for this figure were similar to those of Fig. 99, except that the section is considered cut at an angle of $\lambda = 10°$, $\mu = 10°$ (small circle coordinates from the center) with one of the optic binormals.

FIG. 101.—This figure differs in construction from Figs. 99 and 100 only in the fact that the section is considered cut at an angle $\lambda = 20°$, $\mu = 20°$ with one of the optic binormals.

The curves of Figs. 99 to 101 are constructed for minerals with a refractive index $\beta = 1.60$, while those of Figs. 102 to 105 are orthographic projections of the actual positions of the directions of 0° extinction within the crystal as obtained by the graphical construction when the refractive index of the mineral and the surrounding medium are equal. It is evident from the figures that the differences in curvature of the axial bars for the different values of $2V$ are sufficient to warrant their use in measuring optic axial angles approximately. The accuracy of the method depends on the accuracy with which the points A_1 and H (Fig. 86) can be determined. The positions most favorable for these points are located one-half to two-thirds the distance from the center of field to its margin. Near the center of the field the errors of construction increase rapidly, while near the margin errors due to the decrease in distance between equal angular intervals, to imperfections in the lenses, and to elliptical polarization tend to modify the interference figures and decrease the accuracy to be attained.

The actual diameter of the field covered by the micrometer-screw movements of the writer's ocular measures about 600 micrometer-screw divisions. The distance covered by the extremes of the curves for 0° and 90° is less than 200 divisions, or about 2 divisions for 1° on an average. Taking into

consideration the indistinctness and width of the axial bars, it is easily possible to make an error of three or four divisions of the micrometer scale in these readings, so that a greater accuracy than ±2° to 3° can not be

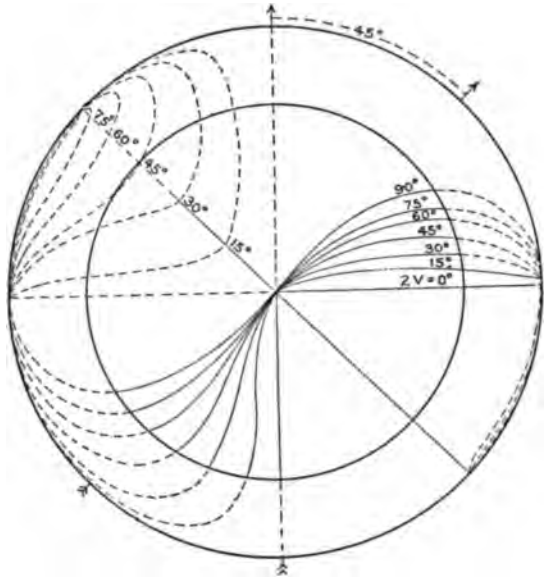


FIG. 102.

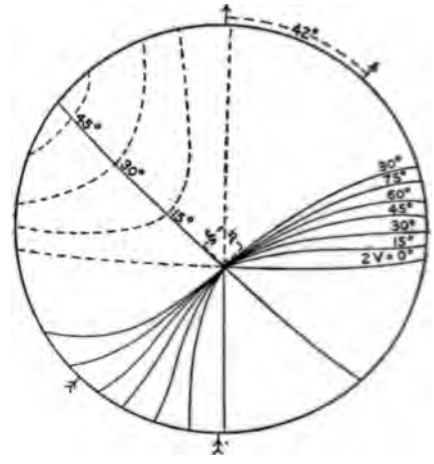


FIG. 103.

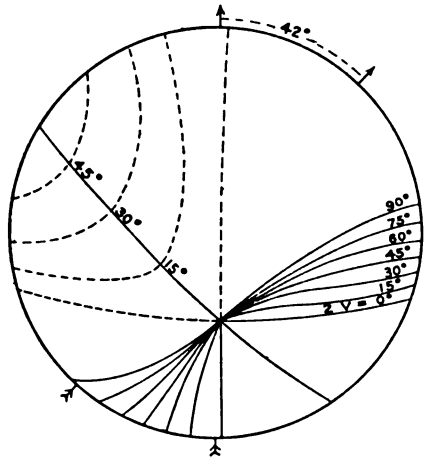


FIG. 104.

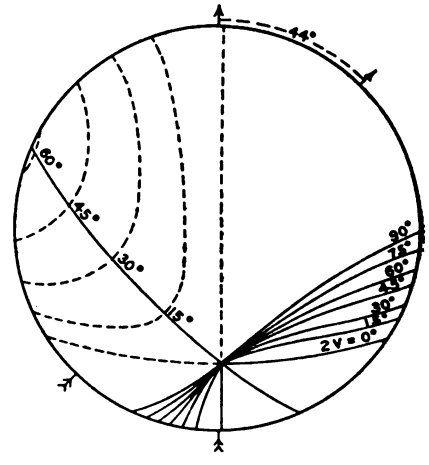


FIG. 105.

FIGS. 102–105.—In Figs. 102 to 105 the axial curves (zero isogyres) are constructed under the assumption that the mean refractive indices of both the mineral and the medium in which its interference phenomena are observed, are identical; in short, an orthographic projection of the phenomena as they appear within the crystal is given. In Fig. 102 the section is normal to an optic axis; in Fig. 103 the section makes angles, $\lambda = 5°$, $\mu = 6°$ (small circle coordinates) with the optic binormal; in Fig. 104 the angles are $\lambda = 12°$; $\mu = 13°$, while in Fig. 105 the section makes angles $\lambda = 20°$, $\mu = 20°$ with the optic axis; the angle of rotation of the stage with consequent new position of the trace of principal plane of lower nicol in each case is indicated by the arrows. The area included by the inner circle of Fig. 102 indicates the relative field of view of ordinary microscopes.

claimed for this method. With the drawing-table this probable error is about $\pm 5°$ under the same circumstances.

In place of the expensive double-screw micrometer ocular, a positive ocular with 0.1 mm. coordinate micrometer scale in its front focal plane (Fig. 106) has been used and found in practice to be equally satisfactory and nearly as accurate.* Plate 6, Fig. 3, shows another form of this ocular, in which the coordinate scale (Fig. 107, only millimeter and not 0.1 mm. divisions shown) is supported in a metal carriage, which in turn is inserted in a specially constructed holder. The same holder is used to support a number of other different plates and wedges and has been found so serviceable that in the microscope of Plate 2, Fig. 1, it has been built directly into the microscope. The coordinate scale ocular, Fig. 106, is also useful in estimating the relative volumes of minerals present in a thin section (statistical method of rock analysis) and serves the purpose of an ordinary micrometer ocular as well.

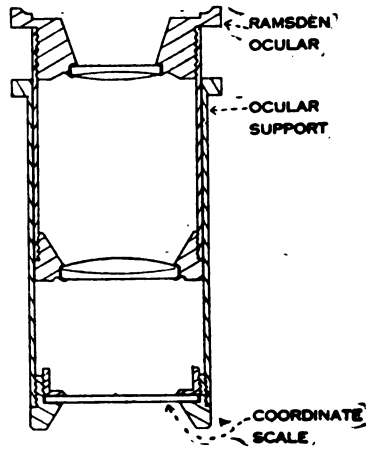


FIG. 106.

After insertion of the Amici-Bertrand lens, the secondary image of the interference figure is brought to focus in the focal plane of the ocular, where the location of any point can be read off directly in coordinates, which in turn are to be reduced (just as the readings of the double-micrometer screw ocular) to angle directions within the crystal and then plotted in suitable projection.

The ocular of Czapski and also Klein's lens, which was first described by Becke, can be changed to fit the new conditions by simply introducing the above fine coordinate micrometer scale in place of the single micrometer scale.

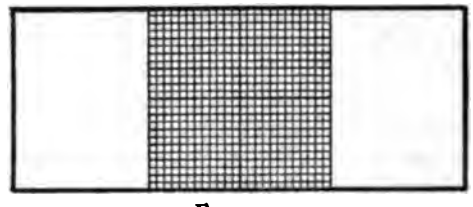


FIG. 107.

By the use of such oculars with fine coordinate scales, one has the entire field of the interference figure under control, and by use of projection plats can readily measure optic axial angles on all sections which are so cut that one optic axis at least is in the field. If two optic axes appear within the field of vision, their positions can be read at once from the coordinate scale of the ocular and after proper reduction can be plotted in stereographic, orthographic, or angle projection where their angular distance can be determined directly. In the measurement of optic axial angles by these methods the errors arise chiefly from the determination of the exact position of the dark axial bar rather than from the reading of the scales.

*Constructed for the writer by R Fuess & Co., Steglitz, Germany; also by Bausch & Lomb Optical Co., Rochester, N. Y. See Jour. Wash. Acad. Sci., 1, 60–61, 1911.

THE CONDENSER APERTOMETER.

In 1905 E. Sommerfeldt* suggested that a micrometer scale be attached to the lower focal plane of a specially constructed condenser lens system consisting of three plano-convex lenses and so calculated that the scale appears simultaneously with the interference figure.

Later, and without knowledge of Sommerfeldt's work, Dr. H. Kellner, of the Bausch & Lomb Optical Company, suggested that a micrometer scale might be fitted in the lower focal plane of a highly corrected Zeiss aplanatic achromatic condenser of numerical aperture 1.4. The writer accordingly had constructed, by the Bausch & Lomb Optical Company, a coordinate scale, the divisions of which read directly to angles in air (the distance of any division on the scale from the center being $f \cdot \sin E$, where f is the equivalent focal length (E. F.) of the condenser and E is the angle in air). This scale in the condenser is conjugate to the upper focal plane of the objective and is viewed therefore simultaneously with the interference figure, either as it appears in the objective alone or magnified by the Bertrand lens and ocular. Actual experience with this new scale indicates that the distortion is practically negligible in the central two-thirds of the field and increases only toward the margin, where the measurements at best are not very satisfactory. The great advantage of this device lies in the fact that the values of E are read off directly without regard to the optical system of the draw-tube, with the result that the determination of the Mallard constant or the angle equivalent curve is unnecessary and any optical system can be used directly. The coordinate divisions on the present scale are 5° apart. In applying this method to actual measurements it is essential that the object be located in the upper focal plane of the condenser; otherwise distortion results and incorrect readings are obtained. Since in ordinary work with parallel polarized light the coordinate ruling is not observed, the scale may remain below the condenser permanently and is ever ready for use in optic axial angle measurements or as an apertometer. A careful comparison of the readings of the Zeiss-Abbe apertometer with the condenser apertometer has shown that the readings of the latter are satisfactory for all parts of the field except near the margin.

After having located the points in the interference figure with the condenser apertometer, the angular values are first reduced by means of the sine formula to corresponding angles within the crystal, and then plotted, after which the measurements are made by the graphical methods outlined above.

SUMMARY STATEMENT OF COURSE OF PROCEDURE IN MEASURING THE OPTIC AXIAL ANGLE OF A MINERAL PLATE IN CONVERGENT POLARIZED LIGHT.

In measuring the optic axial angle, when only one optic axis appears in the field of the interference figure the following steps are necessary:

(1) Locate the points A_1 and H (Fig. 86), by use of either the double-crew micrometer ocular, or the coordinate micrometer ocular, or the Becke drawing-table, or the condenser apertometer.

(2) Reduce the angles thus obtained to corresponding angles within the crystal by means of the sine formula $\sin i = \beta \sin r$, where i = observed angle of incidence, r = angle of refraction, and β = average refractive index.

*Zeitschr. wissen. Mikroskop., 22, 356–362, 1905.

(3) Draw a great circle, DA_1E, in stereographic or angle projection or on the Nikitin graduated hemisphere, through A_1 (Fig. 88) parallel with the direction of vibration of the lower nicol in the first position. This is the plane of the optic axes.

(4) Draw the plane of vibration FOI of the lower nicol in the second position (after rotation through some known angle, as 30° or 45°).

(5) Draw the polar circle PK to H and let the point of its intersection with FI be C; draw the great circle HA_1 and let A'_1 be its point of intersection with PK.

(6) Lay off on PK the angular distance $CA'_2 = CA'_1$. Draw the great circle HA'_2; its point of intersection A_2 with the great circle DA_1E determines the position of the second optic axis. The angle A_1A_2 is the optic axial angle, $2V$.

MICHEL-LÉVY METHOD.

For sections normal to the acute bisectrix of a mineral with a large optic axial angle, Michel-Lévy has suggested a method which, although theoretically interesting, is not of great practical value, owing to the indistinctness of the phenomena to be observed. His method consists in reading the angle of rotation of the stage necessary to bring the interference figure from the crossed position to that in which the emerging axial bars of the interference figure are tangent to a given circle.*

H. Tertsch has recently† described a method, requiring the Becke drawing-table, with which approximate results of a fair order of accuracy can be obtained on a section cut normal to one of the bisectrices. The accuracy of the method, however, at best is not great, as is emphasized by Tertsch, and it decreases rapidly if the section be not cut precisely normal to a bisectrix. In view of the limited application of these methods and the low degree of accuracy obtainable by their use, they will not be described further.

METHODS WITH THE UNIVERSAL STAGE.

In practice it frequently happens that a given section is not favorably cut to show the optical phenomena to the best advantage, and that by tilting it a certain angle the interference figures can be improved materially. This is particularly the case with fine-grained artificial preparations where, although individual crystals and cleavage fragments can frequently be obtained, they do not rest in the section in the most advantageous position. Such crystals and crystal plates can be tilted either by means of an axial angle apparatus for the microscope, as that described by Bertrand‡ many years ago, or by use of the glass hemisphere of Schroeder van der Kolk,|| or by the new upper condenser lens of ten Siethoff.§ The last two methods are qualitative methods only, while that of Bertrand, although quantitative, permits of rotations in one plane only. To supply the want of a universal condenser lens on which angular movements can be accomplished and measured in any

*Michel-Lévy et Lacroix, Les Minéraux d. Roches, 94–95, 1888. For a modification and simplification of his formula, see F. E. Wright, Amer. Jour. Sci., **22**, 289, 1905.
†T. M. P. M., **27**, 589–594, 1908.
‡E. Bertrand, Bull. Sco. Min. Fr., **3**, 97–100, 1880.
||Schroeder van der Kolk, Zeitschr f. wiss. Mikroskopie, **8**, 459–461, 1891, and **12**, 188–189, 1895.
§E. G. H. ten Siethoff, Centralblatt f. Min., 657, 1903

direction, the writer modified* the Fedorow-Fuess universal stage by having a brass disk, *L*, constructed in the workshop of the Geophysical Laboratory to fit in the Fedorow-Fuess stage (large model) in place of the inner ring bearing the glass with cross-hair (Plate 6, Fig. 2). Into this ring the upper lens of the ten Siethoff condenser lens system is inserted. The partially beveled upper surface of this condenser lens has a radius of 1.5 mm., and permits, even with a No. 9 Fuess objective, angular movements of about 30° on either side of the normal. By means of a proper cylinder of brass resting on the cylinder containing the lower nicol, the remaining lenses of the condenser system are raised to the required distance from the upper lens. This type has proved extremely serviceable in work with artificial preparations, since by its use sections can be so placed that the most favorable measurements possible can be accomplished with the cross-line micrometer ocular; in case the optic axial angle is small, it can be measured directly by means of the universal stage in convergent polarized light.

Wladimir Arschinow† has recently described a glass hemisphere, 50 to 60 mm. in diameter, which rests on the microscope stage and carries two graduated metal semicircles attached on pivots above the plane surface of the hemisphere. The mounted section is placed on the centered glass hemisphere and the coordinate angles of tilting are read off on the graduated semicircles. By means of this device the section can be readily tilted into any position, but can not be turned about any axis after having been tilted, as is the case with the universal stage.

The writer has recently had constructed a graduated hemisphere (Fig. 108, *a*, actual size 63 mm. in diameter)‡ which rests on the microscope stage after the manner of the Schroeder van der Kolk hemisphere|| and on which the crystal plate or mounted thin section is placed. In the writer's microscope the edge of the circular opening of the stage in which the glass hemisphere rests coincides exactly with the 20° circle of the hemisphere. Two notches (Fig. 108, *c*) cut in the edge of this stage opening serve as points to indicate the zero meridian, and the polar and longitudinal angles of tilting are read off directly from these points through the hemisphere, 20° being subtracted from the polar angle thus determined. The graduations on the spherical surface of the hemisphere are indicated in Fig. 108, *a* (orthographic projection). The divisions are 5° apart and 1° intervals can be readily estimated. On the flat surface of the hemisphere fine cross-lines are cut (Fig. 108, *b*) and aid in centering the hemisphere and in adjusting a crystal plate in the primary position. This graduated hemisphere may well serve, in place of the expensive universal stage, for most measurements of the optic axial angle and of extinction angle in zones, and for tilting crystal plates any desired angle. It may also be used to advantage as an apertometer (refractive index of the glass $n_{Na} = 1.519$) for ascertaining the numerical aperture of objectives and for determining the angular equivalent of the divisions of the cross-line micrometer ocular. To increase the angle of view, a small glass hemisphere may be attached with cedar oil above the section or crystal plate after the method suggested by Fedorow.

*Amer. Jour. Sci. (4), **24**, 342, 1907.
†Zeitschr. Kryst., **48**, 225–229, 1910.
‡This hemisphere was ground and graduated by the Bausch & Lomb Optical Co., of Rochester, N. Y., and has been found satisfactory.
||Z. wissen. Mikroskop., **12**, 188, 1895.

Although the measurements accomplished by the universal stage methods of Fedorow are made in parallel polarized light and with low-power objectives, the same objectives can be used for weakly convergent polarized light with the Bertrand lens and the position of the optic axes may be thus determined if it be possible to bring them within the field of vision and if they are sufficiently distinct for accurate location under these conditions. For general work, however, with thin sections in convergent polarized light, the methods requiring the double-screw micrometer ocular or the coordinate scale ocular are the most accurate and easy of application.

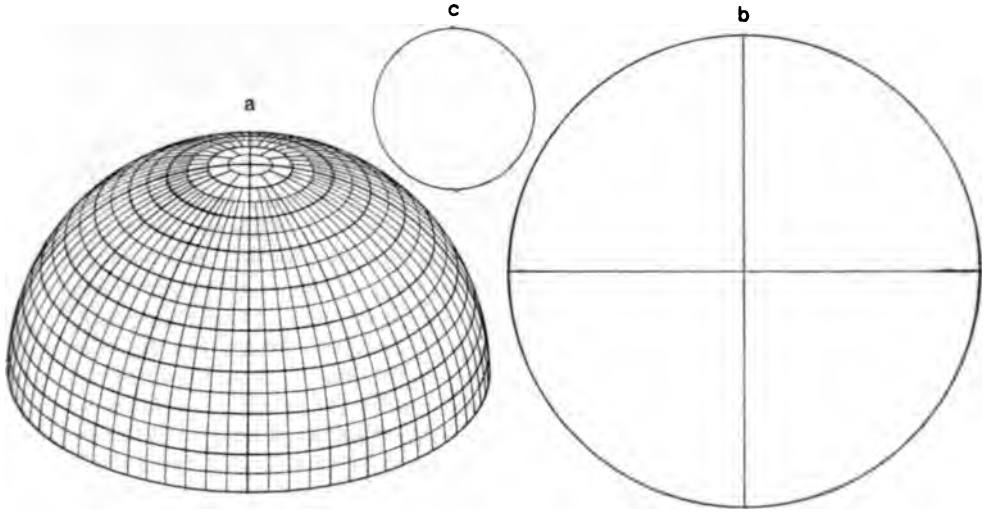


FIG. 108.

PARALLEL POLARIZED LIGHT.

The introduction of the universal-stage methods by Fedorow, in 1893* and succeeding years, placed a powerful instrument of attack in the hands of petrologists. With his methods it is now possible to obtain the optic properties of mineral sections which before were considered practically useless. The universal stage (Plate 6, Fig. 1) can be attached securely to any suitable petrographic microscope; parallel-polarized light only is used. By means of horizontal and vertical axes of rotation, a crystal section can be brought to any given position and rotated about any axis for optical examination.

In parallel-polarized light an optic axis is recognized by the fact that when placed parallel to the axis of the microscope it remains uniformly dark during a complete rotation about that axis. By plotting these directions graphically in projection, and by determining extinction angles in given zones, it is possible not only to measure the optic axial angle, but also to determine the position of the optic axes with reference to the crystal plate, even though it may happen that neither optic axis appears within the field

*E. v. Fedorow. Zeitschr. Kryst., 21, 574–678, 1893; 22, 229–268, 1894; 25, 225–261, 1895; 27, 337–398, 1897; 29, 604–568, 1898; see also F. Wallérant Bull. Soc. Min. Fr., 19, 356–363, 1896; J. W. Evans, Miner. Mag., 14, 157–159, 1906.

of vision. Fedorow has also shown how it is possible with his methods to measure the refractive indices and the birefringence approximately of a mineral from any section.

The values for $2V$ thus obtained on different sections, however, are not all of the same order of exactness, as will appear later in the more detailed discussion of the different sections. It should be noted that in the Fedorow methods, as in the convergent polarized light methods, the measured angles are reduced by means of the average refractive index β to corresponding angles within the mineral before plotting in stereographic or angle projection. Here also the combination of tracing-paper with the projection plat as a base, as suggested by Wulff, is to be recommended as the best and most efficient scheme for obtaining results rapidly and accurately. The new graduated hemisphere of Nikitin can also be used for the same purpose and is equally efficient.

In these methods the rule of construction of Biot-Fresnel, that the planes of vibration of light-waves propagated in any given direction bisect the angles between the two planes containing an optic axis and the given direction, is used constantly, since the two factors on which the universal stage methods are practically based are the directions of the optic axes, as they may be determined directly, and extinction angles for certain zones and directions. If the crystal plates be observed, as is usually the case, mounted in Canada balsam between the object-glass and cover-slip, the rotatory effects of the boundary surfaces on the planes of vibration of the transmitted waves become noticeable and render the results inaccurate to just that extent. In case the Fedorow glass hemispheres described below be used above and below the crystal mount, the rotatory effects of the boundary surfaces are decreased and the results obtained are correspondingly more accurate. Even under these favorable conditions elliptic polarization is often noticeable on steeply inclined plates, even in monochromatic light, but it is usually so slight that it may, in general, be neglected.

It may be stated that, although the methods of Fedorow involve the use of a stereographic or angle projection plat and are in part graphical in nature, they are not difficult of application and often furnish results where other methods fail. In ordinary microscopic work it frequently happens that one method will yield more accurate data in a shorter time than a second, and that particular method should then be chosen in preference to all others.

In general, the Fedorow methods are indirect methods and frequently involve a large expenditure of time to complete the observations on a single plate. For these reasons, chiefly, petrologists have not adopted them so rapidly and generally as might have been anticipated, particularly as the old, tested methods accomplish about what is desired by the busy petrologist who uses the microscope simply as a means to an end—to aid him in interpreting geological phenomena and relations.

When attached to the microscope, the Fedorow-Fuess stage (Plate 6, Fig. 1) possesses, when in the o° (primary) position, three horizontal circles, H_1 (microscope stage), H_2, and H_3, each circle graduated into degrees with verniers attached to H_1 and H_2; each of these circles can be rotated about a vertical axis; the horizontal axes of rotation and equivalent vertical circles are V_1 and V_2 (also divided into degrees) and V_1 with vernier attached.

On the original stage described by Fedorow and made by Fuess, the partial scales V_2 are wanting and have been attached by the writer.* These scales have been found essential and of practical service in several of the methods described below, especially those involving the principal sections of the triaxial ellipsoid of any mineral. Each partial scale of V_2 is accurately divided and carefully adjusted to the instrument. When not in use, the scale segments of V_2 can be inclined to a horizontal position V_{2b} (Plate 6, Fig. 2) and are then entirely out of the way. Measurements given below will be referred to this modification of the Fedorow-Fuess universal stage.

To increase the angle of vision of the field, two glass hemispheres, A_1 and A_2 (in Plate 6, Fig. 1, A_1 only appears, A_2 being hidden by H_2), are usually employed; between these the preparation is placed, either cedar-wood oil or glycerin being used to stick the same together and to reduce the effects of total reflection. For general work with the universal stage, it is advisable to follow the suggestion of Fedorow and use special circular (2 cm. diameter) object-glasses on which to mount the preparations in place of the ordinary rectangular (26 × 46 mm.) thin-section object-glasses.

With the universal stage of this type it is possible not only to bring a crystal section to any given position, but also to rotate the section about any axis; in short, by its use one has control over all possible directions and zones or axes of rotation of a crystal.

THE DETERMINATION OF THE CRYSTAL SYSTEM OF A GIVEN MINERAL BY MEANS OF THE UNIVERSAL STAGE.

The fact that the universal stage allows the observer to study the different effects of a given mineral section on light-waves transmitted through it in different directions enables him to determine at once the crystal system to which the crystal belongs. This is accomplished most readily by means of extinction angles along certain directions, since the term extinction angle implies a definite relation between a given crystallographic and a given optical direction in any mineral. These relations vary with the crystal system of the mineral, and in fact are such definite functions of the same that, as Brewster† was the first to show, it is possible from extinction angles alone to determine definitely the crystal system of a given mineral. Briefly, an isometric mineral is isotropic for all directions of light-wave propagation. Uniaxial minerals (hexagonal and tetragonal) appear isotropic for light-waves passing along the principal crystallographic axis. For all other directions they are anisotropic, but even then can generally be distinguished from biaxial minerals at once by the fact that every section of a uniaxial mineral contains the ω ellipsoidal axis, parallel with and normal to which it extinguishes. If the section be placed, therefore, in the position of darkness between crossed nicols and be rotated about a horizontal axis, V_1, it will continue to remain dark if the ellipsoidal axis ω coincides with the axis of rotation, while if the ellipsoidal axis ω be normal to the latter the crystal will exhibit interference colors of polarization on rotation except for sections of the prism zone. Biaxial minerals, on the other hand, do not

*Amer. Jour. Sci. (4), 24, 345, 1907. The improved form of universal stage is now furnished by R. Fuess & Co., Germany.

†D. Brewster, Phil. Trans., 187–218, 1814; 199–272, 1818.

in general remain dark for either axis of rotation, and only do so for sections in the principal zones of the optical ellipsoid. Biaxial minerals show, moreover, two directions of apparent isotropism, those of the optic axes (optic binormals). To trace out the relations obtaining for orthorhombic, monoclinic, and triclinic minerals and their distinguishing features is not a difficult matter, but one for which space is not here available. They are, in effect, those used for the same purpose with ordinary methods.

THE ACCURATE DETERMINATION OF THE POSITION OF AN OPTIC AXIS IN THE FIELD OF VISION.

Although the underlying principles of determination by means of the universal stage are the same for all sections of a mineral, it has been found by experience that, for certain sections, definite courses of procedure for measuring the optic axial angles are best adapted to produce the best results. Fedorow has divided the possible sections of any biaxial mineral into four convenient groups, each of which has its special characteristics and to each of which certain methods are best suited. The relative positions of the optic axes to and in the field of vision have been made the criteria for distinguishing these different groups; thus, in group (1) both optic axes are within the field of vision; (2) one optic axis is within the field of vision and makes an angle of less than 20° with the normal to the section; the second optic axis can not be brought within the field of vision by any rotation of the stage; (3) one optic axis only appears in the field and makes an angle of over 20° with the normal to the section; the second optic axis lies entirely outside the field; (4) both optic axes lie outside of the microscopic field, the section in question being cut more or less nearly perpendicular to the optic normal, or about parallel to the plane of the optic axes, or approximately normal to the obtuse bisectrix of a mineral with small optic axial angle.

In case one or both optic binormals of a biaxial mineral section can be brought by rotation to coincidence with the axis of the microscope, it is necessary to determine these angles of rotation with the greatest possible accuracy. In all cases an approximate determination is first effected by rotating the section about V_1 and H_2 until it is dark and remains dark during a complete rotation of the microscope stage H_1. In weakly convergent polarized light the optic axis can be seen in the center of the field. In ordinary microscopes, where absolutely plane parallel polarized light can not be obtained, the section in such a position will not be perfectly dark, owing to internal conical refraction and to the fact that the incident light is not strictly parallel,* but will preserve the same degree of slight uniform illumination for all positions of the microscope stage.

More accurate determinations of the position of an optic axis can then be made by means of extinction angles along definite directions, which, when plotted in projection, give rise to curves, all of which pass through the optic axis. The average point of intersection of a set of such curves is then the correct position of the optic axis in projection (Fig. 109).

*See E. Kalkowsky, Zeitschr. Kryst. (9), 486–497, 1884; W. Voigt, Verh. deutsch. phys. Gesel., 7, 340–345, 1905; Ann. d. Phys. (4), 19, 14–21, 20, 108–126 1906; Physik. Zeitsch., 6, 672-673, 1905.

Such curves have been called optical curves by Fedorow and are obtained most readily by first placing the crossed nicols in any given but fixed position, then turning H_2 through angles of 5° respectively, and for each position of H_2 determining the angle of inclination about V_1 for which the section is in the darkest position (0° extinction) (Fig. 109); the same results can also be attained by first turning the preparation about V_1, a specified angle,

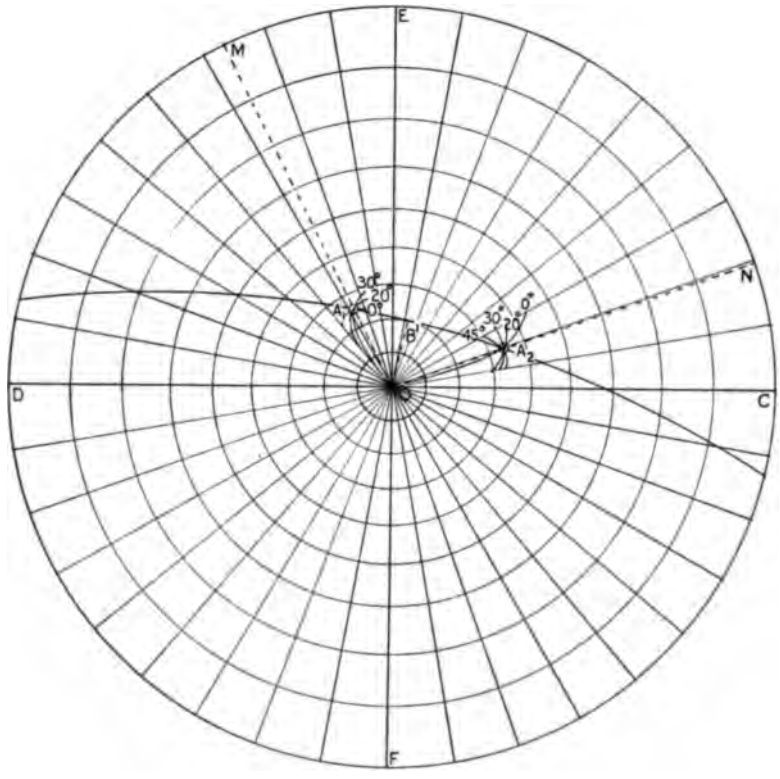


FIG. 109.—In this figure the method for locating the position of the optic axes by means of optical curves is illustrated. The figures 0°, 20°, 30°, and 45° opposite the curves indicate the angles which the plane of vibration of the polarizer at the time of observation made with the plane of symmetry of the microscope.

and then about H_2 until darkness ensues. By this method those directions in the crystal are obtained (after proper reduction of observed angles to crystal directions by means of the refractive index) for which the extinction is zero for a given position of the nicols. The curve uniting these directions in projection is the optical curve for the particular position of the nicols to the axes of rotation. Analogous curves for other and different positions of the nicols are to be obtained and plotted in similar manner. All such curves pass through the optic axes. Their points of intersection in the projection determine, therefore, with considerable accuracy, the exact position of the optic axis or of both axes, in case both axes can be brought within the field of vision. Since such optical curves are intended solely to increase the accu-

racy of the determination of the positions of the optic axes, their approximate positions being known from the preliminary determination, it is necessary, in actual practice, to take readings of H_2 only 5° or 10° on either side of the approximately correct position of the optic axis, as determined by the preliminary direct observations. Convenient positions of the nicols for optical curves are at 0°, 45°, 15°, and 30° from the V_1 axis of revolution. If

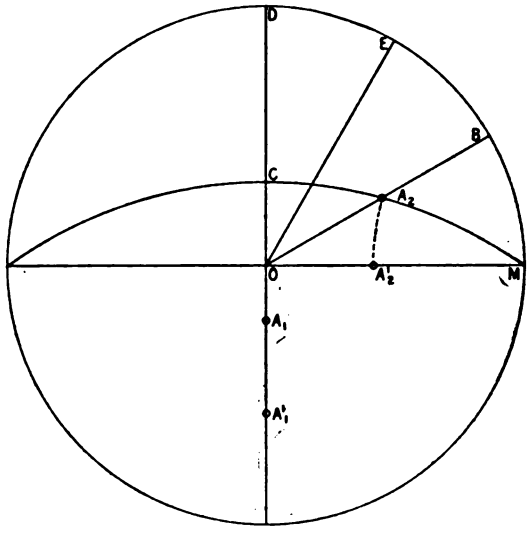


FIG. 110.—In the method illustrated by this figure, the visible optic axis A_1 is brought to coincide with the plane *DCO* and the angle of the position of extinction *DOE* measured while the stage is in the horizontal position. A_2 must lie then in the plane *OB*, the angle *BOE* having been made by construction equal to *DOE*. The section is then rotated about *OM* (axis V_1) until the extinction angle becomes 45°, in which case the plane *OM* contains A_2, since A_1 has remained during this rotation in the plane *DCO;* on turning the specimen back to its original position, the line *OM* becomes the great circle CA_2M and the intersection of this great circle with the plane *OB* fixes A_2 definitely in the projection. In practice, the great circle CA_2M need not be drawn, since on placing the tracing over the projection plat it is only necessary to find that small circle $A_2A'_2$, the arc of which intercepted between *OB* and *OM* is equal to the angle of rotation.

both optic axes appear within the microscopic field of vision, the most satisfactory method for measuring the optic axial angle by means of the universal stage is to determine the exact position of each axis by the above method and to plot the same (after proper reduction to corresponding angles within the crystal) in suitable projection, in which the angle can be measured directly by graphical methods rather than by calculation.

The results obtained by the use of optical curves can be checked and verified by several of the methods described below, which are of general application and can readily be applied to this special case.

Fedorow has shown that in actual practice with minerals of weak to medium birefringence the errors can be disregarded which are due to the reduction of all observed angles by means of the average refractive index of the crystal in place of the true refractive indices for each given direction.

The method of measuring the optic axial angle by means of optical curves can be used only when both optic axes appear within the field of vision. In other cases other methods are to be employed, which involve either the measurement of extinction angles in zones or the determination of the positions of the principal planes of the ellipsoid, these latter to be plotted in appropriate projection. In most cases, however, one optic axis can be determined directly by optical curves, while the second optic axis makes a large angle with the normal to the section and must be determined indirectly. A simple but comparatively accurate method to accomplish this consists in first turning the stage about H_2 until the known optic axis comes to lie in the plane normal to the axis V_1 (OCD, Fig. 110), and in determining the angle of the position of extinction ($\sphericalangle EOD$) when the stage is in the horizontal position and also at such an inclination about V_1 that the extinction angle is 45°; this can be recognized most readily by placing the nicols in the 45° position and then rotating the preparation about V_1 until darkness ensues. By thus ascertaining the angle of rotation necessary to attain the required 45° extinction angle, the great circle CA_2M is fixed with reference to the horizontal diameter, which in turn represents the plane in which the unknown optic axis A_2' must rest when the extinction angle is 45°. The intersection A_2 of the great circle CA_2M with the radius OB drawn at an angle, with the vertical line, of twice the angle of extinction ($\sphericalangle EOD$) for the plate in the horizontal position, fixes the position of the second optic axis in the projection.

This method, however, is not always applicable, owing to the indistinctness of extinction phenomena on steeply inclined sections (effect of elliptical polarization), and a second method of extinction curves, of which the above is only a special case, can be used to advantage. Having first placed the known optic axis in the plane normal to the axis V_1 as in the above method, measure the extinction angles for different inclinations of the stage about V_1 (the angles, as usual, to be reduced to real angles within the crystal by means of the average refractive index), and plot these directions of extinction in stereographic or angle projection (Fig. 111). Under these conditions the radii, which make an angle with the vertical diameter OM equal to twice the extinction angle, are evidently the planes containing the second optic axis A_2, whose exact location can be readily found by noting (for two given radii, as OA_2 and OA_2') the small circle, whose arc A_2A_2' intercepted between the radii is equal to the angle of rotation of the stage. In practice it is advisable to repeat the determinations of the extinction angles and to take as angles of inclination those equivalent to 0°, 10°, 20°, 30°, 40°, and 45° in the crystal on both sides of the normal to the section.

In actual work with this method it happens occasionally that the determination of the location of A_2 is not accurate because of the acute angle between the radius and the small circle A_2A_2'. In such cases the writer has been able to apply with favorable results one of the two following methods,* which, like the preceding method, are based on the measurement of extinction angles for different angles of inclination about one of the horizontal axes of rotation of the universal stage. The new circle V_2 renders hereby valuable assistance.

*Amer Jour. Sci. (4), 24, 351, 1907

In the first of these methods the horizontal position of the section is exactly that of the above method (Fig. 112). A_1, having been previously located accurately, is brought to coincidence with ON, and the extinction angle of the specimen in the horizontal position is ascertained; and then, instead of being rotated about the horizontal axis V_1 (the line OL in projection), the section is rotated about V_2 (or ON in the projection) as an axis,* a given angle (apparent angle in air corresponding to angle in crystal) A_1 travels during the rotation of stage to A_1' in the projection, the direction

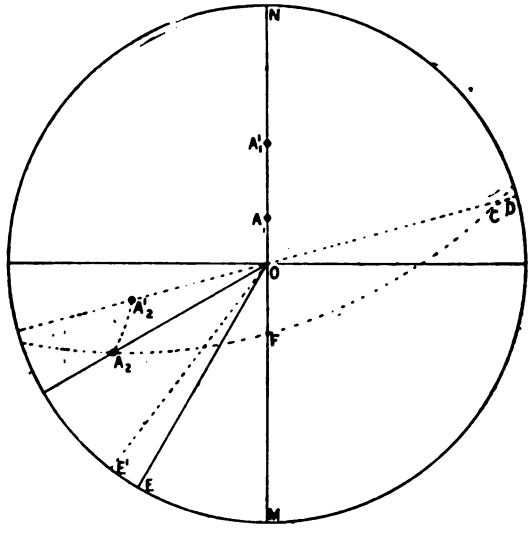


FIG. 111.—The general method of extinction curves shown in this figure is applicable to all sections in which one optic axis A_1 can be brought to coincidence with the axis of the microscope. After the determination of the exact position of A_1 by means of optical curves the specimen is rotated about H_1 until A_1 coincides with the plane NO normal to the axis V_1 of the universal stage. The extinction angle MOE of the specimen in the horizontal position is then determined; by construction EOA_2 is made equal to MOE; the specimen is then rotated about V_1 a convenient angle (apparent angle observed to be reduced to true angle in crystal), and the new extinction angle MOE' is ascertained. In the new position, the optic axis is contained in the plane OA_2', angle $E'OA_2'$ having been made equal to MOE'. The exact position of A_2 is then determined on the drawing on tracing-paper by noting the small circle of the underlying projection plat, whose arc A_2A_2' intercepted between OA_2 and OA_2' is equal to the angle of rotation. This determination can be checked by drawing the great circle CF, which marks the position which the plane OA_2' would assume were the specimen turned back to its original position. In practice the position of A_2 is determined for different angles of rotation about V_1 and the mean position of all determinations taken as the most probable and correct location of A_2.

of extinction wanders from OE to OE', and the plane OA_2 (containing A_2) wanders from OA_2 to OA_2', the angle $E'OA_2'$ being by construction $= E'OA_1'$. By recording the angle of rotation of the stage about ON (V_2) required to bring the section to its new position, it is not difficult to find in the projection that small circle, parallel to OL, whose arc A_2A_2' (intercepted by the lines OA_2 and OA_2') is equal to the above angle of rotation, and thus to locate A_2. To insure accuracy, this measurement should be repeated for several different angles of rotation and A_2 determined in each case. As in

*The same effect can be produced by rotating the specimen 90° about H_1 and then about V_1 as an axis.

the first method, the great circle CF, indicating the original position of the plane OA_2, can be constructed and should pass through A_2 on the line OA_2.

The second new method differs from the first only in the fact that instead of placing the optic axis A_1 in the plane OE (Fig. 113) and then measuring the extinction angle of the section in the horizontal position, the actual direction of extinction OE is brought to coincidence with the axis of rotation of the universal stage (V_1 or V_2); the section is then rotated a given angle about this axis and, from the extinction angles, the lines OA_2 and OA_2' are determined; the arc $A_2\,A_2'$ is equal to the angle of rotation. The point A_2 is then the desired direction of the second optic axis.

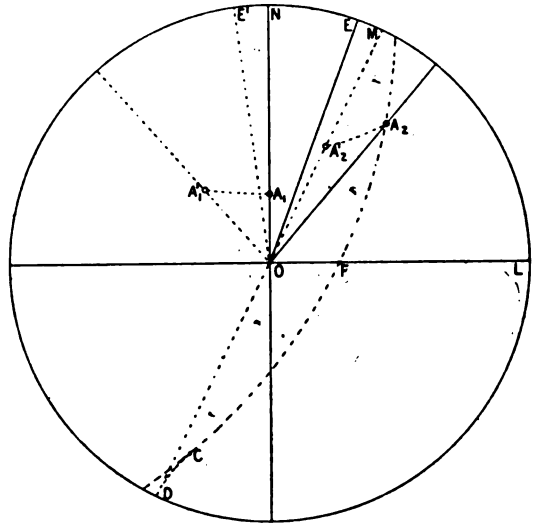


FIG. 112.

In both new methods the determination can be varied by inclining the specimen first about V_1 as an axis and then determining a series of extinction angles for different angles of inclination about V_2 (V_2 in this case being normal to V_1) and thus locating A_2 afresh with each extinction. By establishing a set of observations about V_2 for each new position of V_1 it is possible to extend the number of observations indefinitely and thus to locate A_2 with great accuracy. In fact, the position of A_1 in the projection is immaterial so long as its position be definitely known with respect to the axes of rotation (V_1 and V_2), since with A_1' located at any point in the projection it is still possible to locate A_2 by means of extinction angles for different angles of inclination about V_1 and V_2. This method, involving the use of both V_1 and V_2, is therefore a method of general application and is capable of furnishing reliable data on all sections so cut that one optic axis at least falls within the field of vision.

Still another method, which furnishes trustworthy results and is of general application, consists in determining first the positions of the planes of symmetry and the axes of the ellipsoid within the crystal (Fig. 114). In this method, practically all the graduated circles of the stage are brought

into play, since not only must extinction angles be observed, but also the section rotated about the ellipsoidal axes and the exact position of each axis noted. The method of procedure consists in first placing the stage in the zero (primary) position, H_3, H_1, H_2, and V_1 in zero position, and V_2 normal to V_1; the section having any orientation and position. The section is then inclined about V_2 until darkness between crossed nicols ensues; if this be not the case, it is turned about H_3 a small angle, and the attempt made a second time, and so on until darkness is observed at a definite angle of inclination about V_2. The preparation is then rotated about V_1, and, if

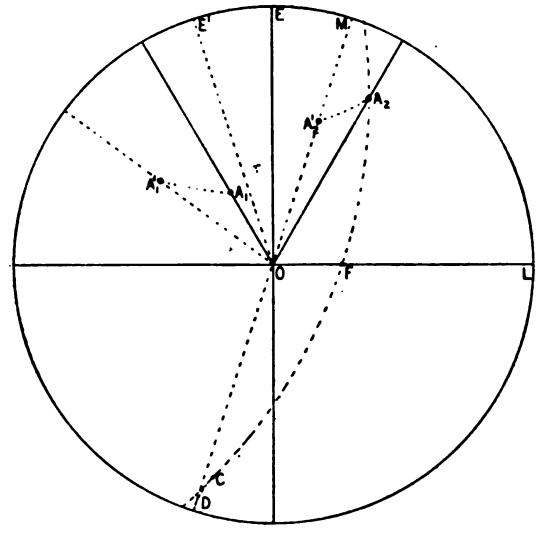


FIG. 113.

by chance the correct position be obtained, darkness will continue for every angle of inclination about V_1. This is usually not the case; by repeated trial that position of H_3, H_2 is to be found for which the preparation remains dark for every angle of rotation about V_1. The angle of inclination V_2 and the directive angle H_3 determine then the position of one of the planes of symmetry of the ellipsoid within the crystal, *e. g.*, the plane $\alpha\beta'\gamma$ of Fig. 114, this being fixed by the line $O\beta'$; in similar fashion the planes $\alpha\gamma'\beta$ and $\gamma\alpha'\beta$ are located and plotted in the suitable projection. This method of locating the planes of symmetry of the ellipsoid within the crystal is comparatively rapid and sensitive, and a fair degree of accuracy can be attained by its use. The new circles V_2 (Plate 6, Fig. 1), attached to the large Fedorow-Fuess universal stage, have proved extremely serviceable and time-savers in this method.

Having once determined the position of either α or γ by this method, and that of one optic axis A_1 by optical curves, the position of the second optic axis A_2 is readily obtained, since the angle $A_1\alpha$ or $A_1\gamma$ is by definition equal to $A_2\alpha$ or $A_2\gamma$ respectively.

After some practice, the exact relative positions of H_2, H_3 can be found without difficulty, for which darkness remains for all angles of inclination about V_1. To insure accuracy, however, the fact of remaining dark should

be scrutinized very sharply, since the correct position is not always that of absolute darkness, but rather that for which the same degree of darkness or intensity of uniform illumination obtains throughout.

From the complete determination (by this method) of the positions of α, β and γ, which should be mutually 90° apart, Fedorow has shown that

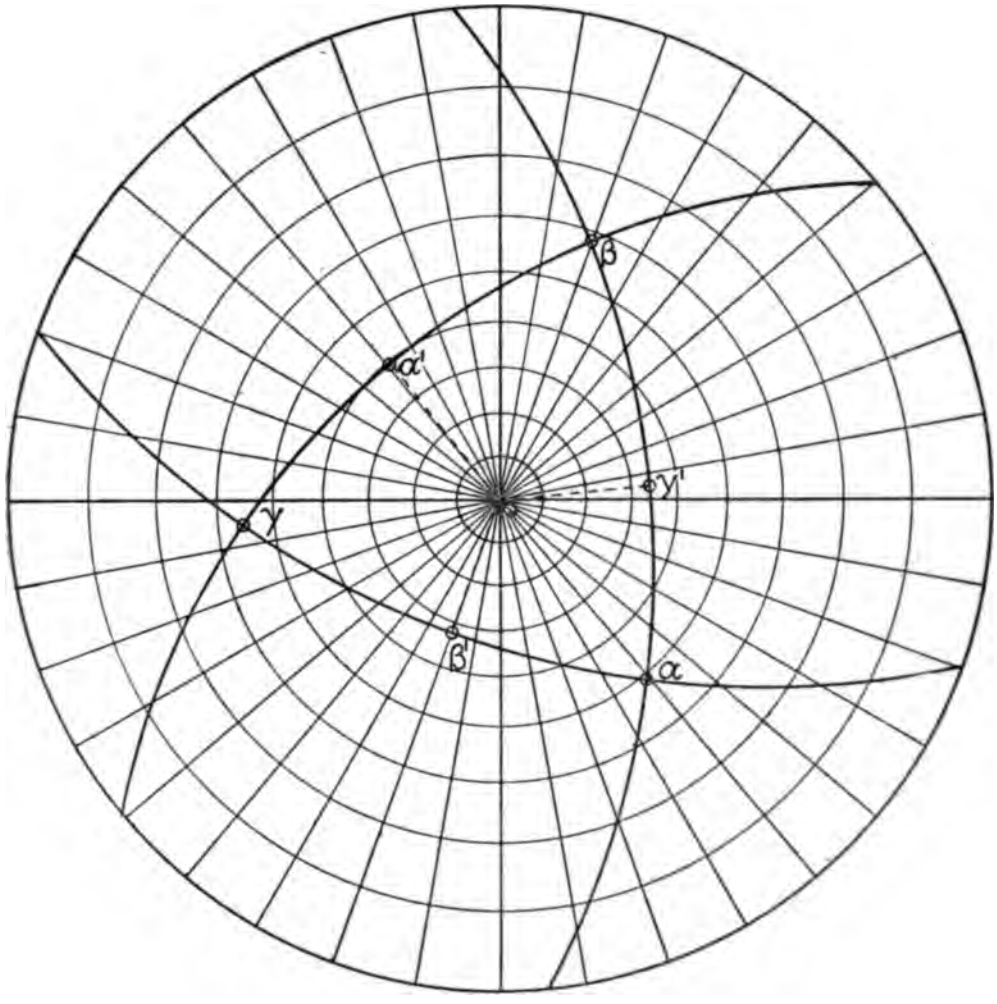


FIG. 114.—In this figure the great circles $\alpha\beta'\gamma$, $\gamma\alpha'\beta$ and $\beta\gamma'\alpha$ of the stereographic projection denote the traces of the principal planes of the optical ellipsoid within the crystal. They are fixed in position by determining the positions of H_1 and V_1 for which the section remains dark for all positions of inclination about the horizontal axis V_1 (V_1 being normal to V_1); the lines $O\beta'$, $O\gamma'$ and $O\alpha'$ are thus fixed both in direction and length and also the great circles $\alpha\beta'\gamma$, $\gamma\alpha'\beta$, and $\beta\gamma'\alpha$, the planes of symmetry of the ellipsoid, the intersections, α, β, and γ, of which are in turn the ellipsoidal axes.

the average refractive index of the mineral can be derived approximately by use of this fact, although the determination is not of sufficient accuracy to be of great practical value.

By this method of determining the positions of the principal sections of the ellipsoid, the distinction between uniaxial and biaxial minerals is greatly facilitated and the general problem solved for all possible sections. In case the position of neither optic axis can be determined directly, both optic axes lying outside the field of vision, the methods for measuring the optic axial angle are based solely on the determination of extinction angles along certain directions, and are of such a nature that by their use only very rough approximations to the true value of $2V$ can be obtained, errors of $\pm 10°$ and over being easily within the range of possibility. Fedorow has suggested one principal method applicable to such cases and the writer has had occasion to use several others. They are not so satisfactory, however, as the above methods, and are not of equal practical value. For the sake of completeness they are described briefly below.

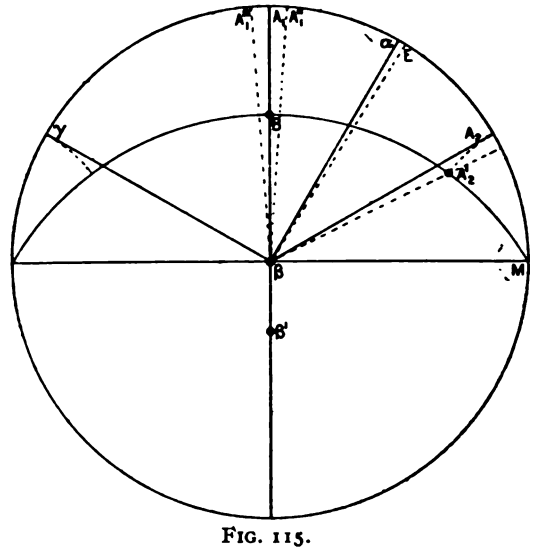


FIG. 115.

SECTION NEARLY PERPENDICULAR TO THE OPTIC NORMAL β.

In case the section of a mineral is so cut that it makes an angle of 30° or less with the plane of the optic axes, neither optic axis appearing, in consequence, within the field of vision, the above method places the observer in a position to measure the optic axial angle without even seeing either optic axis. The exact position of β can first be determined by this method, and then brought to coincidence with the microscopic axis, in which case the plane of the optic axis is horizontal. In this position the circles V_1 and H_1 are free and the section can be revolved about V_1 and extinction angles determined on H_1 (Figs. 115 and 116).

Since the exact positions of α and γ have been determined and the two optic axes make equal angles with these bisectrices, it is possible by trial to bring one of the optic axes A_1 to coincidence with the normal to V_1 (Fig. 115) and to test the accuracy of its position by means of extinction curves for different inclinations of the section about V_1. Thus let α be the acute bisectrix (Fig. 115) and assume that one optic axis A_1 coincides pre-

cisely with the normal to axis V_1; A_2 is then the second optic axis and angle $A_1\beta a$, equal to angle $A_2\beta a$ and $A_1\beta a$, is the extinction angle. On rotating the section now, about V_1, the optic axial point A_1 is brought to B and the extinction angle $B\beta E$ for the new position of the section should bisect exactly the angle $B\beta A_2'$. If this be not the case and the extinction angle be too large or too small, the section should be rotated about H_2 either counter-clockwise (A_1'' to A_1) or clockwise, A_1''' to A_1, through a small angle and a new set of measurements should be made, until after repeated trials the corrected position is to be found for which observation and construction agree precisely. The angle $A_1\beta a$ is then half the desired optic axial angle.

In certain cases this method of placing the one optic axis A_1 in the plane normal to the axis of rotation V_1 has been found unsatisfactory, and a new method* has been used, which consists in first bringing by trial the one optic axis to coincidence with the axis of rotation and then measuring the extinction angles for different angles of inclination about V_1 (or V_2) and testing the results of observation and construction until they coincide. The method is shown in Fig. 116 and is so similar to the foregoing method (Fig. 115) that further description is unnecessary.

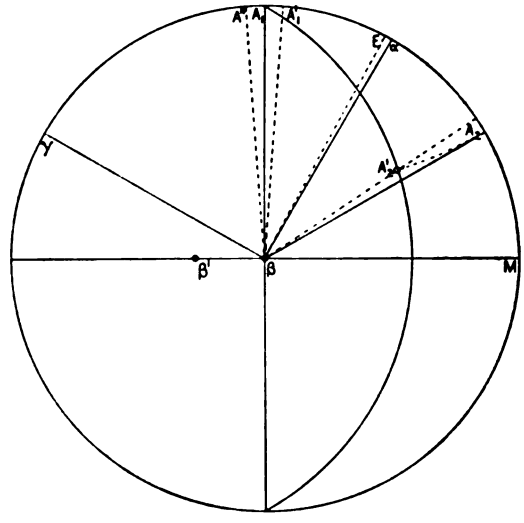


FIG. 116.

SECTION NEARLY NORMAL TO THE OBTUSE BISECTRIX.

For a section nearly normal to the obtuse bisectrix of a mineral both optic axes lie again outside the field of vision and the optic normal β can not be brought to coincidence with the axis of the microscope. The above methods do not apply, therefore, and new ones are required to meet the new conditions, and of these the following has been found practicable by the writer:†

Place the universal stage in the primary position, the axis of V_2 normal to that of V_1 and the circles H_1, H_2, and H_3 all in the horizontal position; determine the exact position of the obtuse bisectrix (a or γ, as the case may be) by the method of principal ellipsoidal planes (page 183), and bring it to

*Amer. Jour. Sci. (4), 24, 355, 1907. †Amer. Jour. Sci. (4), 24, 355, 356, 1907.

coincidence with the axis of the microscope, the plane of the optic axes being then parallel to the vertical cross-hair (Fig. 117). Rotate the section some convenient angle about axis V_2 and then about V_1 (as shown in Fig. 117), also through any suitable angle. Measure accurately the extinction angle of the section in its new position. Plot the data of observation in angle or stereographic projection (after proper reduction of observed angles to true crystal angles); and find those two points A_1'' and A_2'' contained in the optic axial plane and equidistant from the obtuse bisectrix a'', which are so located that the observed extinction angle OE_1 bisects the angle $A_2''OA_1''$ (Fig. 117). The angle $A_1''A_2''$ is then the desired optic axial angle, $2V$.

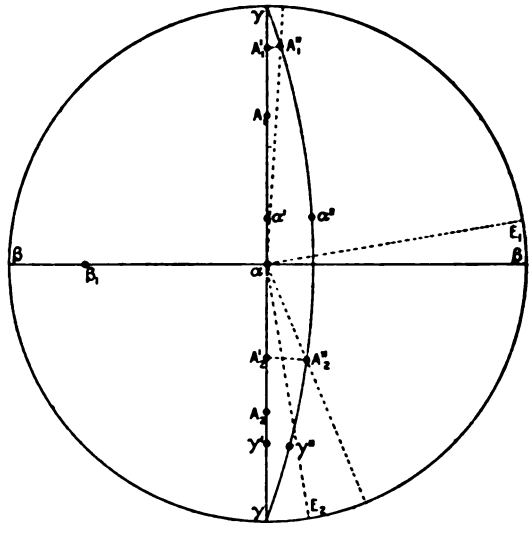


FIG. 117.—To use the method indicated by this figure, turn the section so that its obtuse bisectrix coincides with the axis of the microscope (center of the projection plat) and the optic axial plane is parallel to the vertical cross-hair; turn the preparation about axis V_2 a convenient angle (reduce to true crystal angle equivalent to apparent angle observed in air or glass), and then about axis V_1 (normal to V_2) any suitable angle and measure the extinction angle of section in its new position. Plot these data in stereographic or angle projection and find those two points A_1'' and A_2'' equidistant from the obtuse bisectrix and contained in the plane of optic axes in its new position, for which the observed line of extinction OE bisects the angle included between OA_1'' and OA_2''. O is the center of the sphere of projection.

With the universal stage in its present form it is not always possible to execute the movements indicated in the above method, since, when the obtuse bisectrix is brought to coincide with the axis of the microscope, the axis of V_2 is in general no longer horizontal and the rotation about V_2 is therefore along an inclined axis. In plotting the observed data, this fact should be carefully noted, otherwise errors may nullify the results.

With the universal stage it is thus possible to measure the optic axial angle of any grain of any transparent birefracting substance in the thin section and to distinguish the biaxial and uniaxial minerals. The degree of accuracy of this measurement, however, is not of the same order of magnitude for all sections, but differs very materially with different sections. As a matter of experience, it has been found that the most accurate results can be obtained on sections in which both optic axes appear within

the field of vision; that good results can be had from sections which show only one optic axis within the field, while for sections in which neither optic axis appears within the field the determination is uncertain and at best only a rough approximation.

To summarize briefly the different methods best applicable to the four different possible cases cited above:

(1) *The optic axes are both within the field of vision and inclined between 15° and 55° with the normal to the section.* Determine the approximate positions of the two optic axes by bringing each one, by means of H_2 and V_1, into the vertical position.

Determine the position of each optic axis more accurately by means of optical curves in projection and check by means of extinction curves and exact location of principal planes of ellipsoid, especially the plane containing the optic axes.

(2) *The section is nearly normal to an optic axis; one optic axis A_1 inclined less than 20° to section normal.* Place the stage in the horizontal position (H_2 and H_3 in horizontal position and V_1 normal to V_2), turn H_3 and incline about V_2 until the optic axis coincides with the axis of the microscope; then rotate the section about V_1 and turn H_2 until darkness is attained, and thus determine plane of optic axes and β. Incline V_2 back to o° position, rotate about H_2 until the optic axis coincides with the plane normal to V_1 and determine the extinction curve, the intersection of which with the plane of the optic axes in projection fixes the position of the second optic axis accurately. Check by determining α and γ both from projection and observation; also by the extinction curve for the rotation about V_2.

(3) *One optic axis inclined 20° to 55° within the crystal to the normal of the section, the second entirely out of the field of vision.* Determine visible optic axis by optical curves and second optic axis by means of extinction curves, both about V_1 and V_2. Verify results by determination of α, β, and γ.

(4) *Both optic axes are entirely outside of the field of vision,* i.e., *are inclined at an angle of more than 65° in air with the normal to the section.* In such instances the location of the optic axes is effected by means of extinction angles alone and the values obtained are not accurate, since an error of 1° in the determination of the extinction angle may affect the value of the optic axial angle up to 30°. For accurate work, therefore, such sections are of little value at the present time for measuring the optic axial angle by the universal-stage methods; but in case the section be about normal to the obtuse bisectrix, the measurement of the optic axial angle is much more certain and satisfactory.

As noted previously, experience has shown that the best and most rapid method of projection is that of Wulff, who uses an accurate projection plat as a base and tracing-paper on which to sketch the great circles and to execute the actual measurements. The graduated hemisphere of Nikitin can also be used to advantage for the purpose.

PROBABILITY OF ENCOUNTERING SECTIONS SUITABLE FOR OPTIC AXIAL ANGLE MEASUREMENTS.

Since the accurate measurement of the optic axial angle can be accomplished only on sections in which at least one optic axis is within the field

of vision, it is of interest to note the probable relative frequency of occurrence of such sections in a rock section. The microscopic field of the universal stage fitted with glass segments includes an angle of about 120°, and the area on the surface of the unit sphere thus covered for a biaxial crystal is evidently

$$s = 4\pi \cdot 2(1 - \cos\phi) = 4\pi \cdot 4\sin^2\frac{\phi}{2}$$

2ϕ being the angle of vision of the field reduced to the true value within the crystal; if the observed angle 2ψ be used, the average refractive index of the mineral β and that of the glass segments n should be introduced into the formula

$$s = 4\pi \cdot 2\left(1 \pm \sqrt{1 - \frac{n^2}{\beta^2} \cdot \sin^2\psi}\right)$$

The probability, P_1, that a section will show an optic axis is evidently measured by the relative surfaces s to S, the surface of the sphere itself:

$$P = \frac{s}{S} = \frac{4\pi \cdot 4\sin^2\frac{\phi}{2}}{4\pi} = 4\sin^2\frac{\phi}{2} = 2\left(1 \pm \sqrt{1 - \frac{n^2}{\beta^2} \cdot \sin^2\psi}\right)$$

In case the areas covered by the two optic axes overlap, the formula should be changed, as Césaro has shown,* to

$$P = 4\sin^2\frac{\phi}{2} - \frac{2}{\pi}\left[\cos^{-1}\left(\frac{\sin V}{\sin\phi}\right) - \cos\phi\,\cos^{-1}\left(\frac{\tan V}{\tan\phi}\right)\right]$$

in which $2V$ denotes the angle between the optic axes. The probability of encountering a proper section with minerals of average refractive index 1.65 and with glass hemispheres of refractive index 1.52 ranges from 2 to 5 in uniaxial crystals to 4 to 5 in biaxial crystals for which the fields for the optic axes do not overlap. The degree of probability is high, and one should be able to find suitable sections in every slide for measuring the optic axial angle of each mineral present.

METHODS BASED ON THE RELATIVE BIREFRINGENCE OF KNOWN SECTIONS.

In 1883 Michel-Lévy† suggested that the optic axial angle could be determined approximately by measuring the path-differences on two oriented sections perpendicular to one of the two bisectrices or to the optic normal in the same slide. The path-difference is dependent both on the birefringence $(\gamma' - \alpha')$ and on the thickness of the section, but if two sections be chosen from a uniformly thick slide the ratio of their path differences in monochromatic light can be used directly in the calculation of the optic axial angle from any one of the following standard approximate equations:

$$\sin^2 V_c = \frac{\Delta_c}{\Delta_b} = \frac{d(\beta-\alpha)}{d(\gamma-\alpha)} = \frac{\beta-\alpha}{\gamma-\alpha} \text{ or } \cos^2 V_c = \frac{\gamma-\beta}{\gamma-\alpha} \text{ or } \mathrm{tg}^2 V_c = \frac{\beta-\alpha}{\gamma-\beta}$$

in which V_c is the angle between the least ellipsoidal axis $\mathfrak{c}$ and one of the

*G. Césaro, Mem. de l'Acad. Roy. d. Sci. d. Belgique, 54, 1895.
†Bull. Soc. Min. Fr., 6, 147, 1883; Les Minéraux des Roches, 53, 1888.

optic axes. In 1890, A. C. Lane* described practically the same method, discussed its accuracy under different conditions, and gave a graphical solution of the above equation. He applied the method especially to the study of the amphiboles and pyroxenes.

In 1896 E. von Fedorow† proved that it is possible to measure the birefringence $\gamma-\beta$ and $\beta-\alpha$ by use of the universal stage fitted with glass segments of specially high refractive index and with the Fedorow mica comparator, and thus to ascertain the optic axial angle from the same formula either by graphical means or by calculation. Recently J. Uhlig‡ has redescribed the method of Michel-Lévy and Lane and discussed its accuracy under different conditions. He determined the path-difference by means of the Michel-Lévy color chart. The chief sources of error in this method are: (1) sections are rarely found cut precisely normal to a bisectrix or optic normal; (2) the thickness of the thin section is usually not the same throughout; (3) if white light and the color chart be used for determining the order of the interference color, the determination of the path-difference is only approximately correct. If the mineral be weakly birefracting the determination by this method is of little value. Under favorable conditions rough approximations to the correct optic axial angle can be obtained in a short time. To facilitate such determinations Plate 9 has been drawn, which is a graphical solution of the equation

$$\sin^2 V_c = \frac{\beta-\alpha}{\gamma-\alpha} = \frac{\Delta_c}{\Delta_b}$$

the abscissæ indicating directly $\gamma-\alpha$, the ordinates, $\beta-\alpha$, (indicated by $\gamma'-\alpha'$ in Plate 9), and the curves the corresponding axial angle V. Lane's application of this method to parallel intergrowths of different amphiboles and pyroxenes has proved especially valuable.

G. Césaro|| has described a method for ascertaining the optic axial angle on a section parallel to the plane of the optic axes. He measures the path-difference of two points along the diagonals in adjacent quadrants of the interference figure and from this calculates the optic axial angle. The method, however, is in general too inaccurate to be of much service.

EXTINCTION ANGLES OF PLATES IN ZONES WHOSE AXES LIE IN THE PLANE OF THE OPTIC BINORMALS.

This method is particularly adapted to monoclinic minerals, as amphiboles and pyroxenes, and may be of service in a rough estimation of the optic axial angle of such a mineral. The underlying principle of this method is again the rule of Biot-Fresnel, and mathematical formulæ suitable for its solution have been developed by Michel-Levy,§ Césaro,¶ Harker,** Lane,†† Daly,‡‡ Ferro,|||| Souza Brandao§§ and others. These formulæ show that

*Amer. Jour. Sci. (4), **39**, 53–58, 1890; **43**, 79, 1892.
†Zeitschr. Kryst., **25**, 349–356, 1896.
‡Centralblatt f. Min., 1911, 305–312.
||Bull. Acad. roy. Belg. Classe des Sciences, 1906.
§Michel-Lévy et Fouqué, Minéralogie Micrographique, 72–76, 1879.
¶A. Césaro, Mem. de l'Acad. Roy. d. Sci. d. Belg., **54**, 26, 1895.
A. Harker, Miner. Magazine, **10, 239, 1894.
††See Daly, Proc. Amer. Acad. Arts and Sci., **34**, 314, 1899
‡‡R. A. Daly, Proc. Amer. Acad. Arts and Sci., **34**, 314–323, 1899.
||||A. A. Ferro, Rivista di Mineralogia, etc. Padua, **20**, 1–14, 1908; Zeitschr. Kryst., **32**, 532, 1899.
§§V. de Souza Brandao, Communicacoes da Dirreccao dos Servicos Geologicos, 4, 1–28, 41–56, 1900; **6**, 339–364, 1907.

for the exact determination of the optic axial angle the method of extinction angles on different faces in the same zone is not well adapted to optic axial-angle determinations, especially when the optic axial angle of the mineral is small. In certain cases it is possible to express this relation, as Lane has shown, in a slightly different form better adapted for measurements. Lane's method, as applied to the pyroxenes and amphiboles, consists in measuring the angle between the clinopinacoid and that face of the prism zone which has the same extinction angle. The trigonometrical relations which obtain for this particular case can be readily deduced from Fig. 118, in which the plane of the optic axes is represented by the vertical line LM,

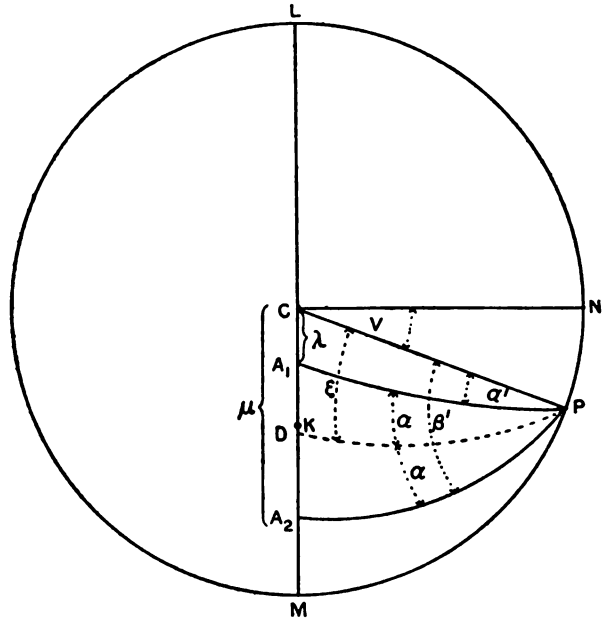


FIG. 118.

the optic axes by A_1, A_2, the zonal axis by C, and a given plane by P which includes the angle v with the normal CN; let the angle $CA_1=\lambda$ and $CA_2=\mu$; also $CPA_1=\alpha'$, $CPA_2=\beta'$, $CPD=\xi$ and by construction $A_1PD=A_2PD=\alpha$. Then in the triangle PA_1C

$$\text{tg } \alpha' = \cos v . \text{tg } \lambda;$$

similarly in triangle PA_2C

$$\text{tg } \beta' = \cos v . \text{tg } \mu.$$

But $2\xi=\alpha'+\beta'$; accordingly

$$\text{tg } 2\xi = \frac{\text{tg } \alpha' + \text{tg } \beta'}{1 - \text{tg } \alpha' \text{tg } \beta'} = \frac{\cos v \ (\text{tg } \lambda + \text{tg } \mu)}{1 - \cos^2 v . (\text{tg } \lambda \text{ tg } \mu)} \qquad (1)$$

For that section for which 2ξ is equal to the extinction angle

$$CK = \frac{\lambda + \mu}{2}$$

in the plane of the optic axes

$$\text{tg}2\xi = \text{tg}2 \,.\, CK = \frac{\cos v(\text{tg}\,\lambda + \text{tg}\,\mu)}{1 - \cos^2 v \,\text{tg}\,\lambda\,\text{tg}\,\mu} = \frac{\text{tg}\,\lambda + \text{tg}\,\mu}{1 - \text{tg}\,\lambda\,\text{tg}\,\mu} \qquad (2)$$

$$\therefore \cos v = -\cot\lambda \,.\, \cot\mu. \qquad (3)$$

In order that this equation be valid the product cot λ cot μ must be less than unity and either λ or μ must be negative and one or both greater than 45°. The optic axial angle, $2V$, and the extinction angle, ξ, in the plane of the optic binormals are related to λ and μ by the equations

$$\lambda + \mu = 2\xi \qquad \lambda - \mu = 2V$$

With the aid of these values equation (3) can be written

$$\text{tg}^2 \frac{v}{2} = -\frac{\cos 2V}{\cos 2\xi}$$

an equation which is valid only for $2V$ or 2ξ greater than 90°.

In table 8, the values of v are given for different optic axial angles ($2V$) and different extinction angles (ξ), the extinction angle being considered taken invariably to the acute bisectrix of the optic binormal angle. It is evident from this table that for small optic axial angles this method has no practical value for even rough measurements. The larger the axial angle, however, the more sensitive the method becomes.

TABLE 8.

$2V$	$E_1=50$ $E_2=40$	$E_1=55$ $E_2=35$	$E_1=60$ $E_2=30$	$E_1=65$ $E_2=25$	$E_1=70$ $E_2=20$	$E_1=75$ $E_2=15$	$E_1=80$ $E_2=10$	$E_1=85$ $E_2=5$
10°	134°27′	118°58′	109°03′	102°08′	97°10′	93°41′	91°21′	90°00
20°	133 29	117 47	107 47	100 49	95 50	92 10	90 00	88 39
30°	131 46	115 43	105 32	98 31	93 31	90 00	87 40	86 19
40°	129 08	112 30	102 08	94 01	90 00	86 29	84 10	82 50
50°	125 04	107 47	97 11	90 00	85 59	81 29	79 11	77 52
60°	118 58	100 49	90 00	82 49	77 52	74 28	72 13	70 57
70°	109 03	90 00	79 11	72 13	67 30	64 17	62 13	61 02
80°	90 00	70 57	61 02	54 56	50 52	48 14	46 31	45 33

MEASUREMENT OF THE OPTIC AXIAL ANGLE ON THE TOTAL REFRACTOMETER.

Pulfrich,* Soret,† Viola,‡ Cornu,|| and Wallérant§ have shown that it is possible on a single section of a biaxial or uniaxial mineral to determine, not only the three principal refractive indices α, β, and γ, but also, by observing the planes of polarization of each wave corresponding respectively to $\alpha\beta$, $\beta\gamma$ and $\gamma\alpha$, to determine accurately the relative position of the principal planes of the ellipsoid to the given section; and, from the accurate refractive indices thus ascertained, to calculate the optic axial angle with great exactness. These methods, however, require specially ground and polished sections and

*C. Pulfrich, Das Total Reflektometer, Leipzig, 1890.
†Soret, Zeitschr. Kryst., 15, 43, 1899.
‡C. Viola, Zeitschr. Kryst., 31, 40-48, 1889; 36, 245-251, 1902.
||Cornu, Compt. Rend., 133, 125; Bull. Soc. Min., 25, 7.
§F. Wallérant, Bull. Soc. Min., 20, 12-26, 1889.

are not, in general, microscopic methods, although the total refractometer of Wallérant is attached directly to the microscope and is employed on thin uncovered and polished sections of rocks. Unfortunately, the writer has had practically no opportunity to work with the total refractometer of Wallérant and is, therefore, not in a position to judge personally of its fitness for optic axial-angle determinations. With the Abbe-Pulfrich total refractometer the probable error of the values obtained with monochromatic light on highly polished crystal plates of at least 1 sq. mm. surface area should not exceed 3 in the fourth decimal place. From these refractive index values the optic axial angle can be calculated by the usual formula and a check on the direct measurements of the optic axial angle can be thus obtained.

THE RELATIVE ACCURACY OF THE DIFFERENT METHODS.

In this section only a brief summary of the results of observation with the different methods will be given, together with a short statement of the relative accuracy and applicability of the several methods under test.

Different minerals (as aragonite, topaz, muscovite, etc.) were first chosen and oriented sections cut to show the different phenomena required by the several methods. The correct optic axial angle for each mineral was then measured in sodium light on a Wülfing-Fuess axial angle apparatus,* the angle obtained thereby being adopted as the standard of comparison for all methods. For each mineral a series of measurements of the optic axial angle was taken for different sections and by the different methods; the relative degree of accuracy of each method was judged, not only by the results obtained, but also by the factors, on which the method itself is dependent, and by their influence under the different conditions of observation.

MEASUREMENTS WITH THE AXIAL-ANGLE APPARATUS.

The optic axial angles obtained in sodium light on the Wülfing axial-angle apparatus varied slightly and the average of five determinations of each angle is given below:

Topaz, Willard Co., Utah: $2E = 126° 13'$. $2V = 66° 42'$.
Aragonite, Bilin, Bohemia: $2E = 31° 09'$. $2V = 18° 22'$.
Muscovite: (*a*) $2E = 71° 40'$; (*b*) $2E = 59° 42'$.

MEASUREMENTS WITH THE BECKE DRAWING-TABLE.

To economize space, the results are given below in their reduced form ready for plotting directly in projection, the angle ϕ denoting the equatorial angle from the horizontal line of the projection and ρ the polar distance; A_1, as usual, denotes the visible axial point and P_1 any point on the dark axial bar.

Topaz.

	ϕ	ρ
A_1	0°	5°.0
P_1	+65	20.5

*E. A. Wülfing, Neues Jahrb. Beil. Bd., 12, 405–446, 1898; Rosenbusch-Wülfing, Mikros. Physiogr. 4th ed., I, 1, 217, 1904.

By projecting these angles and performing the requisite mechanical operations, the optic axial angle thus determined on this section was $2V=62°.5$. For a second section the values were:

	ϕ	ρ	
A_1	$-8°$	$3°.8$	$2V=70°$
P_1	-72	$19\ .6$	

For a third section:

	ϕ	ρ	
A_1	$+27°$	$8°.8$	$2V=63°$
P_1	-58	23	

The average of these three values is $65°.2$.

Aragonite.

In aragonite the optic axial angle is so small that both axial bars A_1 and A_2 are visible and the direct determination of $2V_{A_1A_2}$ should in all cases be accurate within 1°. The birefringence is so strong, however, that the measurements involving a point P_1 or P_2 on the dark axial bar and consequent introduction of the refractive index β for that point, may be decidedly incorrect. The use of the refractive index β presupposes only slight differences between the refractive indices of the mineral in order that the errors thus caused may not be too large.

		ϕ	ρ	
(1)	A_1	$-80°$	$17.5°$	$2V_{A_1A_2}=17°.4$
	A_2	-130	23	$2V_{P_1A_1}=17\ .5$
	P_1	$+15$	$18\ .4$	
(2)	A_1	$-10°$	$8°$	$2V_{A_1A_2}=18°.3$
	A_2	-17.1	$10\ .6$	$2V_{P_1A_1}=21$
	P_1	$+42$	21	
(3)	A_1	$+44°$	$28°.3$	$2V_{A_1A_2}=18°.0$
	A_2	$+85$	$19\ .5$	$2V_{P_2A_2}=17\ .6$
	P_2	-15.5	$18\ .4$	
(4)	A_1	$-56°$	$11°.2$	$2V_{A_1A_2}=17°.5$
	A_2	-140	$14\ .3$	$2V_{P_1A_1}=16$
	P_1	$+29$	$23\ .4$	$2V_{P_2A_2}=23$
	P_2	$+158$	$26\ .0$	

The values for $2V_{A_1A_2}$ do not differ over 1° from the correct value, while those for $2V_{P_1A_1}$ differ as much as 5° from the correct value.

Muscovite (b).

	ϕ	ρ	
A_1	$0°$	$35°.6$	$2V_{A_1A_2}=71°.2$
A_2	0	$-35\ .6$	

MEASUREMENTS WITH THE DOUBLE-SCREW MICROMETER OCULAR.

The data given below appear also only in corrected form ready for plotting in projection, the actual scale-readings having been reduced to equivalent angles in air and these in turn figured to true angles within the crystal by means of the refractive index β. The errors observed above in aragonite sections because of strong birefringence apply equally well here. In the following tables H indicates the horizontal and V the vertical micrometer screw of the ocular.

Topaz.

		H	V	
(1)	A_1	3°.5	0°	$2V = 64°.5$
	P_1	5 .5	17 .5	
(2)	A_1	5°.3	.3	$2V = 65°.5$
	P_1	7 .6	17 .2	

Aragonite.

		H	V	
(1)	A_1	7°	9°.8	$2V_{A_1A_2} = 18°.0$
	A_2	−10 .8	9 .8	$2V_{P_1A_1} = 18\ .5$
	P_1	17 .8	6 .8	$2V_{P_1A_2} = 14$
	P_2	−17 .5	6 .8	
(2)	A_1	4°.6	20°	$2V_{A_1A_2} = 17°.9$
	A_2	−13 .20	20	$2V_{P_1A_2} = 19$
	P_1	5	3	$2V_{P_1A_1} = 23$
	P_2	−23 .5	3	
(3)	A_1	7°.3	1°.8	$2V_{A_1A_2} = 18°.5$
	A_2	−11 .5	1 .8	$2V_{P_1A_1} = 18$
	P_1	20 .6	18 .8	

Muscovite (a).

		H	V	
(1)a	A_1	35°.5	0°	$2E = 71°$
	A_2	−35 .5	0	
(1)b	A_1	0°	36°	$2E = 72°$
	A_2	0	−36	

Muscovite (b).

	H	V	
A_1	30°.1	0°	$2E = 60°.2$
A_2	−30 .1	0	
A_1	0°	30°	$2E = 60°$
A_2	0	−30	

MEASUREMENTS WITH THE UNIVERSAL STAGE.

The angles given below were read directly on the different circles of the universal stage and before plotting in projection require reduction to true crystal angles by means of the refractive index β of the mineral and n (= 1.5239) of the glass hemispheres used. The letters H_1, H_2, H_3 and V_1, V_2 designate the different circles of the universal stage (see Plate 6, Fig. 1) on which the angles were read. The angle after the letter N designates the angle made by the principal plane of the lower nicol with the plane of symmetry of the microscope.

Topaz. Section after 001 (Acute Bisectrix).

A direct preliminary determination of the position of the optic axes in parallel polarized light was first made and the approximate location of each axis determined. These values were later corrected by means of optical curves. The direct preliminary determination was as follows:

	H_1	H_2	H_3	V_1	V_2
A_1.....	180°	90°	294°	35°	−0.5°—$2V$ = 66°.6
A_2.....	180	90	294	−36	−0.5

The following shows the corrections by method of optical curves:

				N = 0° V_1		N = 30° V_1		N = 45° V_1	
H_1	H_2	H_3	V_2	A_1	A_2	A_1	A_2	A_1	A_2
180°	80°	294°	1°	33.5	−33.5	37	−39.5	33	−37
180	85	294	1	35	−36	36	−38	33.5	−37.5
180	90	294	1	35.5	−36	34	−37	34	−36.5
180	95	294	1	34	−34.5	33	−36	34	−37
180	100	294	1	33	−34.5	33	−34	35	−37

After proper reduction of these angles, the corrected angle, obtained directly from the stereographic projection plat, is $2V$ = 66°.5.

Topaz. Section nearly normal to an optic axis.

The determination in this case can be most readily accomplished by first locating A_1 accurately by optical curves and then fixing the position of A_2 in projection by means of the principal ellipsoidal planes. The optical curves for A_1 are as follows:

				N = 0°	N = 30°	N = 45°
H_1	H_2	H_3	V_2	V_1	V_1	V_1
180°	80°	225.5	−1	4.5	4.5	5
180	85	225.5	−1	5	5	5.5
180	90	225.5	−1	6.5	6	5
180	95	225.5	−1	7	6	5
180	100	225.5	−1	8	5	4

After reduction to corresponding crystal angles, the position of A_1 in projection was found to be

$$H_1\ 180°,\ H_2\ 90°,\ H_3\ 225°.5,\ V_1\ 6°\ \text{and}\ V_2\ 1°$$

The $\gamma\beta$ ellipsoidal plane was located by

$$H_1\ 180°,\ H_2\ 0°,\ H_3\ 315°,\ V_1\ -,\ V_2\ 26°$$

while for the $a\gamma$ ellipsoidal plane, the readings were

$$H_1\ 180°,\ H_2\ 90°,\ H_3\ 225°.5,\ V_1\ -,\ V_2\ -1°$$

The optic axial angle thus determined in projection plat is $2V=64°$. In such cases, where the section is nearly normal to an optic axis, the method of extinction curves is not of practical value, owing to the difficulty of determining extinction angles with the requisite accuracy.

Topaz. Section nearly normal to the obtuse bisectrix.

The optic axial angle was found by first locating the principal ellipsoidal plane $a\beta$ and $a\gamma$ and then measuring the extinction angle of the section when a coincided with the microscope axis and after rotation of the section from that position through known angles about V_1 and V_2.

For the ellipsoidal planes the readings were:

	H_1	H_2	H_3	V_1	V_2
$a\beta$ plane.....	180°	90°	236°.5	..	0°
$a\gamma$ plane.....	180	90	326	..	−17.5

For a in coincidence with the microscope axis, the readings were found from the projection to be

$$H_1\ 185°,\ H_2\ 90°,\ H_3\ 326°,\ V_1\ 30°,\ V_2\ 22°.5$$

After the rotation about V_1 and V_2 the angles recorded were

$$H_1\ 189°.5,\ H_2\ 90°,\ H_3\ 326°,\ V_1\ 40°,\ V_2\ -15°$$

From these angles $2V$ was measured in projection and found to be 66°.

By direct observation of the optic axis, the same angle was also obtained. This method may, in favorable instances, give reliable results, but in general it can not be considered accurate, owing to the undue influence in projection of slight deviations of the extinction angle on the value of the optic axial angle.

For the second section the readings were:

	H_1	H_2	H_3	V_1	V_2
$a\beta$ plane.....	180°	90°	140°	..	+0°.5
$a\gamma$ plane.....	180	90	234	..	−32.5
Optic axis A_1.....	180	90	140	4°	+0.5

The optic axis A_1 was determined by direct readings. After proper reduction to true crystal angles the value $2V=63°$ was obtained from the projection plat.

Topaz. Section about perpendicular to the optic normal.

In this instance the principal ellipsoidal planes were first determined and ellipsoidal axis β brought to coincide with the microscope axis and the extinc-

tion angle measured in that position. By trial that position of H_2 was found, for which A_1 coincided with the principal plane of the lower nicol, and the optic axial angle thus ascertained by measuring the extinction angles of the section in different positions of V_1 and comparing the data of observation with those obtained by graphical methods from the projection plat on the assumption that A_1 did actually coincide with the principal plane of the lower nicol. In like manner the section was rotated about V_2 and extinction angles measured until theory and observation furnished identical results.

The principal ellipsoidal planes of the section were determined by the readings:

	H_1	H_2	H_3	V_1	V_2
$\beta\gamma$ plane.....	180°	90°	333°	..	+16°
βa plane.....	180	90	241 .5	..	− 1

For the different positions of H_2, the extinction angles for a given angle of rotation about V_1 were:

H_1	H_2	H_3	V_1	V_2
145°	123°	333°	17°	31°
145 .5	122	333	17	31
144 .5	124	333	17	31
144 .3	123 .5	333	17	31

On plotting these values in projection, it was found that $2V$ was about 64° to 67°, but a more decisive result was not attainable. The method is not accurate and can only furnish very rough approximations.

In the second method, which involves rotation about an axis normal to that of the above, the values observed were:

H_1	H_2	H_3	V_1	V_2
148°.5	33°	332°	17°	31°
148 .5	34	332	17	31
149	32	332	17	31

and from these angles, $2V$ was found to be between 64 and 68°.

The determination can not be termed satisfactory, and this method, like the above, can furnish only rough approximations to the correct values of $2V$.

RECAPITULATION.

(1) The optic axial angle of minerals in the thin section can be determined under the microscope in either convergent or parallel polarized light.

(*a*) In convergent polarized light, methods for the measurement of the optic axial angle are available for all sections in which at least one optic axis appears within the field of vision. Of these, the method requiring the use of the Becke drawing-table is of general application and furnishes results of a fair degree of accuracy—the usual probable errors being about ± 1° if both optic axes be visible, and ± 5° if only one optic axis be visible. More accurate and somewhat simpler in manipulation and of the same general application is the method involving the new double-screw micrometer ocular

or the coordinate micrometer ocular described above. This ocular combined with the method of projection of Professor Wulff, or with the graduated hemisphere of Professor Nikitin, is a general extension of the Mallard method, and like the Becke method utilizes the rule of Biot and Fresnel, which defines the planes of vibration for any direction of wave propagation. With this ocular the probable errors of determination on sharp interference figures should not exceed 1° if both optic axes are visible, nor 3° if only one optic axis appears in the field.

(*b*) In parallel polarized light the methods involving the Fedorow-Fuess universal stage are used and furnish satisfactory results, provided the position of one optic axis can be determined directly. If both optic axes are outside of the field of vision, the results obtained are usually unsatisfactory and inaccurate. Theoretically, it is possible to measure the optic axial angle of any biaxial transparent mineral on any section by means of the universal stage. If both optic axes appear within the field of vision, the error of determination should not exceed 1°, and if only one of the optic axes be visible the accuracy may decrease to $\pm 5°$. The exact location of a visible optic axis is assisted somewhat by use of the method of optical curves. Having once fixed the location of one optic axis, that of the second is determined by the method of extinction curves. If both optic axes lie entirely outside of the field, special methods must be resorted to, but in general without marked success, owing to the great difference in the value of $2V$ caused by a very slight deviation in the measured extinction angle.

The range of the field of vision of the universal stage is greater than that of any possible interference figure; the Fedorow universal-stage methods are, therefore, applicable to a greater number of sections than the methods with convergent polarized light and may furnish results on sections otherwise useless for ordinary methods.

Both experience and theory show that for all these methods the accuracy of the determination varies considerably with the section and mineral in question. The most accurate results can be obtained on sections for which both optic axes appear within the field of vision; less accurate but still satisfactory measurements can be made when only one optic axis appears, particularly when it is situated about midway from the center to the margin of the field.

INDEX.

Abbe, apertometer, 51, 172; apochromatic objectives, 50; compensating eye-piece, 33; definition of aplanatism, 28; image formation in microscope, 10, 42, 43; numerical aperture, 37, 42; ocular spectroscope, 71; sine condition, 27; total refractometer, 97, 194.
Abbott, intensity of solar spectrum, 109.
Aberrations, chromatic, 31; spherical, monochromatic light, 21.
Absorption, 7, 69, 71.
Achromasy, 32.
Achromatic objectives, 33, 49, 51.
Achromats, 49.
Adjustment of petrographic microscope, 11, 12, 61, 128.
Ahrens prism, 55, 60, 62.
Ambronn, immersion method, 93.
Amici lens, 147, 171.
Amman birefractometer, 102; wedge, 101, 102.
Analyzer, 11.
Analyzer, distortion due to, 53, 54, 55.
Angle, optic axial, 147.
Angle projection, 63, 67.
Angular aperture, 34.
Angular field of view, 34.
Angular magnification, 15.
Anhydrite, 162, 164.
Apertometer, Abbe, 51, 148, 149, 172.
Apertometer, condenser, 172.
Aplanatic points of a single refracting spherical surface, 23, 37.
Aplanatism, Abbe's definition of, 28.
Apochromatic objectives, 33, 50, 51.
Apochromats, 49.
Arago, rotatory power of quartz, 131.
Aragonite, optic axial angle of, 194, 195, 196.
Arons, chromoscope, 70.
Arschinow, graduated hemisphere, 174.
Artificial illumination, 77.
Astigmatic difference, 29.
Astigmatic flattening of image, 30.
Astigmatism, 21, 29, 30, 49, 50, 52.
Axial angle apparatus, measurements with, 194.

Babinet compensator, 101, 114, 140.
Back focus, 19.
Beck, Conrad, 10.
Becke, determination of optical character, 79, 81; dispersion of quartz, 172; drawing table, measurements with, 194; drawing table method, 152, 153, 154, 173; Klein lens, 171; lens for interference figures, 147; line, 53, 87, 92; line method, 95, 96; measurement of optic axial angle, 155, 156, 157, 158, 159, 160, 163, 164, 166, 167; oblique illumination, 85.
Becquerel, refractive index determination, 97.
Behrens, microchemical reactions, 69.
Bertin, refractive index determination, 97.
Bertrand, axial angle apparatus, 173; lens, 11, 13, 59, 63, 77, 147, 148, 171; ocular, 63, 138, 146; plate, 137.
Bi-nicol ocular, 137, 145.
Biot, compensator, 101; rotatory polarization of quartz, 131; rule of, 68, 80, 154, 167, 168, 176, 191, 200.
Bi-quartz wedge plate, 63, 140, 145, 146.
Birefractometer, Amman, 102.
Birefringence, 7, 8, 69, 101; average, 105; calcite, 112; measurement of, 58, 102, 103, 104, 114; probable error in determination of, 106; quartz 112; selenite, 112.
Bisectrices, dispersion of, 82.
Blur circle, 49.
Bocke, stereographic projection, 64.
Brace, dispersion of quartz, 112.
Bravais-Biot compensator, 101, 103.
Bravais, sensitive tint plate, 103.
Bravais-Stöber plate, 134, 137, 145.
Brewster, determination of crystal system by optical properties, 177.
Brezina, plate for determining extinction angles, 142.
Brill, refractive indices, 98.
Brucke, schistiscop, 70.
Brun, refractive index determination, 85.
Calcite, birefringence of, 112.
Calderon, calcite plate, 131, 135; ocular, 146.
Calkins, refractive index of Canada balsam, 91.
Cahours, refractive index determination, 97.
Canada balsam, refractive index of, 91.
Cap stop for observation of interference figures, 59.
Centering screws, 12, 60.
Césaro, determination of optical character, 79, 81, 82; interference cross in objective, 76, 159; mathematical formulæ for extinction angles, 191; measurement of birefringence, 113; probability of finding sections, 190; wedge, 101, 102.
Character of principal zone, 71.
Chaulnes, method, 83, 97, 114.
Chief axis, 26.
Christiansen, refractive index determination, 92, 93.
Chromatic aberration, 31, 49; correction of microscope objectives, 33; difference of magnification, 30, 32, 50, 52; difference of spherical aberration, 33, 49; overcorrection, 31; overcorrection in eye-piece, 32; undercorrection, 31; undercorrection in eye-piece, 32.
Chromoscope, Arons, 70.
Chrustschoff compensator, 101.
Chwolson, text-book, physics, 10.
Circle of confusion, 33, 49.
Circular polarization of quartz, 112.
Cleavage, 71; angle, 8, 69.
Color, 7, 69; primary, 51.
Color chart, 101; Michel-Lévy, 108, 111.
Color scale, Kraft, 111; Newton, 108, 112, 113, 133; Quincke, 111; Radde, 69; Rollet, 111; Wertheim, 111.
Colorimeter, Ives, 69; Nutting, 70.
Coma, 27, 51.
Combination wedge, 71, 72, 73.
Comparator, Fedorow mica, 103; Michel-Lévy, 101, 111.
Compensating eye-piece, 52; Abbe, 33; ocular, 52.
Compensator, Babinet, 101, 114, 140; Chrustschoff, 101; Bravais-Biot, 101, 103.
Condenser, 11, 52.
Condenser apertometer, Kellner; Sommerfeldt, 172.
Condenser lens, ten Siethoff, 173.
Condition for aplanatism, 24.
Conjugate foci, 14; planes, 15; points, 14.
Constant, Mallard, 148, 149.
Construction of petrographic microscope, 11.
Convergent polarized light, 13.
Coordinate micrometer ocular, 155, 166, 171, 175.
Cornu, A., dispersion of minerals, 112, 193; refractive index determination, 99.
Cover glass, spherical aberration due to, 51.
Craig, on projections, 63.
Critical illumination, 52.
Cross-hairs, adjustment of, 62.
Crystal habit, 7, 69, 71.
Crystal system, determination of, 177.
Curvature of field, 21, 30, 49, 52.
Czapski, location of diafram in microscope, 59; observation of interference figures, 147; ocular, 171.

Daly, mathematical formulæ for extinction angles, 191.
Dark-ground illumination, 53, 95.
Diafram, iris, 12.
Diaframs in the microscope, 34
Dick, Allan, petrographic microscope, 10.
Dick, device for rotation of nicols, 57, 58.
Differences of magnification, chromatic, 30.
Diffraction grating, 43.
Dispersion, chromatic, 31; of bisectrices, 7, 69, 82, 128; of minerals, 113; of optic axes, 7, 69, 82; relative, 85.
Distortion, 21, 30, 49, 52; produced by analyzer, 53, 55.
Double combination wedge, 134.
Double-screw micrometer ocular, 155, 166, 175, 196.
Drawing table, Wright, 153.
Draw tube, 11.

Drude, rotatory polarization of quartz, 131; text-book, optics, 10.
Dufet, optical constants, 112.
Dupare, Traité de Techn. Min. et Pétrog., 9, 99, 105, 112.

E. F. (Equivalent focal length), 19.
Emergence port, 34.
Emergence pupil, 34.
Empty magnification in microscope, 47.
Entrance port, 34.
Entrance pupil, 34, 35.
Equal surface projection, 63.
Equator angle projection, 67.
Equidistant projection, 66, 67.
Equivalent focal length, 19, 50.
Equivalent planes, 19.
Equivalent projection, 63.
Euler, mechanical quadrature, 66.
Evans, determination of refractive index, 96; sign of reaction, 81; universal methods, 175; wedge, 101, 102.
Exit port, 34, 35.
Exit pupil, 34, 35.
Exner, microrefractometer, 95.
Extinction angle, 7, 8, 69, 115; methods for measurement of, 132.
Extinction direction, 68.
Eye-circle, 35; diameter of, 38; stop, 78.
Eye-piece, 11, 30; Abbe compensating, 33; chromatic undercorrection in, 32; chromatic overcorrection in, 32; compensating, 52; function of, 30; Huygens, 52; micrometer, 52; Ramsden, 52.
Eye-point, 35.

Ferraris, G., geometrical optics, 10.
Fedorow, determination of refractive index, 176; dispersion of optic axes, 82; graphical plat, 151; measurement of birefringence, 176, 191; mica comparator, 103; stereographic projection, 64; universal stage, 147; universal stage methods, 174, 175, 177, 179, 180, 184, 185, 189, 200.
Feldspars, plagioclase, extinction angles of, 132.
Fenner, acknowledgment to, 9.
Ferro, extinction angles, 191.
Field of microscope, 42.
Field of view diafram, 34.
Figures, formation of interference, 147.
Fischer, Radde color scale, 69.
Flett, apertometer, 148, 149.
Focal length, equivalent, 19; of two component lens system, 19; principal, 17,
Focus distance, 19.
Fouqué, character of principal zone, 71.
Fowle, energy in sun spectrum, 109.
Fresnel, rule of, 68, 80, 154, 167, 168, 176, 191, 200; formulas for rotation of plane of polarization, 76.
Friedel, measurement of birefringence, 103.
Fuess objective, 54, 55.

Gauss equations for paraxial rays, 14.
Germain, on projections, 63.
Gladstone's law, 92.
Glan prism, 55, 62, 71.
Gnomonic meridian projection, 64; projection, 63, 64, 65, 66, 67.
Goldschmidt, on projections, 63.
Grabham, oblique illumination, 95, 102.
Graduated hemisphere, Nikitin, 166.
Granularity limits in petrographic microscopic work, 8, 10, 11.
Gretschel, on projections, 63.

Half-shade apparatus, 128, 129.
Half-shade plate, Nakamura, 139.
Harker, extinction angles, 191.
Hastings, C. S., geometrical optics, 10.
Heath, R. S., geometrical optics, 10.
Hecht, measurement of refractive indices, 98.
Helmholz, equation, 15.
Hemisphere, Arschinow, graduated, 174; Fedorow glass, 176; Nikitin, graduated, 166; Wright, graduated, 174.
Himmelbauer, refractive index of liquids, 98.
Hlawatsch, dispersion of quartz, 112.
Hockin, derivation of sine condition, 10, 27.
Hotchkiss, refractive index determination, 97.
Huygens, construction of wave-fronts, 21; ocular, 58.

Iddings, rock minerals, 9.
Illumination, artificial, 77; critical, 52; dark-ground, 53, 95; measure of, 33; oblique, 53, 85, 87, 88, 95; of object, 83.
Image angle, 34.
Image formation in microscope, Abbe's derivation, 43.
Immersion method for refractive index determination, 92, 94.
Intensity of light, mathematical formula for, 80, 115.
Interference colors, abnormal, 128.
Interference figures, device for observation of, 59; factors underlying formation of, 158, 159; formation of, 147.
Iris diafram, 12.
Ives, standard colorimeter, 69, 71; visual sensibility curve, 110.

Jamin, Babinet compensator, 101.

Kalkowsky, internal conical refraction, 178.
Kellner, acknowledgment to, 9; condenser apertometer, 172; drawing for astigmatism, 29; equivalent focal length (E. F.), 19.
Klein, lens, 171; observation of interference figure, 147; quartz plate, 137.
Klement, microchemical reactions, 69.
Kobell, plate, 142.
Koenigsberger, absorption, 71; plate, 103, 134, 137, 145.
Kraft, color scale, 103, 111, 112.

Lacroix, Les Minéraux des Roches, 173; observation of interference figures, 147.
Lagrange-Helmholz equation, 15
La Hire, projection of, 66.
Landolt-Börnstein, tables of, 85, 112.
Lane, A. C., measurement of optic axial angle, 191, 192.
Larsen, acknowledgment to, 9.
Lasaulx, observation of interference figure, 147.
Laspeyres, observation of interference figure, 147.
Lateral magnification, 15.
Lavenir, measurement of refractive indices, 98.
Leiss condenser, 60; Nikitin hemisphere, 166; optical instruments, 10; wedge, 101, 102.
Lenk, observation of interference figure, 147.
Lens, Amici-Bertrand, 171; formulæ, 13, 14; formulæ fundamental for central rays, 18; Klein, 171; refraction through a, 15
Lens system, aplanatic, Abbe's definition, 28; focal length of two component, 19; optical elements of microscope, 49.
Liebisch, absorption and pleochroism, 71.
Light, equations for intensity of, 118–127, 130, 138; for vibration of, 116.
Light, threshold value of, 127.
Lippich, F., geometrical optics, 10.
Liquids, refractive, 97, 98.
Lowry, projection of, 66.
Lummer, geometrical optics, 10; image formation in microscope, 43.

Macé de Lépinay, Babinet compensator, 101; half shade apparatus, 139, 141.
Magnification, 11, 40; Abbe's definition of, 41; angular, 15; approximate measure for, 41; chromatic difference in, 32; chromatic differences of, 30; empty, 47, 48; lateral, 15; measurement of, 38; normal, 41; useful, 47, 48.
Magnifier, 30.
Mallard constant, 148, 149, 172; determination of optical character, 79; measurement of birefringence, 104; measurement of optic axial angle, 150, 151, 152, 154, 200.
Malus, polarization of light-waves, 68.
Mascart, Bravais-Biot compensator, 101.
Maschke, determination of refractive index, 85, 86, 87; oblique illumination, 92, 95.
Mathematical formula for intensity of light, 115.
Mechanical stage, 60.
Mechanical system of petrographic microscope, 11.
Meridian angle projection, 67.
Meridional plane, sine condition, 28.
Merwin, acknowledgment to, 9; dispersion of liquids, 97; graphical plat, 152; ray filter, 87; refractive liquids, 98.
Michel-Lévy color chart, 101, 108, 111; comparator, 101, 111; dispersion of quartz, 112; Les Minéraux des Roches, 78; measurement of optic axial angle, 173, 190; refractive index indicators, 97; sharp focus on a point, 104.

Micrometer ocular, coordinate, 155, 166, 171, 175; cross-line, 163; double screw, 155, 166, 169, 175, 196; eye-piece, 52.
Microrefractometer, 95.
Microscope, field of, 42; lens system, optical elements of, 49; objective, 49; resolving power of 45; shield, convenient form of, 39; stand, 11.
Miculescu, refractive index determination, 83.
Monochromatic flame, 88.
Monochromatic light, 21.
Müller-Pouillet, Lehrbuch d. Physik, 10.
Muscovite, measurements on, 162, 163; optic axial angle of, 194, 195, 196.

Nakamura, quartz half-shade plate, 139.
Newton, color scale, 108, 112, 113, 133.
Nichols, intensity of daylight, 87.
Nicol, lower, 11; prisms, 55; rotation of upper, 141; upper, 11.
Nikitin, compensator, 104; graduated porcelain hemisphere, 166, 173, 176, 189, 200.
Nodal planes, 18.
Normal magnification, 41.
Numerical aperture, 37, 47, 48.
Numerical aperture, determination of, 51.
Numerical aperture, physical significance of, 42.
Nutting, acknowledgment to, 9; colorimeter, 70; color scale, 69; luminous equivalent of radiation, 87; sensibility of half-shade apparatus, 139; specific rotation of quartz, 131; standard white light, 108, 109, 110, 112.

Object field of view stop, 34.
Object glasses, spherical, 177.
Objective, achromatic, 33, 49, 51; apochromatic, 33, 50, 51; clamps, 60; correction of microscope, 30; dry, 50; determination of numerical aperture, 51; immersion, 50; microscope, 11, 49; measure for resolving power of, 45, 46; oil immersion, 38; semi-apochromatic, 50, 51.
Oblique illumination, 53, 96.
Ocular, Bertrand, 146; bi-nicol, 137, 145; Calderon, 146; compensating, 52; coordinate micrometer, 155, 166, 171, 175; Czapski, 171; double-screw micrometer, 155, 166, 175, 196; Huygens, 52; microscope, 11; projection, 65; Ramsden, 52.
Optic axes, dispersion of, 82.
Optic axial angle, 7, 8, 69, 147; measurement of, 152, 155, 172, 194, 195, 196, 197, 198, 199; measurement of, by means of extinction angles, 191; measurement of, by means of relative birefringence of known section, 190; measurement of, on plate perpendicular to optic normal, 186; measurement of, on section normal to obtuse bisectrix, 187; measurement of, on total refractometer, 193; methods for measurement of, 190, 200; relative accuracy of different methods for measurement of, 194; universal stage methods for measurement of, 189.
Optic axis, determination of position of, 178.
Optical character, determination of, 74; of birefracting minerals, 74; of elongation, 7, 69; of mineral, 7, 69; on plate perpendicular to acute bisectrix, determination of, 78; on plate parallel to plane of optic axes, determination of, 79.
Optical curves, 179.
Optical elements of microscope lens system, 49.
Optical system of petrographic microscope, 11, 13.
Orthographic projection, 63, 65, 66, 67.
Orthogonal projection, 65, 68.
Orthomorphic projection, 63.
Overcorrection, chromatic, 31.
Outline picture, 53.

Parallel projection, 65.
Paraxial rays, 14.
Pauly, refractive index of liquids, 97.
Pearce, Trait d. Téch. Min. et Pétr., 99, 105.
Pellétan, A., geometrical optics, 10.
Penfield, stereographic projection, 64.
Perrot, measurement of refractive indices, 98.
Perspective projection, 64.
Petrographic microscope, accurate measuring device, 57; adjustment of, 11, 12, 61; construction of, 11; mechanical system of, 11; optical system of, 11, 13; purpose of design, 10.
Petzval condition, 21, 30.
Photographic lens, correction of, 30.
Plagioclase feldspars, optical determination of, 7.
Plane polarized light, 13.
Planes, equivalent, 19; nodal, 18; principal, 18.
Plate, artificially twinned quartz, 136; Bertrand, 137; bi-quartz wedge, 139, 140, 146; Bravais-Stöber, 134, 145; Brezina, 142; Calderon calcite, 135; half-shade, Nakamura, 139; Kobell, 142; Koenigsberger, 103, 134, 145; quarter undulation of H. Traube, 135; sensitive tint, 58, 133, 145, 146; Traube, 145; twinned quartz, 145; twinned selenite, 136.
Platform, mechanical, 12.
Pleochroism, 7, 69, 71.
Pockels, measurement of refractive indices, 99.
Polarized light, convergent, 13; plane, 13.
Polarizer, 11, 55.
Postel projection, 67.
Preston, theory of light, 116, 118.
Primary color, 51.
Primary spherical aberration, 51.
Principal focal length, 17.
Principal planes, 18.
Principal rays, 34.
Principal zone, character of, 71.
Probability of encountering sections suitable for optic axial angle measurements, 189.
Projection angle, 34; plats, 166.
Projections, 63.
Pulfrich, total refractometer, 193, 194.

Quarter undulation plate of H. Traube, 135.
Quartz, birefringence of, 112; circular polarization of, 112; plate, artificially twinned, 63, 136; wedge, artificially twinned, 63; wedge, graduated, 101, 102.
Quincke, color scale, 111.

Radde, color scale, 69.
Ramsden disk, eye-piece, 35; ocular, 54, 150.
Rays, central, 14, paraxial, 14.
Reflector, 11.
Reiche, image formation in microscope, 42.
Renard, microchemical reactions, 69.
Rendtorff, rotation of quartz, 112.
Refraction at a single spherical surface, 14; through a lens, 15.
Refractive index, 7, 8; determination of, 92, 99; direct measurement of, 98; of Canada balsam, 91; of mixtures, 92.
Refractive indices, 69, 83.
Refractive liquids, 86, 97, 98.
Refractometer, total, 97, 99, 100.
Resolving power of microscope objective, 45.
Rinne, interference cross in objectives, 76, 159.
Rollet, color scale, 111, 112.
Rosenbusch, Mikros. Physiographie, 9, 10, 69, 71, 79, 81, 105, 149, 194.
Rotation of nicols, simultaneous, 57, 58.
Rotation of plane of polarization, 74.
Rotation of upper nicol, 141.

Sagittal plane, astigmatism, 29.
Schaller, refractive index Canada balsam, 91.
Schistoscop, Brücke, 70.
Schmidt, Babinet compensator, 101.
Schnittweite, 19.
Schott, on projections, 63.
Schrauf, Brezina plate, 142.
Schroeder v. d. Kolk, condenser, 173; hemisphere, 174; immersion method, 93, 95; interference figures, 74; oblique illumination, 85, 102.
Schrönrock, half-shade apparatus, 141.
Schuster, A., optics, 10.
Schuster, M., extinction angles, 132.
Secondary spectrum, 32, 50, 51.
Selenite, birefringence of, 112; twinned plate 63, 136.
Semi-apochromatic objectives 49, 50, 51.
Sénarmont, measurement of birefringence, 103.
Sensibility, adjustable 9.
Sensitive tint plate, 58, 73, 133, 145, 146; Bravais, 103.
Sensitiveness of extinction angle methods, 142, 143.
Shagreened surface, 91.
Siedentopf, wedge, 101, 102
Siethoff, ten, condenser lens, 173.
Simultaneous rotation of nicols, 57, 58.
Sine condition, 21, 26, 28, 49, 50.
Skiodroms, 81.
Smith-Helmholz equation, 15.
Sommerfeldt, condenser apertometer, 172; device for rotation of nicols, 57, 58; selenite plate, 63, 136; suggestion for combination wedge, 102.

Sorby, petrographic microscope, 7.
Soret, measurement of refractive indices, 98, 193.
Southall, geometrical optics, 10; Smith-Helmholtz equation, 15.
Souza-Brandao, device for rotation of nicols, 58; dispersion of quartz, 101, 112; graphical plat, 151; measurement of optic axial angle, 191; refractive index indicators, 97.
Spectrum, secondary, 32.
Spherical aberration, 21, 49, 51; chromatic difference of, 33; correction for, 33; for a single spherical surface, 21; for points near the axis, 26; in simple lenses, 24; of oblique rays, 21; overcorrected, 25; primary, 51; proper, 21; secondary, 51; undercorrected, 25.
Spherical overcorrection, 25.
Spherical undercorrection, 25.
Spherical zones, 26, 51.
Spitta, microscopy, 10; correction of objectives, 51.
Stage, microscope, 12.
Stage, universal, 175.
Stereographic projection, 63, 64, 65, 66, 67.
Stöber plate, 137, 145.
Substage, microscope, 12.

Tangent condition, 21.
Tangential plane, astigmatism, 29; sine condition, 28.
Telescope, 30.
Tertsch, determination of optical character, 78; measurement of birefringence, 114; measurement of optic axial angle, 173; ocular for measurement of optic axial angle, 153.
Test objects, 51.
Thompson, polarizing prism, 53, 54, 55, 60, 62.
Thoulet, determination of refractive index, 92.
Topaz, optic axial angle of, 194, 196, 197, 198.
Total refractometer, 99, 100.
Traube, H., quarter undulation plate, 135, 137, 145.
Twinned plate of selenite, 63.
Twinned quartz plate, 63, 136.
Twinned quartz wedge, 63.

Uhlig, measurement of optic axial angle, 191.
Undercorrection, chromatic, 31.
Universal stage 60, 173, 175, 177; measurements with, 197; methods for measurement of optic axial angle, 189.
Upper nicol, rotation of, 141.
Useful magnification in microscope, 47.

Verniers, on microscope stage, 61.
Viola, measurement of refractive index, 97; direct measurement of refractive index, 98, 99; measurement of optic axial angle, 193
Voigt, internal conical refraction, 178.

Walker, measurement of birefringence, 103.
Wallérant, measurement of optic axial angle, 175; total refractometer, 100, 193, 194.
Wedge, Amman, graduated, 101, 102; combination, 71, 72, 73; double combination, 134; Evans, graduated, 101; Leiss, graduated, 101, Siedentopf, graduated, 101; Wright, graduated, 101.
Wedge-plate, bi-quartz, 139, 140, 145, 146; twinned quartz, 145.
Weierstrass, construction for refracted rays 21.
Weinschenk, rotation of upper nicol, 141.
Wertheim, color scale, 111.
Wiedemann, measurement of birefringence, 103.
Wright, A., dark ground illumination, 95; microscopy, 10.
Wright, F. E., adjustment of refractometer, 109; artificially twinned quartz plate, 136; bi-quartz wedge-plate, 63, 139; combination wedge, 72; coordinate micrometer ocular, 155; determination of optical character, 78, 81; double combination wedge, 134; double-screw micrometer ocular, 155; graduated wedge, 101; graphical plat, 151; measurement of optic axial angle, 173; rotation of plane of polarization, 76; rotation of upper nicol 141
Wulff, projection plat, 64, 154, 176, 189, 200.
Wülfing, axial angle apparatus, 194; dispersion of quartz, 112; Mallard constant, 151; Mikroskopische Physiographie, 9, 10, 69, 149; refractive index Canada balsam 91.

Zeiss apochromat, 54, 55.
Zenithal perspective projection, equations for, 64.
Zirkel, refractive liquids 98.
Zones, spherical, 26.

ERRATA.

Page 9, line 15, add "Les Minéraux des Roches, by Michel-Lévy and Lacroix."
10, between lines 28 and 29, add "Rohr, M. v., Die Bilderzeugung in optischen Instrumenten, 1904."
23, l. 20, read "P'" instead of "A'."
33, l. 17, read "achromatic" instead of "chromatic."
57, l. 1 of footnote, read "160" instead of "660."
58, l. 2 of footnote, read "1910" instead of "1909."
64, l. 1 of footnote, read "Amer. Jour. Sci." instead of "this Journal."
64, l. 2 of footnote, read "1893" instead of "q893."
82, l. 1 of footnote, read "1903" instead of "1902."
87, l. 7 of footnote, read "722" instead of "772."
93, l. 1 of footnote, read "1891" instead of "1892."
95, l. 5 of footnote, read "385, 1891" instead of "385, 1892."
95, l. 9 of footnote, read "1891" instead of "1892."
98, l. 4 of footnote, read "iodide" instead of "sulfide."
Page 98, l. 9 of footnote, read "45, 1889" instead of "45, 1888."
98, l. 11 of footnote, read "40, 1899" instead of "40, 1890."
101, l. 5 of footnote, read "**6**, 143, 1883; **7**, 43, 1884" instead of "**7**", 57, 1884."
102, l. 5 of footnote, read "1891" instead of "1892."
112, l. 19, read "reference" instead of "references."
114, l. 1 of footnote, read "1910" instead of "1911."
115, l. 3, read "elliptic" instead of "ellipsoidal."
141, l. 2 of footnote, read "**30**, 198, 1910" instead of "**30**, 1910."
41, l. 13 of footnote, read "1908" instead of "1910."
147, l. 2 of footnote, read "1899" instead of "1898."
149, l. 1 of footnote, read "**24**" instead of "**14**."
152, l. 5 of footnote, read "1895; **16**, 180, 1898" instead of "1894; **16**, 180, 1896."
155, l. 1 of footnote, read "204" instead of "97."
156, l. 7, insert "angle" after "optic axial."
172, l. 6 from bottom, read "H_1" instead of "H."

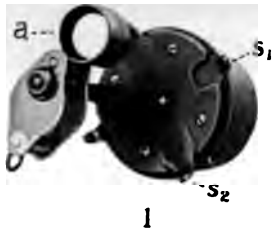
DESCRIPTION OF PLATE 1.

FIG. 1.— Device for cutting down the field when interference figures from small grains are being observed as they form in the objective itself (Lasaulx method). Two sets (S_1 and S_2) of two plates at right angles and sliding in grooves permit the observer to cut off the light from any part of the field. Before observing the interference figure, the image from the objective is first brought to coincidence with the plane of the stops S_1 S_2, by means of the field lens a.

FIG. 2.—Device for obtaining oblique and dark ground illumination, especially with high power objectives. Consists of a brass cap which fits above the eyepiece and carries a rotating plate c, into which small disks of cover glass have been inserted; on these, in turn, small, thin brass disks of different sizes (0.5 to 3.0 mm.) diameter are cemented and serve as central stops when placed in the eye-circle of the ocular.

FIG. 3.—New petrographic microscope constructed for the most part in the Geophysical Laboratory from a large-model Zeiss photographic microscope as base (see also text figure 1.) T (designating letter accidentally omitted from Plate 1, Fig. 3; see Fig. 1 in text), rigid bar connecting two nicols and effecting simultaneous rotation of the same; A, arm connecting upper nicol carriage with T; C, part supporting bar T and rotating about stage; B, arm from lower nicol carriage connecting with bar T; by means of the screw and crossbar at B, this arm can be instantly released from T and the lower nicol either rotated by itself or, after release by a snap-spring, not shown in the figure, thrown out of the field altogether. The total angle of simultaneous rotation of both nicols by this device is 190°. O, new mechanical stage, simple in design and construction and fairly dust-proof. H_1, stage screws with divisions on head reading to 0.01 mm. motion of stage plate. Q, sensitive-tint plate inserted above lower nicol, W, and revolvable about microscope axis by means of containing carriage F. M, combination wedge above objective; a_1 a_2, fine adjustment screws above objective; U, screw of fine adjustment device of upper microscope tube; V, iris diafram below Bertrand lens, diafram opened and closed by turning head V, which is connected with iris diafram by pin and ratchet movement; E, pin for insertion of Bertrand lens which moves in an accurately fitting carriage, supporting iris diafram V, Bertrand lens E and auxiliary lens L, which swings on an arm indicated in text, Fig. 1, and is of such focal length that, together with the ocular, it forms a small microscope used in focussing the image from the objective in the plane of the iris diafram, V. The supporting carriage of V, E, and L, can be moved up and down in the microscope tube and the amount of movement read off on the adjacent scale, thus obtaining different magnifications (6.5 to 15.2 diameters) of the interference figure. G, upper iris diafram directly beneath ocular.

FIG. 4.—Recent model petrographic microscope made by the Bausch & Lomb Optical Co., Rochester, New York. In the design of this microscope, which is largely due to Dr. H. Kellner, of the Bausch & Lomb Optical Co. (with suggestions by the writer), special care has been taken to produce a microscope that is not only convenient but also optically and mechanically satisfactory. The optical system is excellent in every respect and the mechanical workmanship throughout is of a high order of precision. Although simple in design, the plan of the microscope is such that with it most of the measurements required in petrographic microscopic work can be made. The oculars fit snugly into the draw tube, the Bertrand lens, B, and the upper nicol, N_2, carriages have wide bearing surfaces and fit accurately. The upper nicol, N_2, can be rotated through 90°, the lower nicol N_1 through 360°. The arm which rotates the upper nicol and the degree circle which indicates the amount of rotation are permanently attached to the nicol carriage; the connection is in consequence dust proof. Iris diaframs, I_1 and I_2, are used both in the draw-tube and in the substage. The centering screws S_1, S_2, of the objective move parallel with the cross-hairs of the ocular and are satisfactory, as is also the steel objective clamp, C. The fine adjustment screw, A, reads to 0.001 mm. and is free from lost motion. The dust guard, D, serves to protect the tube from dust when the wedges and plates are not in use. Throughout the instrument all bearing surfaces have been made wide and with special reference to rigidity and accuracy. The addition of a mechanical stage would be an improvement, but would add to the cost.



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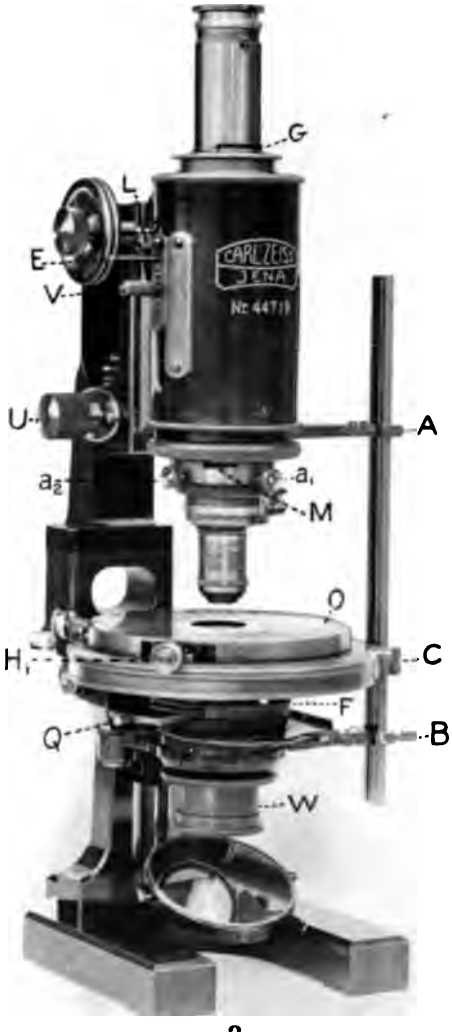

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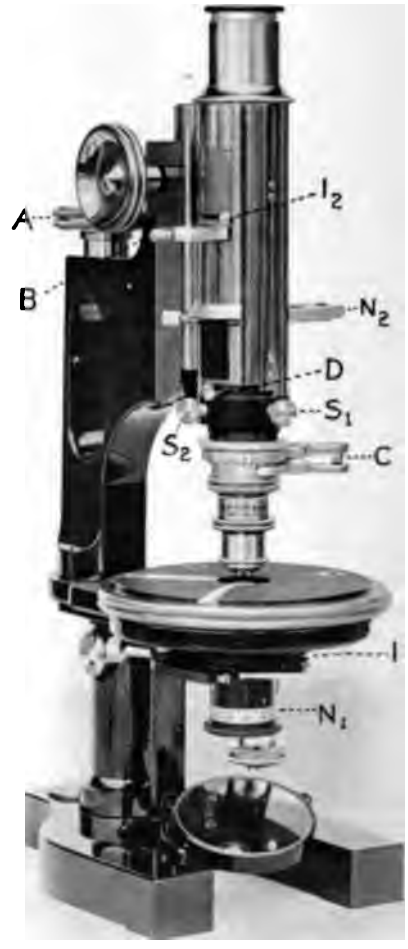


4

1. Aperture stop for interference figures.
2. Eye circle stop for dark ground illumination.
3. Improved petrographic microscope.
4. New Bausch & Lomb petrographic microscope.

DESCRIPTION OF PLATE 2.

FIG. 1.—Improved petrographic microscope (made after specifications by the writer by R. Fuess & Co., Germany), equipped with device for simultaneous rotation of the nicols and with ocular support for different mounted wedges and plates which serve in the accurate determination of the optical constants of mineral grains in the thin section.

FIG. 2.—Double-screw micrometer ocular for use in the measurement of optic axial angles; *H* and *V*, horizontal and vertical micrometer screws respectively; *S*, small stop in the eye-circle of the Ramsden ocular *O* to reduce parallax of rays from interference figure.

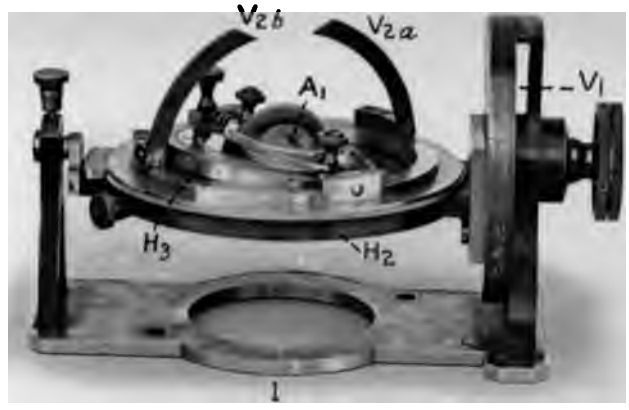
FIG. 3.—Interference figure of muscovite in sodium light. Shows also fine 0.1 mm. division lines of coordinate scale of new cross-line micrometer eyepiece for use in measuring optic axial angles.

DESCRIPTION OF PLATE 6.

FIG. 1.—In its present form the universal stage comprises, when attached to the microscope stage, five graduated circles: H_1, the horizontal circle of the microscope stage; H_2, the large horizontal circle of the universal stage, with H_3, the inner and thin section-bearing circle; V_1, the large vertical circle, and V_2, an inner circle consisting of two segments, V_{2a} and V_{2b}, which serve to measure the angle of rotation of the inner disk H_3 about the horizontal axis. Two glass hemispheres (A_1 being the upper) are usually employed with the stage to increase the angle of view of the microscopic field.

FIG. 2.—Universal stage fitted with upper lens of condenser system, thus producing a universal condenser with which optic axial angles can be measured directly and plates can be tilted definite angles in any azimuth. Especially serviceable with high-power objectives in convergent polarized light.

FIG. 3.—New ocular for use with the petrographic microscope. Consists of holder *B*, which fits into the draw tube of the microscope as an ocular and carries the Ramsden eyepiece *A* and an opening at *B* into which different wedges and plates can be inserted and different optical properties thereby measured. The three plates shown are (*a*) graduated combination wedge used in the measurement of birefringence, the graduations on the wedge giving directly the path-differences in $\mu\mu$ for sodium light; (*b*) bi-quartz wedge plate for measuring extinction angles; (*c*) 0.1 mm. coordinate micrometer scale for measuring optic axial angles on sections in which at least one optic axis appears in the field of view.



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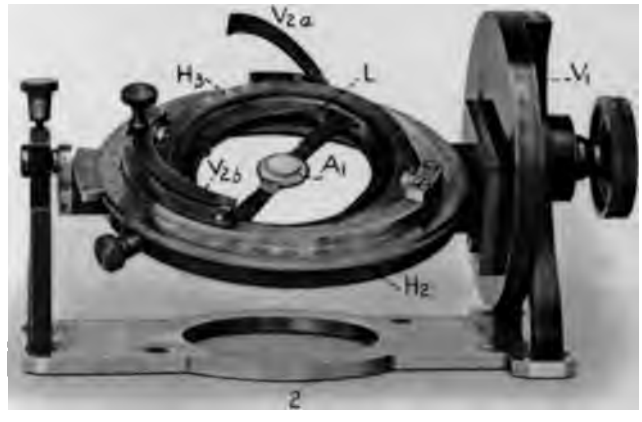

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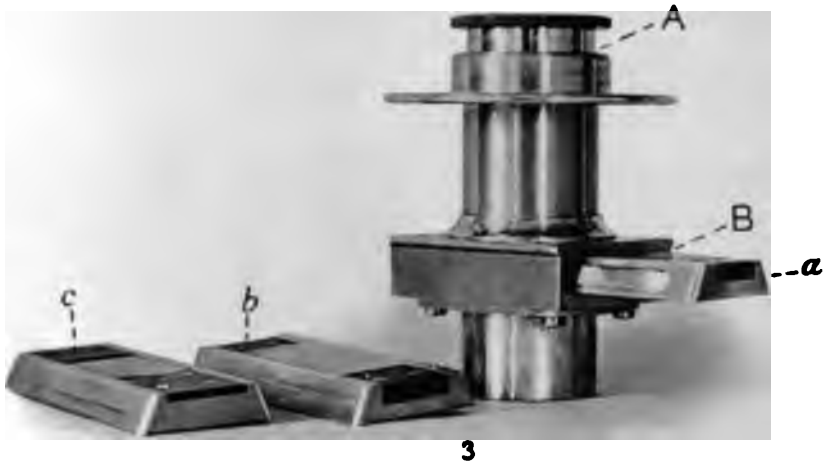
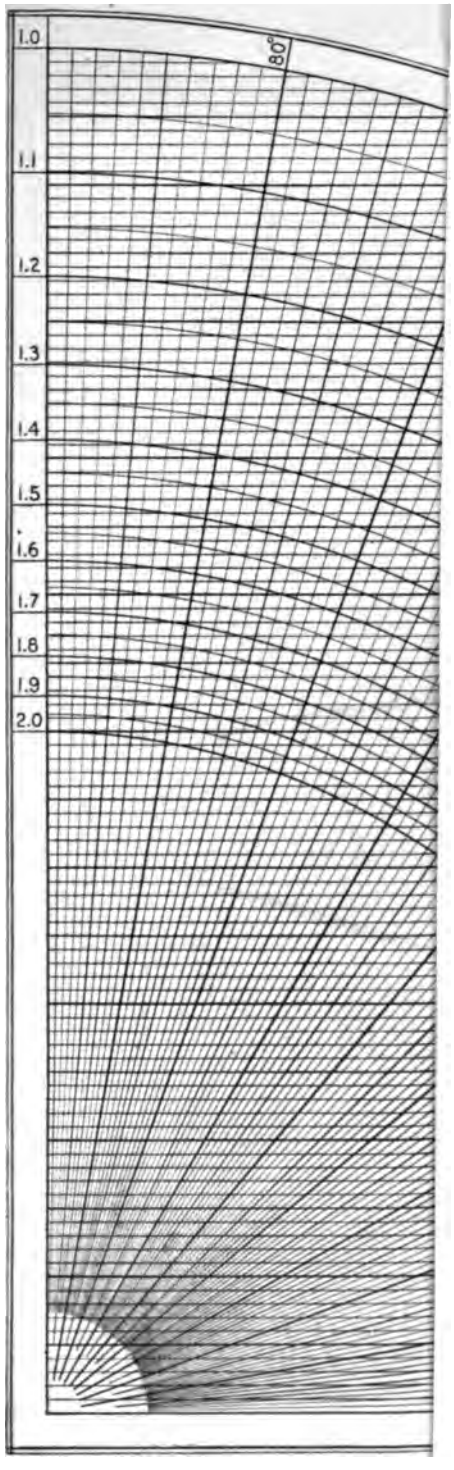
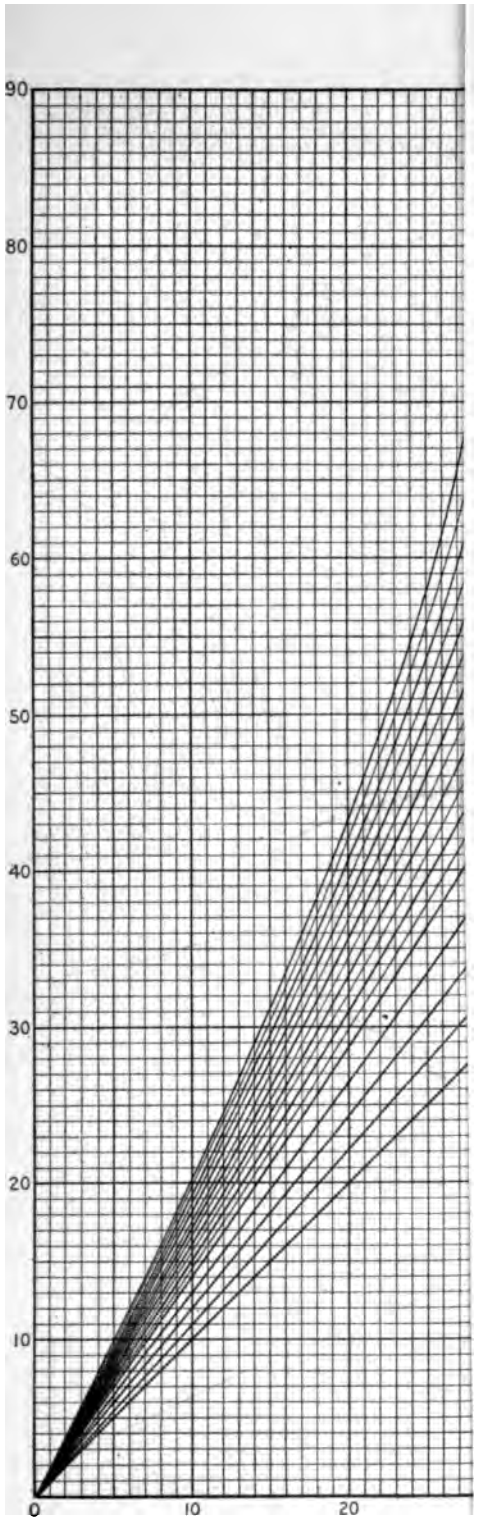


3

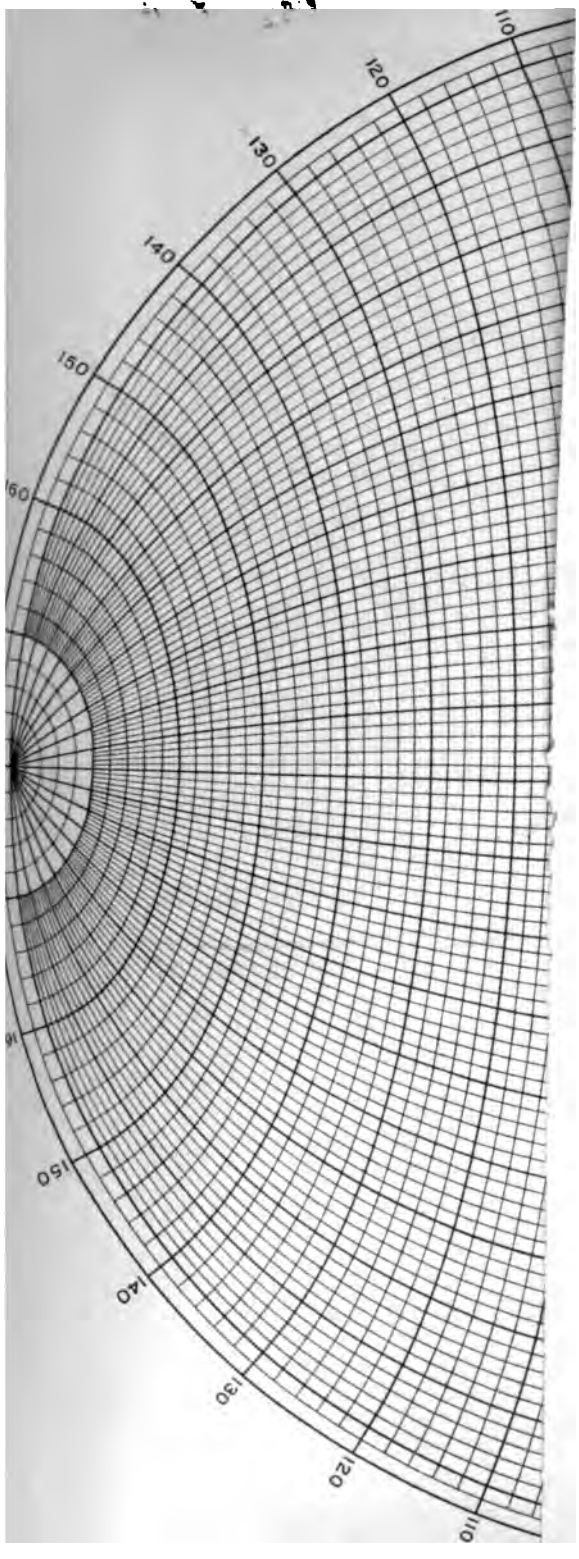
1. Improved Fedorow universal stage.
2. Universal condenser.
3. New ocular for petrographic microscope.



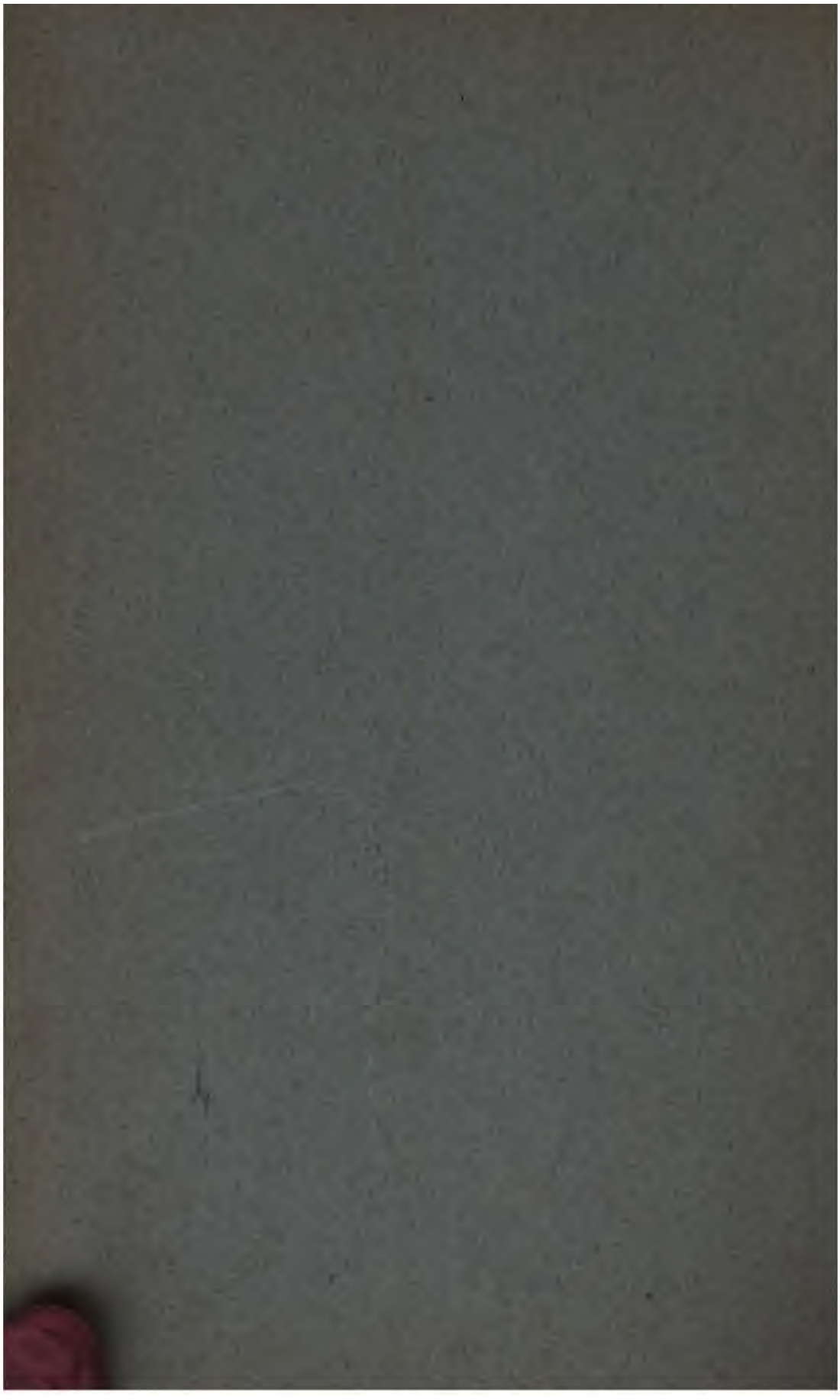
In this plate i = angle of incidence, r = ang
especially in the measurement of optic axi
operandi is the following: to find r when
until it intercepts the n circle ($n = 1.60$).
is the desired angle r. In case n and r ar



This plate serves the same purpose as Plate 7, t
the angles i, and the curves represent the re
To find r, pass along horizontal line at ordin
this plate the angles i and r are represented
coordinates in Plate 7.



For optic axial angle wo
over the entire field is l
oval curves in this proje



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